Nucleon magnetic properties using Landau mode operators with the background field method

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CSSM

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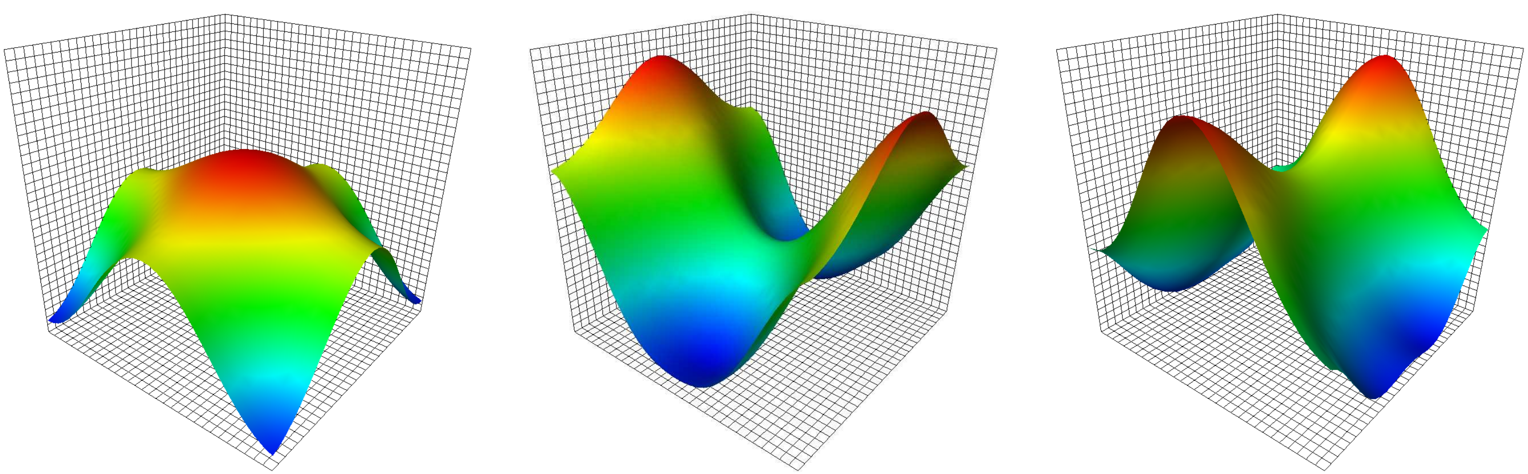
- The Magnetic Polarisability (β) is a fundamental property of a system of charged particles that describes the systems response to an external magnetic field.
- To calculate these with lattice QCD we use,
    - the Background Field Method and a novel implementation of Landau eigenmodes.



Outline

1. How is it done?
   - Background Field Method
   - Simulation Details
   - Quark Operators
2. Magnetic Polarisability
   - Correlator Ratios
3. Results
   - Energy Shifts
   - Energy vs. Field Strength fits

- How is the uniform magnetic field put across the lattice?

$$\mathcal{D}'_\mu = \partial_\mu + g\, G_\mu + qe\, A_\mu, \quad U'_\mu(x) = U_\mu(x)\, \mathrm{e}^{-i\, qe\, a\, A_\mu}$$

- Causes a shift in energy (small field limit) of the baryon.

$$E(B) = M + \vec{\mu} \cdot \vec{B} - \frac{|qe\, B|}{2\, M} - \frac{4\, \pi}{2}\, \beta\, B^2 + \mathcal{O}\left(B^3\right)$$

- Magnetic moment μ and magnetic polarisability β.
- Use of periodic boundary conditions impose a quantisation condition: $\vec{B} = B\, \hat{z}$

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Configurations

- Through the International Lattice Data grid and PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. D79 (2009) 034503.
  - Lattice Volume: $32^3 \times 64$
  - Non-perturbative $\mathcal{O}\left(a\right)$-improved Wilson quark action and Iwasaki gauge action
  - $2 + 1$ flavour dynamical-fermion QCD
  - Physical lattice spacing $a = 0.0907$ fm
  - $m_\pi = 411$ MeV
- Electro-quenched:
  - Dynamical QCD configurations only - 'sea' quarks experience no B field.
- Standard Interpolating Fields: $\chi_{p1} = \left(u^T\, C\, \gamma_5\, d\right) u$, $\chi_{n1} = \left(u^T\, C\, \gamma_5\, d\right) d$

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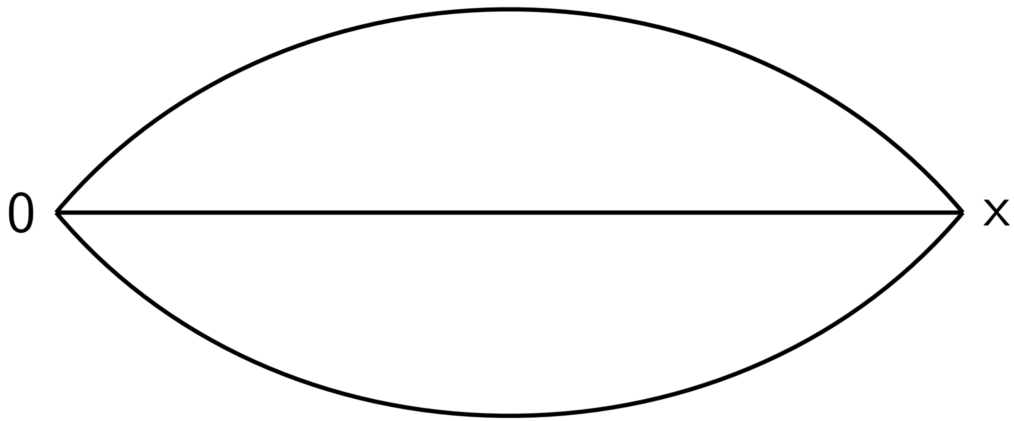
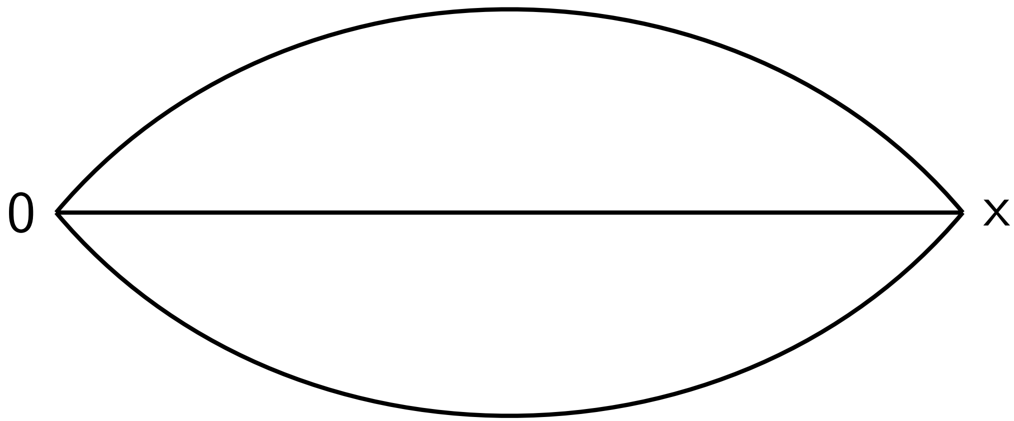


Two point correlation function quark-flow diagram for a baryon

- Construct two point correlation functions using lattice QCD
- These have exponential dependence on energy

$$G(t) \propto e^{-Et}$$

Two Point Correlation Functions



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- A charged particle in an external magnetic field sits in a superposition of energy levels

$$E^2 = m^2 + |qe\,B|\,(2\,n + 1 - \alpha) + p_z^2$$

- Quarks are charged - quarks have Landau levels!
- To what extent does this remain in QCD?
- The Landau levels are closely grouped in energy due to the small fields used.
- Takes longer in Euclidean time for levels above ground state to be exponentially suppressed.

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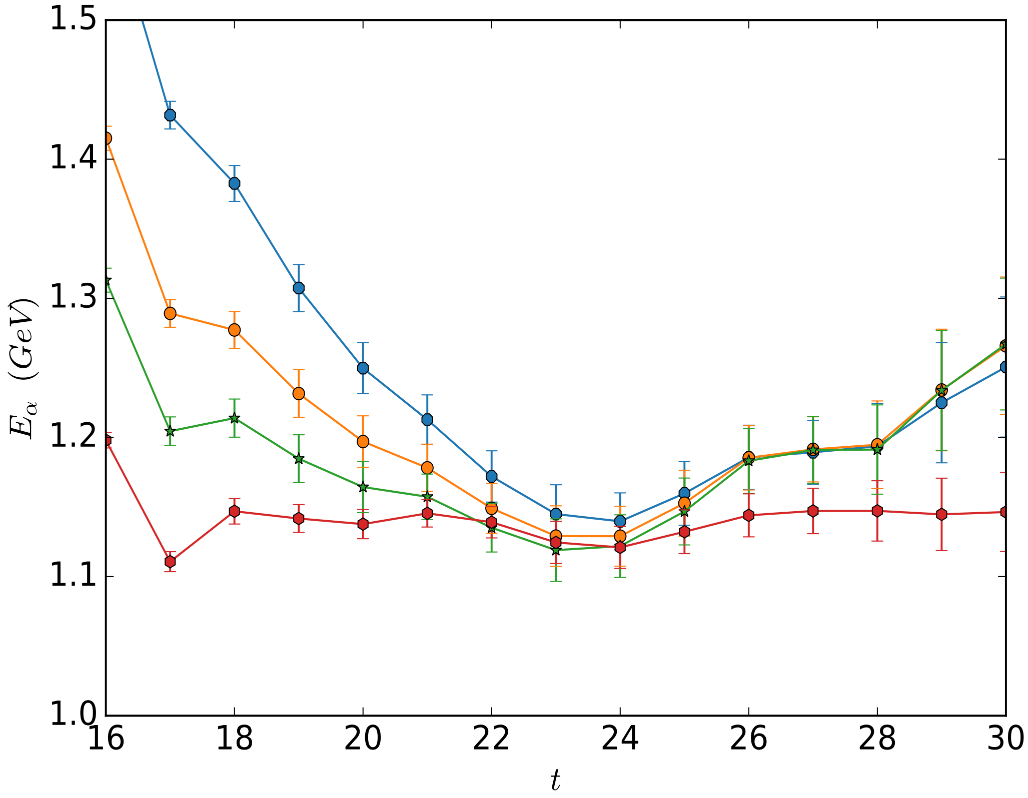
Creating the baryon; $\vec{B} = 0$

- Investigate different levels of source smearing to a point sink. $\mathrm{N}_{sweeps} = 100, 150, 200, 300$



- 300 sweeps is found to provide optimal overlap with the states of interest. This is used for further calculations

Annihilating the baryon; $\vec{B} \neq 0$

- Can account for Landau levels at baryon level.
  - neutron - neutrally charged - standard $\vec{p} = 0$ fourier projection.
  - proton - charged - standard fourier projection produces superposition of Landau levels $\phi_\nu(x)\, \nu = 1, 2, 3 \ldots$
    - Instead project to single Landau level using $\phi_1(x)$.
- What about the quark level though?
  - Include Landau effects in quark propagation through use of Landau eigenmodes.
  - Does a projection to the low-lying Landau levels improve overlap with the states under investigation?

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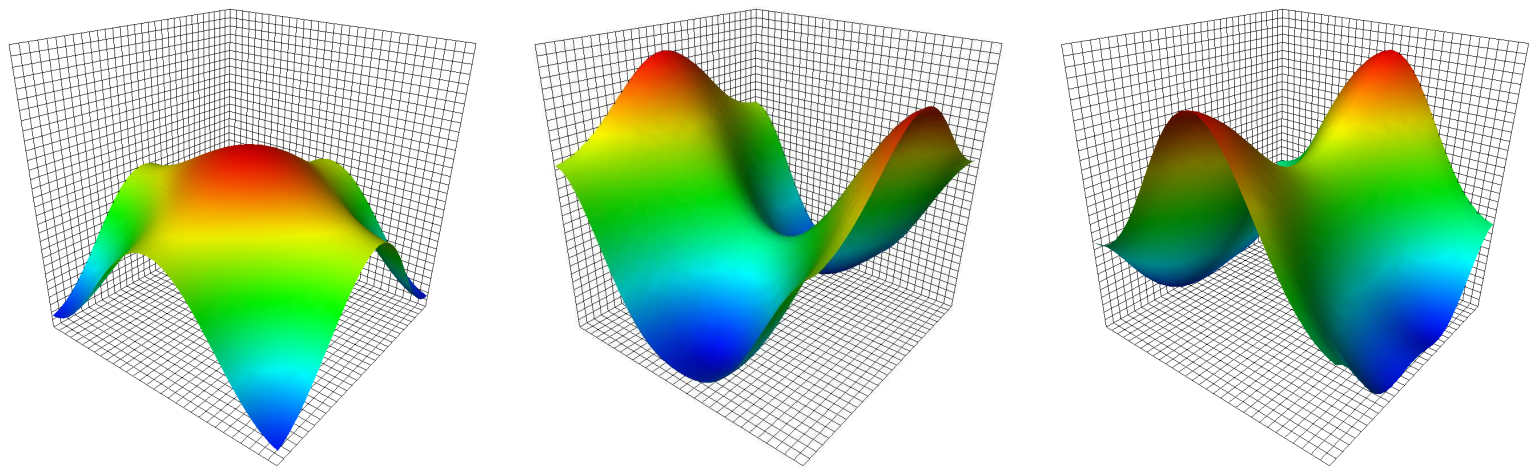
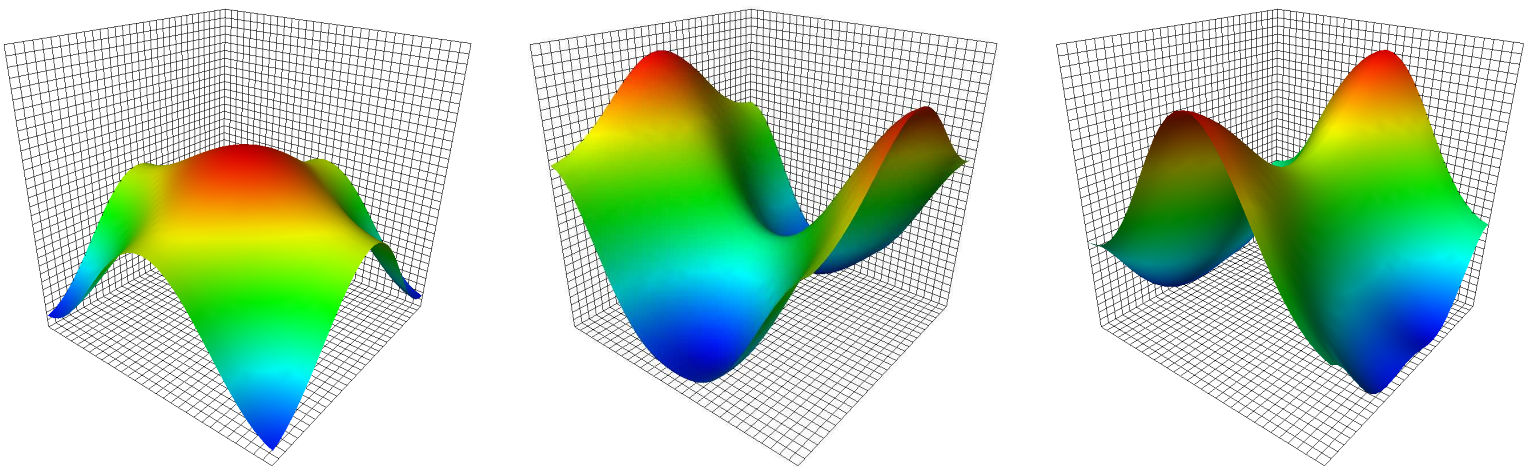


Lowest lying eigenmode probability densities of lattice Laplacian operator.

- Origin is centre of the x-y plane illustrated by bottom surface of the grid.
- Project to these modes, i.e. $\phi_i(x) = \langle x \,|\, \nu_i \rangle$

QED Eigenmodes



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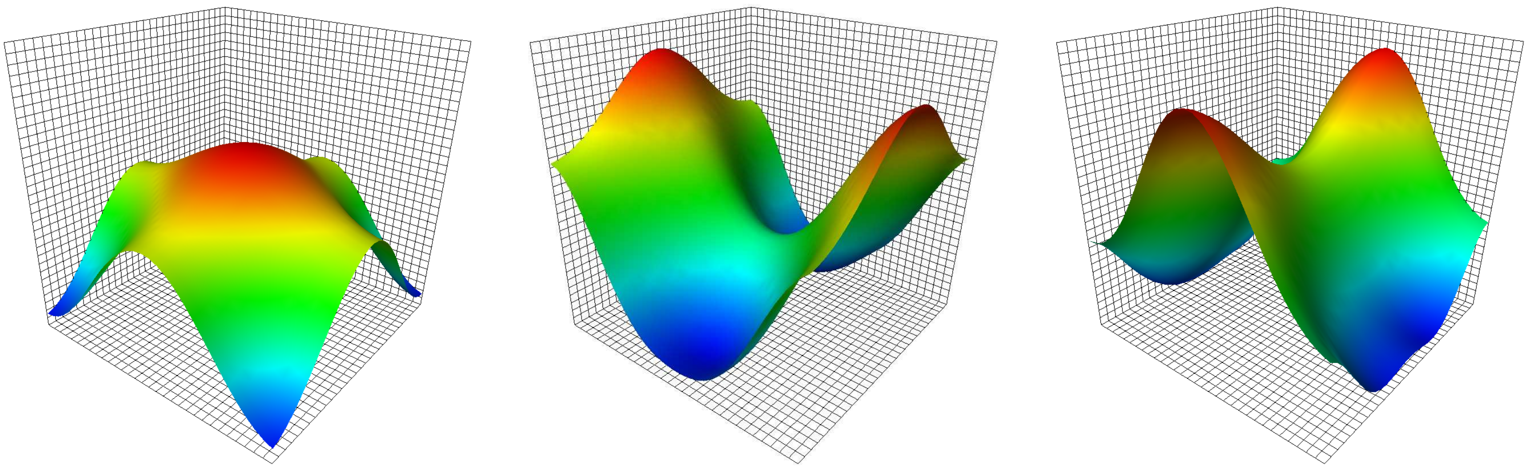

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- Also define QED+QCD eigenmode projection operator

$$P^n_{QED+QCD}(x,y) = \sum_{i=1}^{n=n_{max}} \langle x \,|\, \lambda_i \rangle \, \langle \lambda_i \,|\, y \rangle$$

- and project the propagator at the sink (implicit sum over z)

$$S(y,x,\alpha) = P_\alpha(y,z)\, S(z,x)$$

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$$E(B) = M + \vec{\mu} \cdot \vec{B} - \frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

- Construct ratio of spin and field direction aligned and anti-aligned correlation functions.

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- Then extract an effective energy in the standard manner.

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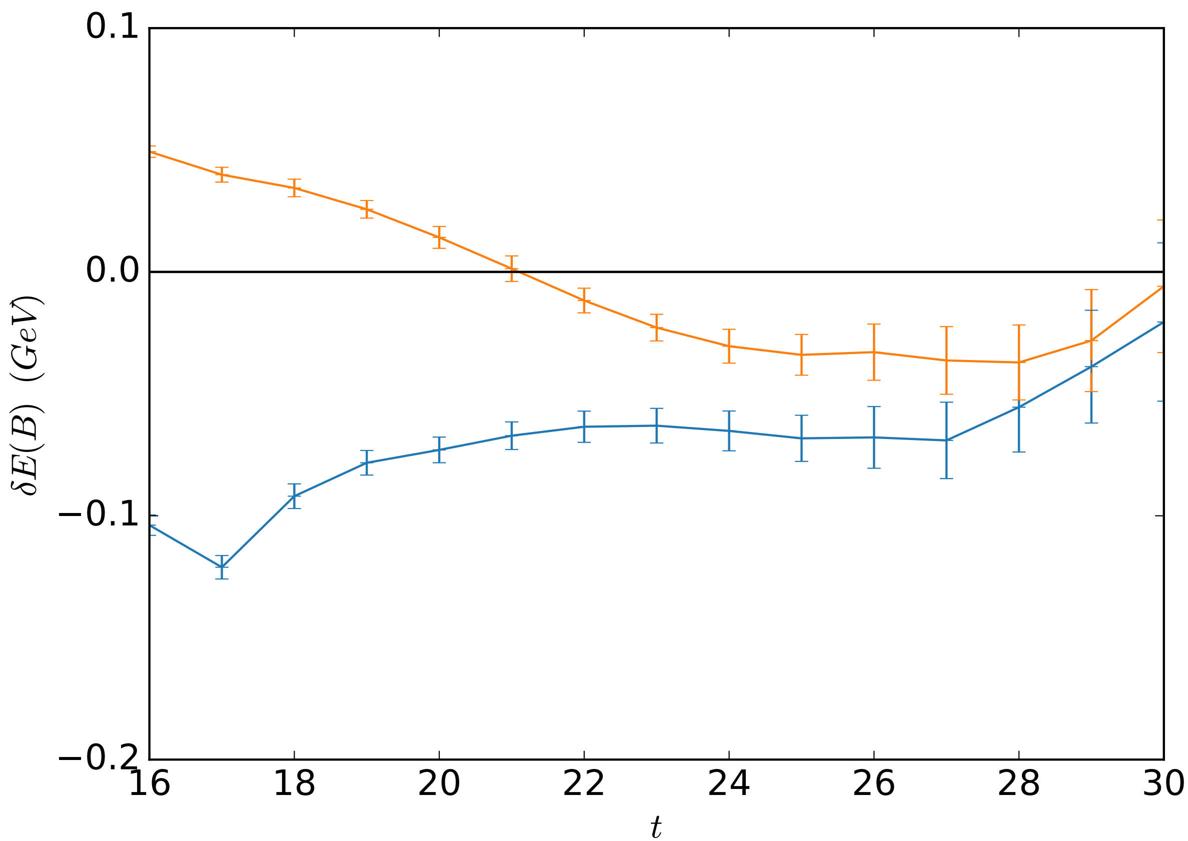
Standard and eigenmode-projected comparision



Neutron energy shift relevant to the magnetic polarisability for largest field strength (BF3).
Standard correlator is in orange, eigenmode-projected is in blue.

Fit window selection

$$\delta E(B) = \left(\frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 \right)$$

- To choose where to fit and obtain polarisability values, a number of factors are considered.
  1. The constant fits to the energy shifts as function of time.
     - We only consider the same fit window across all field strengths.
  2. The fits to energy shifts as function of field strength.
  3. Time window is influenced by δE vs B fits.
- The χ^2_{dof} of each fit in (1), (2) must be in an acceptable range
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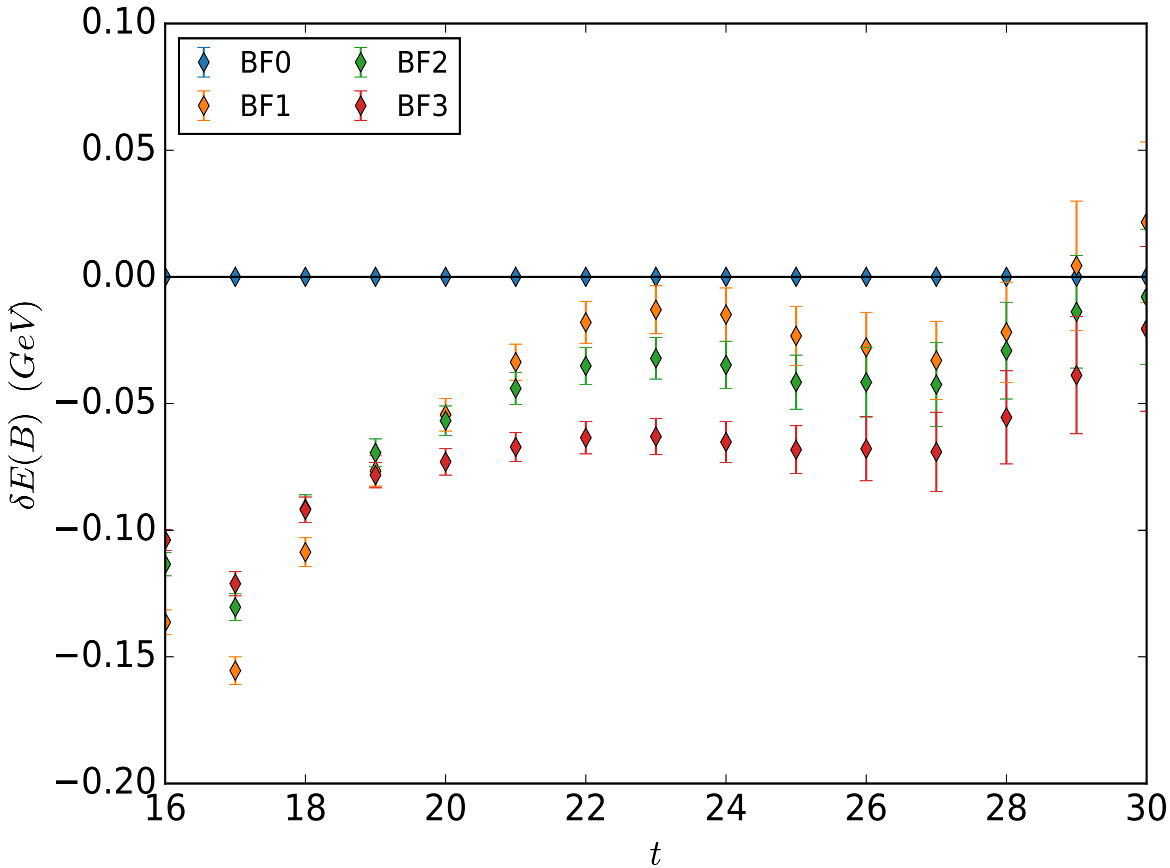
Neutron Energy Shifts for polarisability



Smeared Source to QED eigenmode projected sink neutron energy shift

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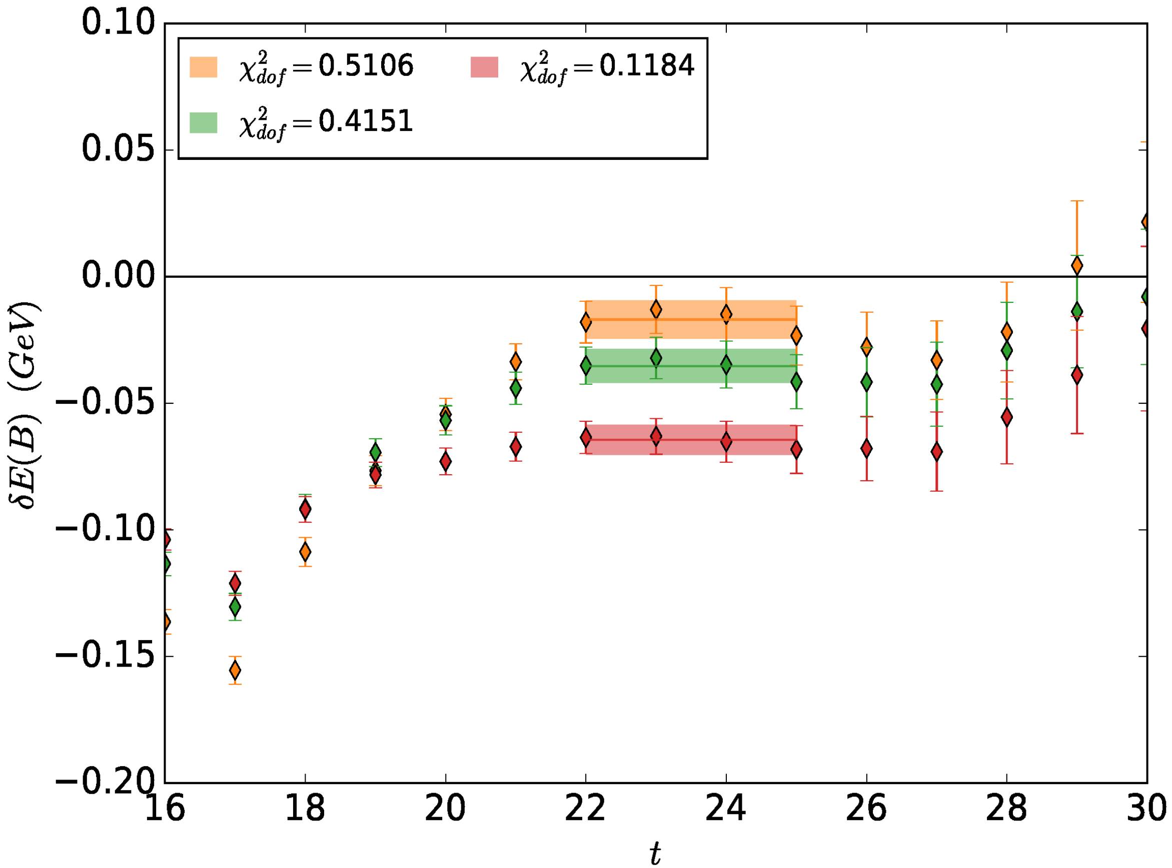

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- Neutron is overall chargeless
  - fit quadratic term only; $\propto c_2\,k^2$
- Proton overall charge $q = 1$
  - Fit linear + quadratic terms; $\propto c_1\,k + c_2\,k^2$
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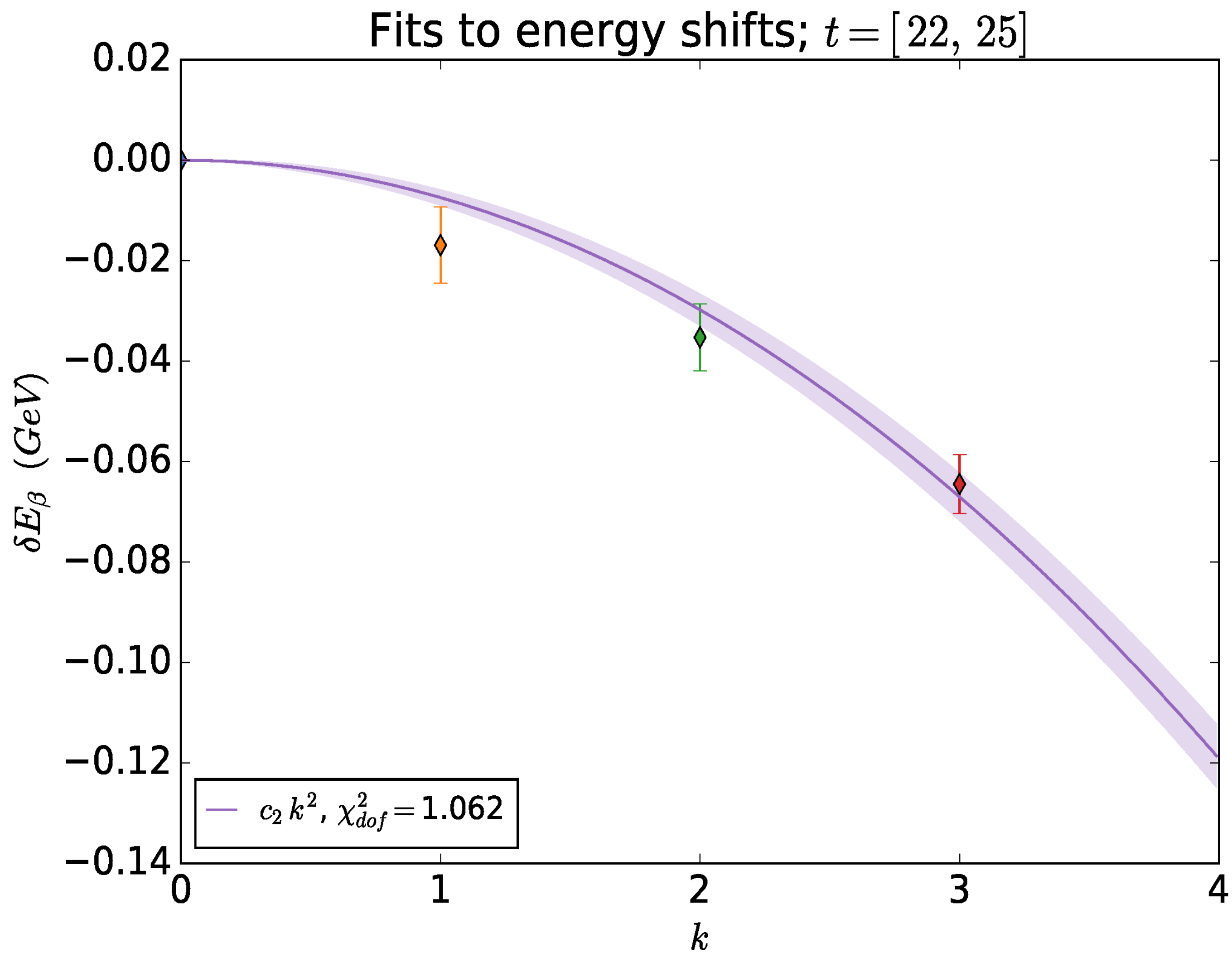
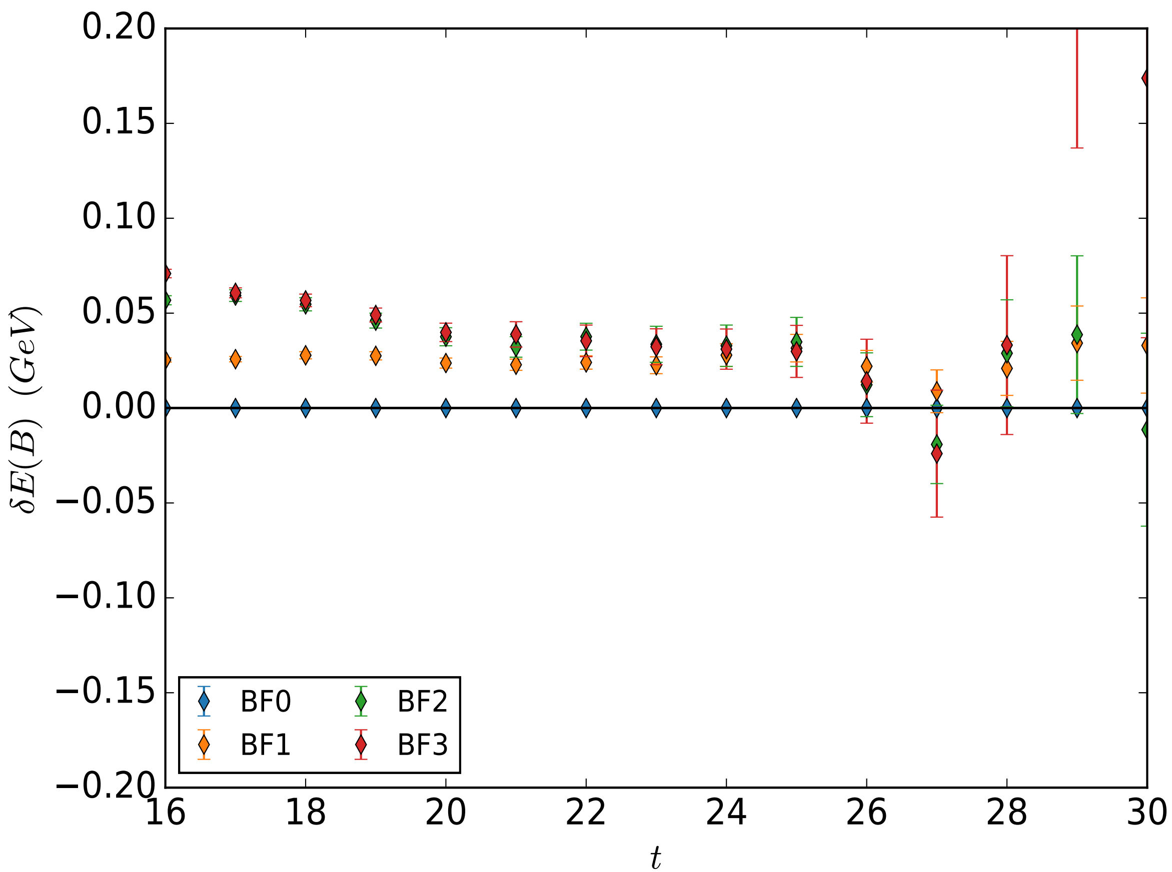
- Proton overall charge $q = 1$
  - Fit linear $+$ quadratic terms; $\propto c_1\,k + c_2\,k^2$
  - Expect charge term produces $q \approx = 1$

Neutron polarisability fit, $\beta_n = 1.39(15) \times 10^{-4}$ fm^3



Proton Energy Shifts for polarisability



QED+QCD eigenmode-projected propagators
Smeared source to Landau projected at baryon level for proton sink.

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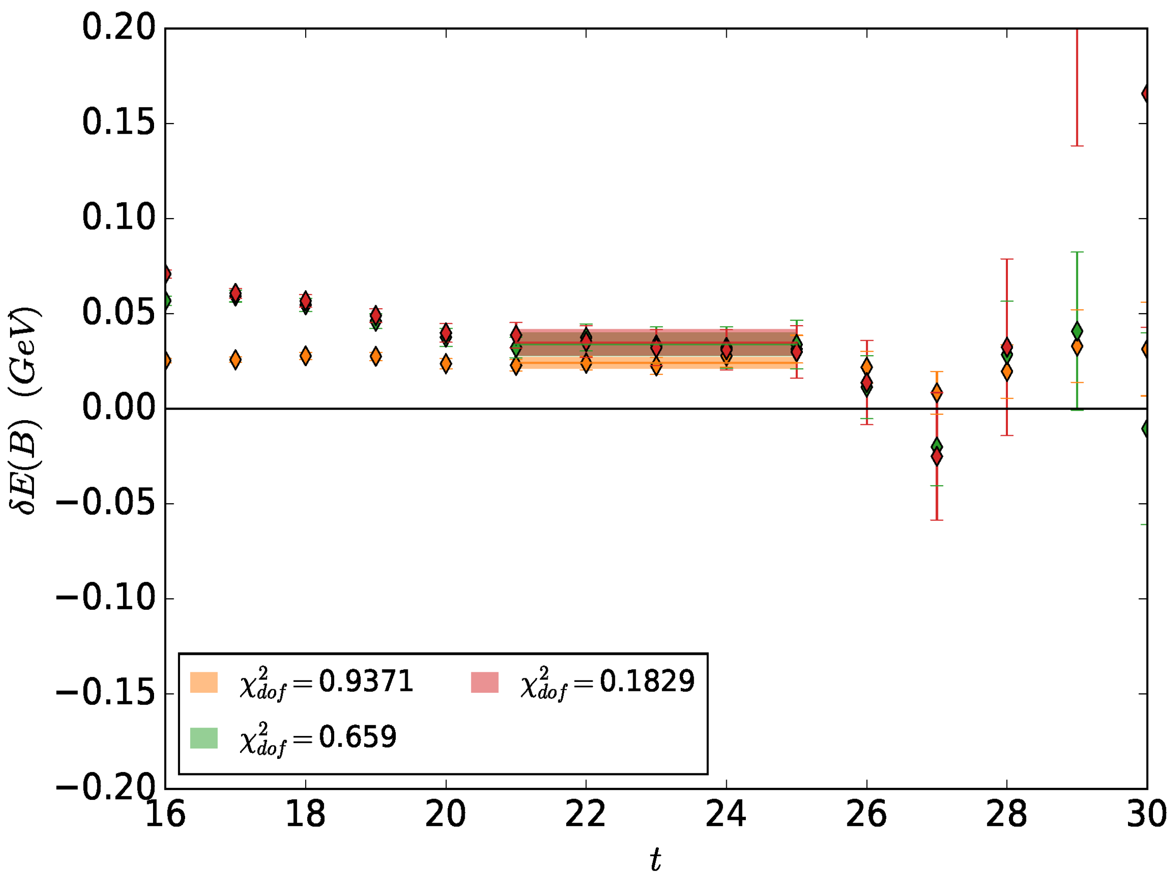

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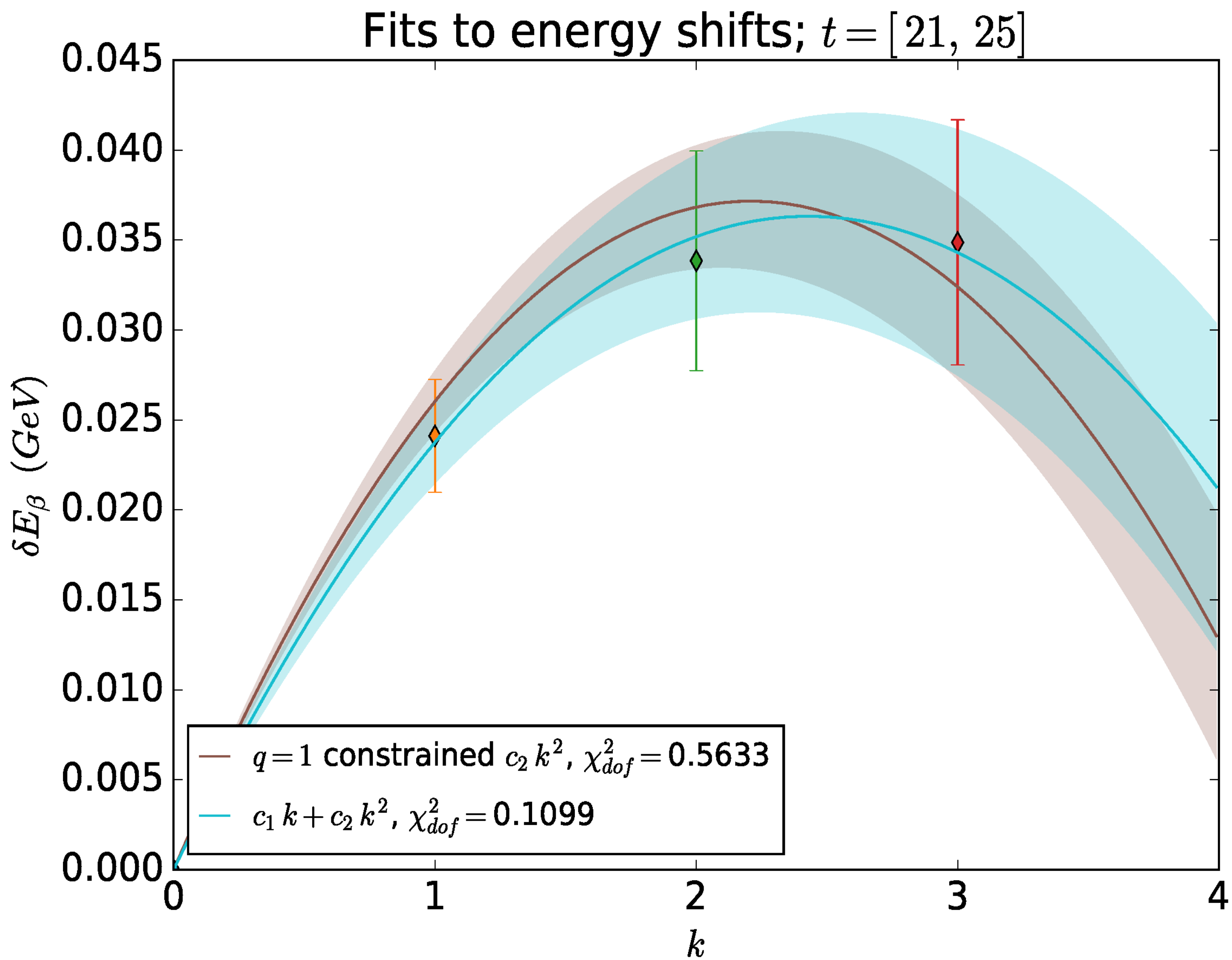
Proton polarisability fit, $\beta_p = 1.15(24) \times 10^{-4}$ fm^3



Nucleon Polarisability

	Experiment ($m_\pi = 138$ MeV)	This Work ($m_\pi = 411$ MeV)
proton	$2.5(4) \times 10^{-4}$ fm^3	$1.15(24) \times 10^{-4}$ fm^3
neutron	$3.7(12) \times 10^{-4}$ fm^3	$1.39(15) \times 10^{-4}$ fm^3

- Relative uncertainties in proton measurements are similar.
- There is potential to make a precise prediction in the neutron case.
- Expect chiral extrapolation to be important - particularly near physical masses.

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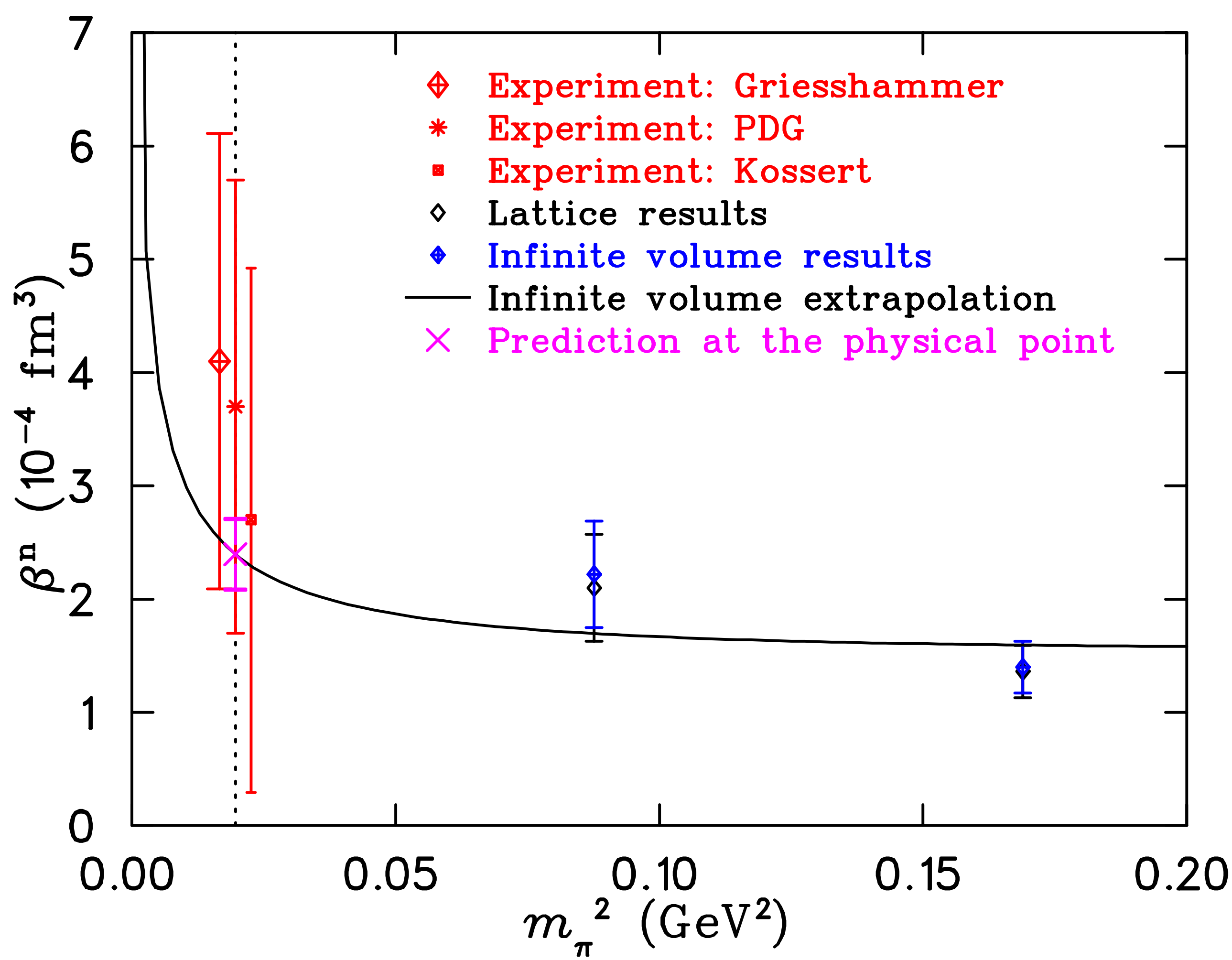

Chiral extrapolation of the magnetic polarisability of the neutron accounting for electro-quenching effect

Summary

1. Explored novel eigenmode-projection operators applied to quark propagators
   - To account for quark-level Landau levels.
2. Investigated two different implementation of projection operators
   (a) QED Eigenmodes
   (b) QED+QCD Eigenmodes
3. Then used the eigenmode-projected propagators and obtain plateaus.
   - These energy shifts are consistent with a small-field energy expansion.
4. Fitted energy shifts such that magnetic polarisabilities are extracted with an accuracy providing an interesting comparision to experiment.

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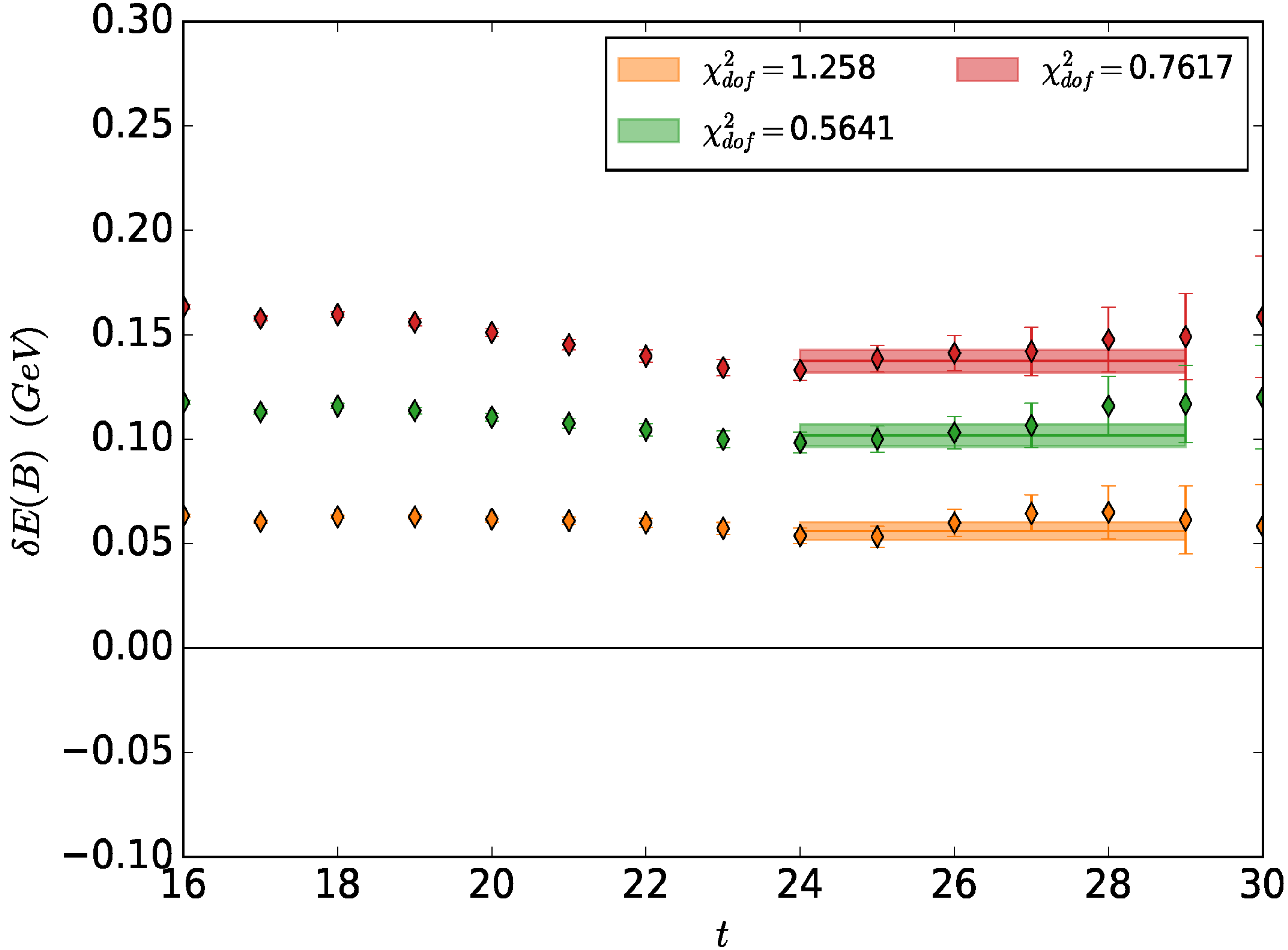
Magnetic Moment

- Considerably easier than magnetic polarisability
- Take a different ratio

$$R(B,t) = \left(\frac{G_{\downarrow}(B-,t) + G_{\uparrow}(B+,t)}{G_{\downarrow}(B+,t) + G_{\uparrow}(B-,t)} \right)$$

- to get an energy shift of

$$\delta E_{\mu}(B) = -\mu\, B + \mathcal{O}\left(B^{3}\right)$$



Energy shift for magnetic moment of the neutron.

Magnetic Moment

- Extract magnetic moment from linear term
- Background field results are preliminary only

	3PT ($m_\pi = 411$ MeV)	BFM ($m_\pi = 411$ MeV)
proton (β_p)	$2.18(2)\,\mu_N$	$2.24(6)\,\mu_N$
neutron (β_n)	$-1.37(2)\,\mu_N$	$-1.36(10)\,\mu_N$