Low-dimensional optical chirality in complex potentials: supplementary material

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1. Planewave solution of a PT-symmetric optical material

To clarify a point of importance in the chirality of a PT-symmetric optical material, here we assume the simplest case of a planewave propagating along the x-axis ($E = E_0 e^{i(\omega t - \beta x)}$). From the PT-symmetric permittivity tensor ϵ_r of the material, Maxwell's wave equation reduces to the following 2-dimensional matrix equation:

$$\begin{bmatrix} \epsilon_{\text{eig}} - \epsilon_{r0} - i\epsilon_{i0} & -\epsilon_{\kappa 0} \\ -\epsilon_{\kappa 0}^* & \epsilon_{\text{eig}} - \epsilon_{r0} + i\epsilon_{i0} \end{bmatrix} \begin{bmatrix} E_y \\ E_z \end{bmatrix} = \mathbf{0}, \quad (\text{S1})$$

where ϵ_{eig} is the effective permittivity of the eigenmodes ($\beta^2 = \epsilon_{\text{eig}}(\omega/c)^2$). The effective permittivity $\epsilon_{\text{eig}1,2}$ and the corresponding eigenmode $\mathbf{v}_{\text{eig}1,2}$ are then obtained as

$$\epsilon_{\text{eig}1,2} = \epsilon_{r0} \pm \sqrt{|\epsilon_{\kappa 0}|^2 - \epsilon_{i0}^2}, \quad (\text{S2})$$

$$\mathbf{v}_{\text{eig}1,2} = \eta_{1,2} \cdot \begin{bmatrix} \epsilon_{\kappa 0} \\ -i\epsilon_{i0} \pm \sqrt{|\epsilon_{\kappa 0}|^2 - \epsilon_{i0}^2} \end{bmatrix}, \quad (\text{S3})$$

where $\eta_{1,2} = [1/(|\epsilon_{\kappa 0}|^2 + |-i\epsilon_{i0} \pm (\epsilon_{\kappa 0}^2 - \epsilon_{i0}^2)^{1/2}|^2)]^{1/2}$ is the normalization factor for each eigenmode. To define the exceptional point (EP) originating from the onset of PT symmetry breaking [1,2], we now introduce the interaction parameter [2] $\lambda_{\text{PT}} = (|\epsilon_{\kappa 0}|^2 - \epsilon_{i0}^2)^{1/2}$, where $\lambda_{\text{PT}} = 0$ at the EP provides the geometric multiplicity of 1 (= the dimensionality of the eigenspace) for $\epsilon_{\text{eig}1} = \epsilon_{\text{eig}2}$. In this case, Eqs (S2) and (S3) are simplified as

$$\epsilon_{\text{eig}1,2} = \epsilon_{r0} \pm \lambda_{\text{PT}}, \quad (\text{S4})$$

$$\mathbf{v}_{\text{eig}1,2} = \eta_{1,2} \cdot \begin{bmatrix} \epsilon_{\kappa 0} \\ -i\epsilon_{i0} \pm \lambda_{\text{PT}} \end{bmatrix}. \quad (\text{S5})$$

For reference, from the condition of zero magneto-electric coupling in the present analysis, the value of $\epsilon_{\kappa 0}$ is real [3] ($\lambda_{\text{PT}} = (\epsilon_{\kappa 0}^2 - \epsilon_{i0}^2)^{1/2}$).

2. Effect of imperfect PT symmetry on the modal chirality

For completeness, here we investigate the effect of imperfect PT symmetry on the modal chirality by considering two different situations related to the condition of PT symmetry: (A) non-symmetric real parts ($\text{Re}[\epsilon_y] \neq \text{Re}[\epsilon_z]$) and (B) non-anti-symmetric imaginary parts of permittivity ($\text{Im}[\epsilon_y] \neq -\text{Im}[\epsilon_z]$).

A. Broken symmetry in the real part of permittivity

To introduce the real-part imperfection ($\text{Re}[\epsilon_y] \neq \text{Re}[\epsilon_z]$), we change Eq. (S1), including the real part difference $\Delta\epsilon_{r0}$, as

$$\begin{bmatrix} \epsilon_{\text{eig}} - \epsilon_{r0} - \Delta\epsilon_{r0}/2 - i\epsilon_{i0} & -\epsilon_{\kappa 0} \\ -\epsilon_{\kappa 0}^* & \epsilon_{\text{eig}} - \epsilon_{r0} + \Delta\epsilon_{r0}/2 + i\epsilon_{i0} \end{bmatrix} \begin{bmatrix} E_y \\ E_z \end{bmatrix} = \mathbf{0}, \quad (\text{S6})$$

from $\text{Re}[\epsilon_y] = \epsilon_{r0} + \Delta\epsilon_{r0}/2$ and $\text{Re}[\epsilon_z] = \epsilon_{r0} - \Delta\epsilon_{r0}/2$. Then, the effective permittivity $\epsilon_{\text{eig}1,2}$ and the corresponding eigenmode $\mathbf{v}_{\text{eig}1,2}$, including the effect of the real part imperfection, are obtained as

$$\epsilon_{\text{eig}1,2} = \epsilon_{r0} \pm \sqrt{|\epsilon_{\kappa 0}|^2 - \epsilon_{i0}^2 + \frac{\Delta\epsilon_{r0}}{4}(\Delta\epsilon_{r0} + 4i\epsilon_{i0})}, \quad (\text{S7})$$

$$\mathbf{v}_{\text{eig}1,2} = \eta_{1,2} \cdot \begin{bmatrix} \epsilon_{\kappa 0} \\ -\Delta\epsilon_{r0}/2 - i\epsilon_{i0} \pm \sqrt{|\epsilon_{\kappa 0}|^2 - \epsilon_{i0}^2 + \frac{\Delta\epsilon_{r0}}{4}(\Delta\epsilon_{r0} + 4i\epsilon_{i0})} \end{bmatrix}, \quad (\text{S8})$$

where $\eta_{1,2}$ is the new normalization factor for each eigenmode satisfying $|\mathbf{v}_{\text{eig}1,2}|^2 = 1$. Due to the complex form inside the square

root of Eq. (S7), perfect coalescence does not occur if $\Delta\epsilon_{r0} \neq 0$, as shown in Figs S1a and S1b. From Eq. (S8), we can also calculate the modal chirality $\chi_{1,2}$ (see Appendix A), which shows the effect of the

imperfection on the modal chirality (Fig. S1c). From those results, we can determine the boundary of the tolerance for the modal chirality (Fig. S1d).

Fig. S1. The effect of the imperfection in the real part of PT symmetry. (a) The real and (b) imaginary parts of the effective permittivity $\epsilon_{\text{eig}1,2}$. (c) The density of chirality $\chi = [\epsilon_{r0} \mathbf{E} \cdot (\nabla \times \mathbf{E}) + (1/\mu_0) \mathbf{B} \cdot (\nabla \times \mathbf{B})]/2$, normalized by the product of the electric field intensity $U_e = |\mathbf{E}|^2$ and $\beta_{r0} = \epsilon_{r0}^{1/2} 2\pi/\lambda_0$. (d) The density of chirality as a function of the defect $\Delta\epsilon_{r0}$ at the EP (red dotted lines in (a-c)). All other parameters are the same as those in Fig. 1 in the main article.

B. Broken anti-symmetry in the imaginary part of the permittivity

For the imaginary-part imperfection ($\text{Im}[\epsilon_y] \neq -\text{Im}[\epsilon_z]$), Eq. (S1) is changed, including the new imaginary part $\Delta\epsilon_{i0}$, as

$$\begin{bmatrix} \epsilon_{\text{eig}} - \epsilon_{r0} - i(\epsilon_{i0} + \Delta\epsilon_{i0}/2) & -\epsilon_{k0} \\ -\epsilon_{k0}^* & \epsilon_{\text{eig}} - \epsilon_{r0} + i(\epsilon_{i0} - \Delta\epsilon_{i0}/2) \end{bmatrix} \begin{bmatrix} E_y \\ E_z \end{bmatrix} = \mathbf{0}, \quad (\text{S9})$$

from $\text{Im}[\epsilon_y] = \epsilon_{i0} + \Delta\epsilon_{i0}/2$ and $\text{Im}[\epsilon_z] = -\epsilon_{i0} + \Delta\epsilon_{i0}/2$, showing the broken anti-symmetry. Then, the effective permittivity $\epsilon_{\text{eig}1,2}$, including the effect of the imaginary part imperfection, is obtained as

$$\epsilon_{\text{eig}1,2} = \epsilon_{r0} + i \frac{\Delta\epsilon_{i0}}{2} \pm \sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2} = \epsilon_{r0} + i \frac{\Delta\epsilon_{i0}}{2} \pm \lambda_{\text{PT}}, \quad (\text{S10})$$

while $\text{veig}_{1,2}$ and the following $\chi_{1,2}$ are the same as Eq. (S3). The modal chirality is thus not influenced by the imperfect anti-symmetry of the imaginary part. Such a result can be understood

within the context of gauge transformation, which has been demonstrated in passive PT-symmetric systems [2,4] ($\Delta\epsilon_{i0} < 0$).

3. Transfer between RCP and LCP modes in the PT-symmetric chiral material

For an incident wave impinging upon the PT-symmetric material ($\mathbf{E}_{\text{inc}} = A_{\text{eig}1} \mathbf{v}_{\text{eig}1} + A_{\text{eig}2} \mathbf{v}_{\text{eig}2}$), the transmitted wave at $x = d$ is expressed as $\mathbf{E}_{\text{trn}} = A_{\text{eig}1} \mathbf{v}_{\text{eig}1} \exp(-i\beta_1 d) + A_{\text{eig}2} \mathbf{v}_{\text{eig}2} \exp(-i\beta_2 d)$, where $\mathbf{v}_{\text{eig}1}$ and $\mathbf{v}_{\text{eig}2}$ are complex eigenmodes in Eq. (S5), $\beta_{1,2} = 2\pi\epsilon_{\text{eig}1,2}^{1/2}/\lambda_0$ is the propagation constant of each eigenmode, and λ_0 is the free-space wavelength of the wave. To investigate the chiral behavior of the wave, we employ the convenient basis of RCP and LCP: $\mathbf{v}_{\text{RL}} = (1/2)^{1/2} (1, \pm i)^T$. From the relation of $\mathbf{v}_{\text{eig}1,2} = A_{\text{RL}2} \mathbf{v}_{\text{R}} + A_{\text{L}1,2} \mathbf{v}_{\text{L}} \equiv [A_{\text{RL}2}, A_{\text{L}1,2}] \mathbf{CP}^T$, a transfer equation between the incident wave $\mathbf{E}_{\text{inc}} = [E_{\text{R}}, E_{\text{L}}] \mathbf{CP}^T$ and the transmitted wave $\mathbf{E}_{\text{trn}} = [E_{\text{RT}}, E_{\text{LT}}] \mathbf{CP}^T$ in CP bases is obtained as

$$\begin{bmatrix} E_{\text{RT}} \\ E_{\text{LT}} \end{bmatrix} = \frac{1}{A_{\text{R}1} A_{\text{L}2} - A_{\text{R}2} A_{\text{L}1}} \begin{bmatrix} A_{\text{R}1} e^{-i\beta_1 d} & A_{\text{R}2} e^{-i\beta_2 d} \\ A_{\text{L}1} e^{-i\beta_1 d} & A_{\text{L}2} e^{-i\beta_2 d} \end{bmatrix} \begin{bmatrix} A_{\text{L}2} & -A_{\text{R}2} \\ -A_{\text{L}1} & A_{\text{R}1} \end{bmatrix} \begin{bmatrix} E_{\text{R}} \\ E_{\text{L}} \end{bmatrix}, \quad (\text{S11})$$

where the explicit expression for $[A_{\text{RL}2}, A_{\text{L}1,2}] \mathbf{CP}^T$,

$$\begin{bmatrix} A_{\text{RL}2} \\ A_{\text{L}1,2} \end{bmatrix} = \frac{\eta_{1,2}}{\sqrt{2}} \begin{bmatrix} \epsilon_{k0} - \epsilon_{i0} \mp i\lambda_{\text{PT}} \\ \epsilon_{k0} + \epsilon_{i0} \pm i\lambda_{\text{PT}} \end{bmatrix}, \quad (\text{S12})$$

is obtained from Eq. (S5). By applying Eq. (S12), Eq. (S11) can be re-expressed using only structural (d) and material (ϵ_{r0} , ϵ_{i0} , and ϵ_{k0}) parameters as $\mathbf{E}_{\text{trn}} = \mathbf{M}_{\text{PT}} \mathbf{E}_{\text{inc}}$ where

$$\mathbf{M}_{\text{PT}} = \begin{bmatrix} t_{\text{R} \rightarrow \text{R}} & t_{\text{L} \rightarrow \text{R}} \\ t_{\text{R} \rightarrow \text{L}} & t_{\text{L} \rightarrow \text{L}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \varphi_1 + \varphi_2 & -i \frac{\epsilon_{k0} - \epsilon_{i0}}{\lambda_{\text{PT}}} (\varphi_1 - \varphi_2) \\ i \frac{\epsilon_{k0} + \epsilon_{i0}}{\lambda_{\text{PT}}} (\varphi_1 - \varphi_2) & \varphi_1 + \varphi_2 \end{bmatrix}, \quad (\text{S13})$$

$\varphi_{1,2} = \exp(-i\beta_{1,2}d)$, d is the propagation distance, $\beta_{1,2} = 2\pi\epsilon_{\text{eig}1,2}^{1/2}/\lambda_0$, and λ_0 is the free-space wavelength. Equation (S13) shows the apparent chiral transfer through two unequal off-diagonal terms. The magnitude of the chirality in the intermodal transfer between CP modes is quantified by C_{IM} as Eq. (2) in the main article.

4. Strength of chiral conversion C_{CS}

After the EP ($\epsilon_{i0} \geq \epsilon_{k0}$), $C_{\text{CS}}^{\text{R} \rightarrow \text{L}}$ and $C_{\text{CS}}^{\text{L} \rightarrow \text{R}}$ can be expressed as

$$C_{\text{CS}}^{\text{R} \rightarrow \text{L}} = \left(\frac{\epsilon_{i0} + \epsilon_{k0}}{\epsilon_{i0} - \epsilon_{k0}} \right)^{1/2} \cdot \tanh \left[\frac{2\pi d}{\lambda_0} \cdot \left(\frac{\sqrt{\epsilon_{r0}^2 + \epsilon_{i0}^2 - \epsilon_{k0}^2} - \epsilon_{r0}}{2} \right)^{1/2} \right], \quad (\text{S14})$$

$$C_{\text{CS}}^{\text{L} \rightarrow \text{R}} = \left(\frac{\epsilon_{i0} - \epsilon_{k0}}{\epsilon_{i0} + \epsilon_{k0}} \right)^{1/2} \cdot \tanh \left[\frac{2\pi d}{\lambda_0} \cdot \left(\frac{\sqrt{\epsilon_{r0}^2 + \epsilon_{i0}^2 - \epsilon_{k0}^2} - \epsilon_{r0}}{2} \right)^{1/2} \right]. \quad (\text{S15})$$

When $\epsilon_{i0} \sim \epsilon_{k0}$, $C_{\text{CS}}^{\text{R} \rightarrow \text{L}} \sim 2\pi L_{\text{eff}}(\epsilon_{i0}/\epsilon_{r0})$ and $C_{\text{CS}}^{\text{L} \rightarrow \text{R}} \sim 0$ near the EP, showing unidirectional chiral conversion (Fig. 1j in the main article).

Because $\text{Re}[\beta_1] \neq \text{Re}[\beta_2]$ before the EP (Fig. 1a, in the main article), C_{CS} becomes oscillatory, in contrast to the case after the EP ($\text{Re}[\beta_1] = \text{Re}[\beta_2]$). The C_{CS} ($\epsilon_{i0} < \epsilon_{k0}$) is then expressed as

$$C_{\text{CS}}^{\text{R} \rightarrow \text{L}} = \left(\frac{\epsilon_{k0} + \epsilon_{i0}}{\epsilon_{k0} - \epsilon_{i0}} \right)^{1/2} \cdot \left| \tan \left[\frac{2\pi d}{\lambda_0} \cdot \frac{\sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2}}{\left(\epsilon_{r0} + \sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2} \right)^{1/2} + \left(\epsilon_{r0} - \sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2} \right)^{1/2}} \right] \right|, \quad (\text{S16})$$

$$C_{CS}^{L \rightarrow R} = \left(\frac{\epsilon_{k0} - \epsilon_{i0}}{\epsilon_{k0} + \epsilon_{i0}} \right)^{1/2} \cdot \left| \tan \left(\frac{2\pi d}{\Lambda_0} \cdot \frac{\sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2}}{(\epsilon_{i0} + \sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2})^{1/2} + (\epsilon_{i0} - \sqrt{\epsilon_{k0}^2 - \epsilon_{i0}^2})^{1/2}} \right) \right|, \quad (S17)$$

which also gives the values of $C_{CS}^{R \rightarrow L} = 2\pi L_{\text{eff}}(\epsilon_{i0}/\epsilon_{k0})$ and $C_{CS}^{L \rightarrow R} = 0$ at the EP, equivalent to those of Eqs. (S14) and (S15).

Figure S2 shows the behaviors of $C_{CS}^{R \rightarrow L}$ and $C_{CS}^{L \rightarrow R}$ with a different ϵ_{k0} as functions of $(\epsilon_{i0}/\epsilon_{k0})$ before the EP ($\epsilon_{i0} < \epsilon_{k0}$) and

after the EP ($\epsilon_{i0} > \epsilon_{k0}$). From the unequal strength of $C_{CS}^{R \rightarrow L} > C_{CS}^{L \rightarrow R}$, there exists an LCP-favored chiral conversion before the EP, although it is much weaker than that at the EP. It is noted that the oscillatory behavior of C_{CS} before the EP is determined by ϵ_{k0} .

Fig. S2. $C_{CS}^{R \rightarrow L}$ and $C_{CS}^{L \rightarrow R}$ as functions of $(\epsilon_{i0}/\epsilon_{k0})$ for the cases of $\epsilon_{k0} = \epsilon_{i0}/1000$ and $\epsilon_{k0} = \epsilon_{i0}/200$. $\epsilon_{i0} = 6.5$ and $L_{\text{eff}} = 10^3$.

5. The state of polarization (SOP) at the EP: Optical spin black hole

For the graphical representation of the SOP, here we apply the Stokes parameter expression (of S_0, S_1, S_2 , and S_3) on the Poincaré sphere, which follows the relations below:

$$\begin{aligned} S_0 &= |E_x|^2 + |E_y|^2, \\ S_1 &= |E_x|^2 - |E_y|^2, \\ S_2 &= 2 \operatorname{Re}[E_x^* E_y], \\ S_3 &= 2 \operatorname{Im}[E_x^* E_y], \end{aligned} \quad (S18)$$

where S_0 is the radius of the Poincaré sphere, and (S_1, S_2, S_3) is the Cartesian coordinate of the SOP on the sphere. To describe the general tendency of the SOP at the EP, we assume the 400 randomly polarized incidences ($E_x = a + bi$ and $E_y = c + di$ for the incidence where a, b, c , and d are real numbers with the uniform random distribution in $[-1, 1]$) on the PT-symmetric optical potential for Fig. 2c in the main article.

6. Low-dimensional linear polarization

Because the formulation of Eq. (S1) is based on the general framework for two-level PT-symmetric potentials [2,4,5], our analysis of the low-dimensional polarization can be extended beyond the case of chirality treated in the main article. For

example, instead of mixing amplifying y-LP and decaying z-LP modes, consider the mixing of amplifying RCP and decaying LCP modes, which is possible with the recent development of active chiral materials [6,7] and circular dichroism [8,9]. The eigenmodes of the PT-symmetric material are then expressed as $\text{veig}_{1,2} = \eta_{1,2} \cdot \{\epsilon_{k0}[1, i] + (-i\epsilon_{i0} \pm \lambda_{PT})[1, -i]\}^T = \eta_{1,2} \cdot [\epsilon_{k0} - i\epsilon_{i0} \pm \lambda_{PT}, -i(-\epsilon_{k0} - i\epsilon_{i0} \pm \lambda_{PT})]^T$, while the eigenvalues are the same as in Figs 1a and 1b in the main article.

Figure S3 shows the corresponding profiles of the eigenpolarizations for points d-h. For the Hermitian case (point d), the eigenmodes are linearly polarized (LP) due to the even and odd couplings of the RCP and LCP modes. When ϵ_{i0} increases ($0 < \epsilon_{i0} < \epsilon_{k0}$, point e), the eigenmodes begin to converge. At the EP ($\epsilon_{i0} = \epsilon_{k0}$, point f), two LP eigenmodes have coalesced, and the reduction to a 1-dimensional LP basis is evident. After the EP ($\epsilon_{i0} > \epsilon_{k0}$, points g and h), each eigenmode is saturated to a CP mode (RCP, amplifying, and LCP, decaying).

Because a low-dimensional eigenstate can be designed deliberately in the form both of CP (main article, poles on the Poincaré sphere) and LP modes (Fig. S3, the equator on the Poincaré sphere), all of the states on the Poincaré sphere can be designed as the low-dimensional EP, by combining the linear loss and circular dichroism. Therefore, the dynamics of optical spin black hole can be achieved on the entire Poincaré sphere.

Fig. S3. Spatial evolutions of eigenmodes in the low-dimensional LP material corresponding to points d-h in Fig. 1a,b in the main article. At the exceptional point f, the complex eigenmode has the singular form of an LP state ($y-z$). $\epsilon_{i0} = 12.25$, and $\epsilon_{k0} = \epsilon_{i0}/10^3 > 0$.

7. Chirality reversal for LP incidences

Focusing on the vicinity of the EP, chirality reversal can be achieved for the SOP of linear polarization (LP) incidences. For the LP incidence $\mathbf{E}_{\text{inc}} = (E_{\text{RL}}, E_{\text{LL}})_{\text{CP}}^T = (1/2)^{1/2} \cdot E_0 \cdot (e^{-i\theta}, e^{i\theta})_{\text{CP}}^T$ of an arbitrary polarized angle θ , the transmitted field $\mathbf{E}_{\text{tm}}$ at the EP is obtained from the transfer matrix $\mathbf{M}_{\text{PT}}$ as

$$\mathbf{E}_{\text{tm}} = \begin{pmatrix} E_{\text{RT}} \\ E_{\text{LT}} \end{pmatrix} = \frac{E_0}{\sqrt{2}} \cdot e^{-i2\pi\sqrt{\epsilon_{\text{R0}}d}/\Lambda_0} \cdot \begin{pmatrix} e^{-i\theta} \\ e^{i\theta} + C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} \cdot e^{-i\theta} \end{pmatrix}. \quad (\text{S19})$$

From Eq. (S19), it is clear that LCP transmission can be controlled by the polarized angle θ of LP and $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}}$ (Fig. S4a), whereas RCP transmission is invariant (see Movie S2 for the SOP evolutions by the degree of PT symmetry with LP incidences; $\theta = 0, \pi/2, \pi$, and $3\pi/2$). In the strong chiral conversion regime of $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} > 2$ (outside the dotted circle in Fig. S4a), the transmitted wave is always left-handed ($|E_{\text{LT}}| > |E_{\text{RT}}|$) for all angles of θ . In contrast, in the regime of $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} \leq 2$, *chirality reversal* to a right-handed output is permitted for input angles of $\cos(2\theta) < -C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}/2$. A pure RCP transmission can also be achieved in the special case of $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} = 1$ and z-LP ($\theta = 90^\circ$, on the solid circle in Fig. S4a), which is counterintuitive regarding the singular existence of the LCP modal helix. This paradoxical result arises from the unidirectional ($\text{R} \rightarrow \text{L}$) intermodal transfer, which leads to a completely destructive interference for the LCP mode only ($e^{i\theta} + C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} \cdot e^{-i\theta}$). Note that the phenomena of CP interference and chirality reversal are absent in conventional chiral materials [10] which are based on *uncoupled* LCP and RCP modes ($t_{\text{L} \rightarrow \text{R}} = t_{\text{R} \rightarrow \text{L}} = 0$), and thus, they maintain the LP state with natural optical rotation during propagation, *a.k.a.* optical activity. In practice, the regime of chirality reversal can be controlled by changing L_{eff} or ϵ_{R0} (Fig. S4b) based on the definition of $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} (=2\pi L_{\text{eff}}(\epsilon_{\text{R0}}/\epsilon_{\text{I0}}))$.

$$\epsilon_{\text{ry}}(\omega) = \epsilon_{\text{poly}} + \frac{\omega_{\text{py}}^2}{\omega_{0\text{y}}^2 - \omega^2 + i\gamma_{\text{y}}\omega},$$

where $\epsilon_{\text{poly}} = 3.238 - 0.144i$ is the permittivity of the polyimide, $\omega_{0(\text{y/z})}$ is the characteristic frequency, $\gamma_{(\text{y/z})}$ is the damping coefficient, and $\omega_{\text{p}(\text{y/z})}$ is the plasma frequency of the metamaterial for y- and z-polarizations. Because the near-field intensity distribution is

$$\epsilon_{\text{rz}}(\omega) = \epsilon_{\text{poly}} + \frac{\omega_{\text{pz}}^2}{\omega_{0\text{z}}^2 - \omega^2 + i\gamma_{\text{z}}\omega} \quad (\text{S20})$$

widely separated for each polarization (Figs S5c and S5d), the Lorentz permittivity curve of each polarization can be detuned independently to realize the required anisotropic metamaterial.

Fig. S5. Crossed I-shaped metamaterials. (a) Metamaterial realization of Fig. 4a in the main article using crossed I-shaped gold structures (tilted by θ) inside the polyimide. (b) 2D cross-sectional schematic view of the structure composed of gold (yellow) inside the polyimide (white). The tilted angle between patches is $\theta = 0^\circ$, and all the other parameters are the same as those in Fig. 4 in the main article. The red dashed box represents the unit-cell metamaterial. The electrical intensity distribution calculated by COMSOL ($|E|^2$ in dB scale) is shown for (c) y-polarized and (d) z-polarized planewave incidences at the peak frequency of the permittivity curves in Fig. 4c in the main article.

Fig. S4. Chirality reversal for LP incidences. (a) The ratio of $I_{\text{L}}/I_{\text{R}}$ at the EP for LP incidences is shown in (a) as a function of $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}}$ and the polarized angle θ of LP and in (b) as a function of coupling permittivity ($\epsilon_{\text{I0}}/\epsilon_{\text{R0}}$). The black dotted arrows denote the points at which $C_{\text{CS}^{\text{R} \rightarrow \text{L}}|_{\text{EP}}} = 1$. All the results are based on the transfer matrix method. $\epsilon_{\text{R0}} = 6.5$ and $L_{\text{eff}} = 10^3$ for (a).

8. Realization of PT-symmetric permittivity in metamaterial platforms

To transplant the point-wise anisotropic permittivity of a PT-symmetric chiral material into the structure composed of isotropic materials, we first consider a metamaterial platform in a THz regime. As a unit element, we adopt the I-shaped patch (Fig. S5a) for each polarization, which induces the enhanced light-matter interaction through the deep subwavelength ($\sim \lambda_0/200$) gap structure [11]. By crossing the y- and z-unit elements (Fig. S5b), we obtain the metamaterial, which supports the well-known electrical response of the Lorentz model both for y- and z-polarizations, as

As noted in the main article, we utilize the low-loss ($|Re[\epsilon]| \gg |Im[\epsilon]|$) and high-loss ($|Re[\epsilon]| \sim |Im[\epsilon]|$) dielectric regimes of the Lorentz response to obtain the passive PT symmetry condition of $Re[\epsilon_y] = Re[\epsilon_z]$ and $Im[\epsilon_z] < Im[\epsilon_y] < 0$ (see Note S2). Because the modal chirality is critically dependent on the condition of $Re[\epsilon_y] = Re[\epsilon_z]$ (see Note S2), we need to intervene the real part of the

Lorentz curves. To achieve this, the effect of varying the physical quantities of the Lorentz model (ω_0 , ω_p , and γ) is shown in Fig. S6. Also corresponding to the Eq. (S20), it is necessary to change the characteristic frequency of ω_0 for the spectral shift (Fig. S6a), while the magnitude of the permittivity is mainly tuned with the plasma frequency ω_p (Fig. S6b compared to S6c).

Fig. S6. The effect of changing the physical quantities (ω_0 , ω_p , and γ) in the Lorentz model. The permittivities $\epsilon(\omega)$ are shown for the changes of (a) the characteristic frequency ω_0 (black: ω_{00} , red: $1.2 \cdot \omega_{00}$, blue: $0.8 \cdot \omega_{00}$), (b) the plasma frequency ω_p (black: ω_{p0} , red: $1.2 \cdot \omega_{p0}$, blue: $0.8 \cdot \omega_{p0}$), and (c) the damping coefficient γ (black: γ_0 , red: $1.2 \cdot \gamma_0$, blue: $0.8 \cdot \gamma_0$). $\omega_{00} = 1.27$ THz, $\omega_{p0} = 8.2$ THz, and $\gamma_0 = 0.11$ THz. Solid (dotted) lines denote the $Re[\epsilon]$ ($Im[\epsilon]$).

By considering the results of Fig. S6, we investigate the role of structural parameters (a , g , L , and w of Fig. S5a) for the design of PT-symmetric permittivity relating to physical quantities (ω_0 , ω_p , and γ). To satisfy the condition of $Re[\epsilon_y] = Re[\epsilon_z]$ and $Im[\epsilon_z] < Im[\epsilon_y] < 0$ with two Lorentz curves in the dielectric regime, it is necessary to introduce the frequency shift between these curves by designing different characteristic frequencies ($\omega_{01} \neq \omega_{02}$) for I-shaped patches 1 and 2. To observe the EP with the spectral stability, we consider only the cases of the broadband design of EP (black circles in Fig. S7a,b), neglecting cases that are too sensitive (green circles in Fig. S7a,b). If I-shaped patch 1 in the high-frequency regime (ω_{01}) supports a lower plasma frequency than that of I-shaped patch 2 ($\omega_{p1} < \omega_{p2}$, Fig. S7a), the condition with broad bandwidth is satisfied at the metallic state (Fig. 4e in the main article), which supports an evanescent mode inside the PT-symmetric metamaterial. Meanwhile, if the design satisfying $\omega_{01} > \omega_{02}$ and $\omega_{p1} > \omega_{p2}$ is applied simultaneously (Fig. S7b), the

propagating mode can be utilized to observe the low-dimensional chirality (Fig. 4c in the main article).

As shown in Fig. S7c-d, increasing the parameters related to the 'length' of the patch (L and a) increases the plasma frequency ω_p due to the large number of participating electrons (note that $\omega_p^2 \sim Ne^2/(m\epsilon_0)$, where N is the density of electrons), while the characteristic frequency decreases due to the weak restoring force. Similarly, decreasing gap g (Fig. S7e) increases the attractive force, yielding the reversed response of $\omega_0 \downarrow$ and $\omega_p \uparrow$. Therefore, the strategy for low-dimensional evanescent waves (Fig. S7a) can be achieved with the manipulation of L , a , or g . However, by widening the patches (Fig. S7f), we can increase the participating electrons ($\omega_p \uparrow$) and the restoring force ($\omega_0 \uparrow$) simultaneously, enabling the case of Fig. S7b, with low-dimensional propagating waves. Now, by changing the tilted angle θ (Fig. S5a), we can obtain the low-dimensional chiral dynamics at the EP by obtaining the coupling between the y - and z -polarizations for both cases.

Fig. S7. The effect of changing the structural parameters. The strategy for satisfying the condition of PT symmetry with (a) $\omega_{01} > \omega_{02}$ and $\omega_{p1} < \omega_{p2}$ (low-dimensional evanescent waves) and (b) $\omega_{01} < \omega_{02}$ and $\omega_{p1} > \omega_{p2}$ (low-dimensional propagating waves). Black (or green) circles denote the condition of low-dimensional EP with broad (or narrow) bandwidths. The COMSOL-calculated permittivities $\epsilon(\omega)$ are shown for the changes of (c) length L , (d) arm length a , (e) gap thickness g , and (f) width w . The unchanged parameters are $L = 30 \mu\text{m}$, $a = 20 \mu\text{m}$, $g = 1.5 \mu\text{m}$, and $w = 5 \mu\text{m}$.

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