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Optimal bicycle design to maximize handling and safety

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1 INTRODUCTION

Generally speaking, bicycles require human intervention (control) to remain upright. The ability to command the vehicle to perform a desired maneuver plays an essential role in its safe operation. The first derivation of equations of motion for the uncontrolled bicycle were presented more than a century ago, independently, by Whipple [1] and Carvallo [2]. These derivations required rather dramatic simplifying assumptions regarding the rider's motion. Although the Whipple-Carvallo equations of motion [1, 2] for the vehicle (with an absolutely rigidly attached rider) can be used to show that this special vehicle can actually be asymptotically stable over a relatively small range of speeds, this must be viewed as a very special and unrealistic case. Furthermore, bicycles are often ridden at speeds outside this stable speed range (e.g. below 4 m/s and/or above 6 m/s in a typical example) and therefore clearly require active rider control to remain balanced. Thus open-loop, uncontrolled stability, although of academic interest, is not dispositive since the assumed conditions (e.g. rigidly attached rider) and special speed ranges under which it occurs are so rarely encountered. [1]

Instead we are forced to consider the active feedback control that the rider exercises on the bicycle to adequately explain how it is maintained upright. The human stabilizes a generally unstable bicycle by providing state dependent inputs: that is what the rider does depends on what is happening. Although bicycles can be ridden "no-hands", the simplest and most widely used mechanism for this control is the handlebars through which the rider can exert a steer torque from the rider onto the front fork assembly to command a desired steer angle. We have presented a description of a proposed general control scheme for rider stabilization of the bicycle using this steering mechanism [4]. This scheme was based on similar earlier work studying the control of aircraft dynamics by pilots [5].

The scheme includes a linear state variable feedback control law and a method for calculating the feedback gains relating the steer torque to the bicycle states (tilt, steering angle, lateral track deviation, etc.) and is based on frequency response methods and successive loop closure [4]. In addition, it provides a quantity called the "Handling Quality Metric" (HQM), a numerical measure of the difficulty of controlling the bicycle

[1] It is interesting that the same competition between the non-obvious relative importance of stability and controllability was a key factor in the historical development of early aircraft design [3].

even when control is essential and able to be applied to ensure stability. The HQM is task independent (e.g. it is the same for a lane change maneuver as for other maneuvers) but depends heavily on the specific physical parameters of the bicycle. Thus the HQM is a property of the vehicle. It comports with an obvious truth: some bicycles are easier to ride than others. Since the HQM is so dependent on the physical parameters of the bicycle, this begs the question: "Can we choose the parameters (i.e. design the bicycle) to make it easy to ride (have good handling qualities) and, indeed, perhaps even easier to ride than *any* other bicycle?" An alternate rephrasing of this question is "Can we design the bicycle to be maximally safe from a handling point of view?"

2 PRIOR WORK

In previous work [6], we developed a numerical method to discover the optimal essential geometry (steer axis tilt, front wheel radius, trail, and wheelbase) for a bicycle to have maximally desired handling qualities as defined by our control theoretic task-independent metric, HQM. The method produced atypical bicycle designs that had features differing from the majority of bicycles used in normal bicycling activities. Additionally, the resulting maximum handling quality metrics for the optimal bicycle designs follow a logarithmic relationship with respect to design speed. These optimal handling bicycles can be ridden at speeds other than the design speed and, in general, handling ability increases with increasing speed. Finally, we showed that there is little evidence of correlation between bicycle open-loop stability and optimal handling.

This study provided much insight but was limited in a number of ways. First, we explored only four of the more than 20 independent physical parameters that fully define the uncontrolled dynamics of the bicycle model. Thus the resulting designs were limited with respect to realistic possibilities, especially with regard to the designer's ability to manipulate the inertial characteristics of the bicycle. This is important because it is well known that the dynamics of the bicycle are a highly non-linear function of *all* the parameters [7]. Secondly, our optimization constraints were limited in scope and breadth. These constraints are in place to bound the parameter search space to the range of parameters that define a physically realizable bicycle which has the functional features that make a bicycle a bicycle. Some examples of these features are having two wheels and two hinged main bodies, being light enough to be comfortably lifted by one adult and narrow enough to be straddled while ridden, etc.

3 OPTIMIZATION IMPROVEMENTS

We have improved the optimization technique to overcome the aforementioned limitations in two ways. The first improvement is to search over twenty-two design parameters as opposed to four. The second is to impose a comprehensive set of optimization constraints that enforce that the design is a physically realizable bicycle. We have redefined the moments of inertia of the four rigid bodies in terms of their radii of gyration and have enforced lateral symmetry with the specification of the products of inertias. The masses of all of the rigid bodies are bounded so that, for the body's size, a minimal amount of mass is needed to make up its structure. The rider is treated as a single rigid body with fixed joints in a predefined configuration, but the entire body can be rotated about the anatomical transverse axis so that the rider can be sitting upright, recumbent, or supine. The inertias of the two wheels are defined by their outer radii and their masses are constrained so that the wheel is ring-like. The essential geometry is constrained to ensure that the wheels do not overlap and that the center of mass location of the vehicle is bounded to not tip forward or backward during specified maximum accelerations. These constraints bound the possible solutions so that the resulting designs are feasible to construct and ride.

Finally, we are now using constrained non-linear quadratic programming together with unconstrained stochastic optimization, as in [6], to find optimal bicycle designs. The stochastic evolutionary methods allow us to quickly find designs that mostly meet the specified constraints and the addition of quadratic programming allows the constraints to be exactly enforced in the solution.

4 CONCLUSION

Bicycle designs that handle optimally will allow users to more easily command the vehicle and thus to avoid situations that contribute to injury and death during bicycling. Our work has the potential to turn bicycle design into a precise method that relies not on evolutionary experimentation with a variety of designs but one that is governed by mathematical optimality that human intuition and experience often cannot reproduce. As with all optimal design techniques, designs that have not been previously imagined may emerge or we may confirm that the evolutionary design of the bicycle is close to optimal.

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