Evanesccently coupled multimode spiral spectrometer: supplementary material

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This document provides supplementary information to “Evanesccently coupled multimode spiral spectrometer,” <http://dx.doi.org/10.1364/optica.3.000956>. It includes a detailed description on the numerical simulation of the spiral spectrometer. The scaling of the spectral resolution and transmission with the spiral size is presented along with simulations of the temperature dependence of the device. © 2016 Optical Society of America

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Numerical modeling of spiral spectrometer

We performed numerical simulation of the interleaved Archimedean spirals. The local radius of curvature, R , is defined as a function of the azimuthal angle θ as $R(\theta) = R_0 - a\theta/\pi$, where R_0 denotes the radius of the spiral, a is the center-to-center arm spacing (the sum of the waveguide width and the coupling gap), and θ increases from 0 to θ_0 . The number of spiral arms is determined by θ_0 . The center position of the multimode waveguides are then defined in Cartesian coordinates as $x = \pm R(\theta)\cos(\theta)$, $y = \mp R(\theta)\sin(\theta)$, where the sign gives the two interleaved spirals. The two spirals are connected at the center by a S-bend that consists of two arcs of a circle [35, 39].

We modeled the wave propagation in the spiral waveguide using the approach in [36, 37]. First we ignored the mixing of waveguide modes with different order, which allowed us to simulate individual modes separately by solving the following two coupled equations:

$$\begin{aligned} \frac{dU_n(\theta)}{d\theta} &= i\beta R(\theta) U_n(\theta) + \kappa_{n+1,n} U_{n+1}(\theta) + \kappa_{n-1,n} U_{n-1}(\theta) \\ \frac{dV_n(\theta)}{d\theta} &= -i\beta R(\theta) V_n(\theta) - \kappa_{n+1,n} V_{n+1}(\theta) - \kappa_{n-1,n} V_{n-1}(\theta) \end{aligned}$$

where U_n and V_n denote the forward (with respect to the input direction) and backward propagating waves in one waveguide mode in the n^{th} arm of the spiral. n ranges from 1 (the outermost arm) to N (the innermost arm), with N given by θ_0/β is the propagation constant for this waveguide mode. $\kappa_{n+1,n}$ is the complex coefficient for evanescent coupling into the n^{th} arm from the $(n+1)^{\text{th}}$ arm. Both propagation constant β and coupling coefficient κ are wavelength dependent. We consider only the coupling between the modes that have the same order and the same propagating direction in neighboring waveguides, as the coupling between modes with different order or opposite direction is negligible. For energy conservation and symmetry reasons, $\kappa_{n+1,n} = -(\kappa_{n,n+1})^* = i\kappa_n$, where κ_n is a real number. Since the fields must be continuous at the junctures between adjacent arms, the continuity relations are $U_{j+1}(0) = U_j(2\pi) \exp[i\beta S_n]$, $V_{j+1}(0) = V_j(2\pi) \exp[-i\beta S_n]$, where S_n is the length of the n^{th} arm of the spiral.

The above equations were solved numerically by dividing the waveguide into many small segments. Within each segment, the waveguide may be approximated as a straight waveguide, because the local radius of curvature $R(\theta)$ is much larger than the length of the segment. The evanescent coupling length L_c for two straight waveguides is obtained from the finite-difference frequency-domain calculation (COMSOL), and

$\kappa_n = R_n/L_c$, where R_n is the radius of the n th arm of the spiral. The higher-order modes of the waveguide experience stronger evanescent coupling, thus having shorter L_c and larger κ_n .

In the spiral structure, the curving of the waveguide introduces mode mixing. Also the sidewall roughness of the fabricated waveguide causes random mixing of the modes. We simulated mode mixing by introducing a random unitary matrix in between any two successive segments of the waveguide. The rate of mode mixing scales inversely with the local radius of curvature. All the modes are completely mixed after propagating through one arm of the spiral. In the center “S-bend”, the modes are randomly mixed, as modeled by a random unitary matrix.

In all four cases shown in Fig. 1(b,c) of the main text, the outer radius of the spiral is fixed to $R_0 = 106 \mu\text{m}$, and the total length of the waveguide is $L = 4.8 \text{ mm}$. The evanescent coupling length L_c varies from 0.55 mm for the lowest-order mode to $6 \mu\text{m}$ for the highest-order mode of the waveguide. Since the coupling lengths are shorter than the waveguide length, all the modes experience evanescent coupling while propagating through the spiral structure. However, the coupling is weaker for the lower-order modes; fortunately, the mode mixing can indirectly enhance the coupling rate by converting them to the higher-order modes (with stronger evanescent coupling) and then back. Therefore, the synergy between mode mixing and evanescent coupling leads to a significant enhancement of the spectral resolution.

We also investigated the scaling of the spectral resolution with the outer radius R_0 of the spiral. Figure S1(a) plots the spectral correlation width $\delta\lambda$ as a function of R_0 when the number of spiral arms is fixed to $N = 8$. Without mode mixing and evanescent coupling, $\delta\lambda$ scales as $1/R_0$. This behavior is expected, as the spiral length L grows linearly with R_0 . In the presence of mode mixing and evanescent coupling, the scaling changes to $1/R_0^2$, indicating the temporal spread of light grows as R_0^2 or L^2 .

The evanescent coupling introduced to the interleaved spiral structure also causes a reduction in the total transmission, as part of the light propagates backward to the input port. To see how much the reduction is, we plot the total transmission T in Fig. S1(b). When the coupling gap width is fixed, all waveguide modes experience more evanescent coupling as R_0 increases. Consequently, T first drops and then levels off, eventually about half of the light is transmitted. To increase the transmission, the input port could be covered by a highly reflecting photonic crystal wall (having a full photonic bandgap in-plane) with a small opening (a defect waveguide for input light). Thus most of the light propagating back to the input port could be reflected to the spiral waveguide. This will enhance not only the transmission, but also the spectral resolution as the temporal spread of the transmitted light will further increase.

Fig. S1 Dependence of spectral resolution and total transmission on the size of spiral spectrometer. (a) Spectral correlation width $\delta\lambda$ vs. the inverse of the outer radius, $1/R_0$, of the spiral waveguide with (solid black squares) and without (open blue circles) evanescent coupling and mode mixing. In the former case $\delta\lambda$ scales as $1/R_0^2$ (black solid line), and in the latter $\delta\lambda \sim 1/R_0$ (blue dashed line). (b) Total transmission vs. $1/R_0$ with evanescent coupling and mode mixing. The error bar is obtained from 50 ensembles of random mode mixing. Due to evanescent coupling, the transmission decreases with increasing R_0 and eventually saturates to 0.5.

Linearity of Speckle Intensity

To check the linearity of the spectral measurements, we show in Fig. 4(b) of the main text two continuous spectra that have the same shape but differ by a factor of 2 in input intensity. After scaling the recovered spectrum from the weaker input by a factor 2, the two spectra overlap, verifying the linearity. We also measured the speckle pattern from the spiral waveguide at a single wavelength while increasing the input light intensity. Figure S2 shows the speckle intensity increases linearly with the input intensity.

Fig. S2: Measured speckle intensity of a spiral spectrometer as a function of the input intensity at wavelength $\lambda = 1520 \text{ nm}$. The spiral outer radius is $250 \mu\text{m}$, the waveguide width is $10 \mu\text{m}$, and the air gap between the neighboring spiral arm is 50 nm wide.

Reconstruction of Sparse Spectrum

We used the compressive sensing algorithm to recover sparse spectra. The total number of independent spectral channels that can be reconstructed simultaneously is P , but only H spectral channels carry signals and the other $P-H$ channels have zero amplitude. For such sparse spectra, P can be significantly larger than the number of spatial channels M when using compressive sensing, as shown in Fig. 4(c,d) of the main text.

To be more quantitative, we analyzed the number of spectral lines that can be reconstructed in a finite range of wavelength by fixing P and M while gradually increasing H . The reconstruction error ε is determined by the difference in

wavelength between the H strongest lines in the recovered spectrum and those in the input spectrum. Figure S3 plots ε averaged over 50 different spectra that have H discrete lines of equal height and random spacing. We set $P = 332$, $M = 40$, and reconstructed the spectrum using the measured transfer matrix of the spiral spectrometer with 50 nm gap. The error ε remains zero for H up to 9, then started growing beyond that. This analysis gave the maximum number of discrete lines that could be reconstructed reliably.

The noise level in the measurement affects the number of spectral lines that could be accurately recovered. To simulate the measurement noise, we added white noise to an initially “noise-less” speckle pattern that was synthesized from the same transfer matrix used by the reconstruction algorithm. The SNR is defined as the ratio of the integrated intensity of the signal (without noise) to that of the noise. The lines in Fig. S3 represent the reconstruction error ε for three different levels of SNR. A higher SNR allows more spectral lines to be recovered accurately. The SNR in our measurement is about 20 dB, and the reconstruction error matches the simulation result for SNR = 20 dB.

Fig. S3: The reconstruction error ε as a function of the number of discrete lines H present within the 2nm bandwidth ($P = 332$, $M = 40$). The solid squares are experimental data averaged over 50 different spectra. The lines are simulated results for three different SNR levels. More discrete lines can be recovered with higher SNR.

Temperature Sensitivity

To quantitatively estimate the temperature sensitivity of the spiral spectrometer, we numerically simulated the change of output speckle pattern with temperature. We assume that the temperature change only modifies the real part of the refractive index of the silicon. Therefore, in the simulation, we vary the refractive index of the silicon gradually and calculate the speckle pattern at a fixed wavelength of 1.5 μm . Then we compute the correlation of the speckle pattern to the original one, and estimate the amount of refractive index change Δn that causes 50% decorrelation of the speckle pattern. From the value of Δn , we find the corresponding change of temperature ΔT using the thermo-optic coefficient of Si at $T = 295$ K and $\lambda = 1.5\mu\text{m}$, where $dn/dT = 1.87\text{E-}04$ [1]. The ΔT can be taken as an approximate measure of the temperature sensitivity.

We simulated the temperature sensitivity for 3 spiral spectrometers with different waveguide length and spectral resolution. As shown in Table S1, the spiral with finer spectral resolution is more sensitive to the temperature change. The temperature sensitivity scales linearly with the spectral

resolution. For the spiral spectrometer with 0.012 nm resolution, which is close to our experimentally measured resolution in Fig. 3(c), a temperature change of 0.16 K is sufficient to decorrelate the speckle pattern by 50%. Existing temperature controllers can provide excellent temperature stability (e.g. Thorlabs TED4015 provides stability of <0.002 K) and would thus provide sufficient thermal control for the spiral spectrometer to operate without recalibration. Alternatively, the thermal stability requirement could be loosened by calibrating the speckle patterns at multiple temperatures and selecting the transfer matrix for the current chip temperature. Typically the temperature variation is slower than the spectrum measurement, and the complexity of reconstruction algorithm would not increase, provided the chip temperature is constantly monitored.

Total length of spiral L (mm)	Spectral correlation function width $\Delta\lambda$ (nm)	Refractive index correlation function width Δn	Corresponding change in temperature ΔT (K)
4.83	0.164	39×10^{-5}	2.10
9.7	0.044	11×10^{-5}	0.59
19.9	0.012	2.9×10^{-5}	0.16

Table S1. Temperature sensitivity of the spiral spectrometer. The amount of refractive index change Δn and the corresponding temperature change ΔT to cause 50% decorrelation of the speckle pattern are estimated for three spiral spectrometers of different waveguide length and spectral resolution.

Comparison of On-chip Spectrometers

Since our spiral spectrometer is designed to be a completely on-chip, integrated spectrometer, we compare its performance to other spectrometers that can operate fully on-chip and would thus compete for the same application space.

Spectrometer	Footprint	Resolution	Bandwidth	reference
Echelle grating	18mm x 17mm	0.8 nm	40 nm (1530-1568nm)	S. Janz, et al., IEEE Photon. Tech. Lett. 16, 503 (2004)
	9mm x 6mm	0.5 nm	15nm (at 845nm)	X. Ma, et al. IEEE Photon. Journal 5,6600807 (2013)
Arrayed waveguide grating	8mm x 8 mm	0.2 nm	10 nm (1539-1549 nm)	P. Cheben, et al., Opt. Express. 15, 2299 (2007)
	860um x 240um	1.5nm	40nm (at 1500nm)	J. Zou, et al., "Appl. Opt. 55, 3531 (2016)
Digital planar holography	~2mm ²	0.15 nm	148 nm (at 600nm)	G. Calafiore, et al., Light: Science & Applications 3, e203 (2014).
Photonic crystal	80um x 220um	8 nm (single lines at ~10 pm)	50 nm (1560-1610 nm)	B. Momeni, et al., Opt. Commun. 282, 3168 (2009)
Microdonet	1 mm ²	0.6 nm	50 nm (1550-1600 nm)	Z. Xia, et al., Opt. Express 19 12356 (2011)
Random Spectrometer	100um x 50um	0.75 nm	25 nm (1500-1525nm)	Redding et al., Nature Photonics 7, 746 (2013)
Fourier-transform spiral waveguide	2mm x 2mm	0.042 nm	0.75 nm (at 1500nm)	A. V. Velasco, et al., Opt. Lett. 38, 706 (2013)

Table S2. Comparison of the footprint, wavelength resolution and bandwidth of our spiral spectrometer to the existing on-chip spectrometers.

Reference:

[1] B. J. Frey, D. B. Leviton, and T. J. Madison. *SPIE Astronomical Telescopes+ Instrumentation*. International Society for Optics and Photonics, pp. 62732J, 2006.