Nonclassical correlations between a C-band telecom photon and a stored spin-wave: supplementary material

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1. THEORETICAL MODEL FOR THE SECOND-ORDER CROSS CORRELATION FUNCTION

As mentioned in the main text, the second order cross-correlation function between the frequency converted write photons and the read photons, is related to the photon detection probabilities as $g_{cw,r}^{(2)} = p_{cw,r}/(p_{cw}p_r)$. During the frequency conversion process, the write photons experience two kinds of imperfections: the first one is imperfect transmission and the second one is that they are mixed with noise photons (coming mainly from residual pump light and detector dark counts). Considering these two effects, we can rewrite the photon detection probabilities as $p_{cw} = \eta_{QFC}p_w + p_N$ and $p_{cw,r} = \eta_{QFC}p_{w,r} + p_Np_r$. In these expressions η_{QFC} is the total efficiency of the quantum frequency conversion device (QFCD), p_w is the probability that a write photon arrives at the input of the QFCD and p_N is the probability to detect a noise photon. These effects can be included in the expression of the cross-correlation function.

$$g_{cw,r}^{(2)} = \frac{\eta_{QFC}p_{w,r} + p_Np_r}{(\eta_{QFC}p_w + p_N)p_r} = \frac{\frac{p_{w,r}}{p_w p_r} + \frac{p_N}{\eta_{QFC}p_w}}{1 + \frac{p_N}{\eta_{QFC}p_w}} \quad (S1)$$

From the previous expression we can identify two terms. The first one is the cross-correlation of the fields without any frequency conversion $g_{w,r}^{(2)} = p_{w,r}/(p_w p_r)$ and the other one is

the signal-to-noise-ratio of the frequency converted photons $SNR = (p_{cw} - p_N)/p_N = \eta_{QFC}p_w/p_N$. Introducing this terms in eq. (S1) leads to the expression of eq. (1) from the main text

$$g_{cw,r}^{(2)} = \frac{g_{w,r}^{(2)} + SNR^{-1}}{1 + SNR^{-1}} \quad (S2)$$

2. MEASUREMENTS WITHOUT FREQUENCY CONVERSION

The blue shaded area in Fig. 3 of the main text shows the cross-correlation values that we expect after the frequency conversion of the write photons. As mentioned in the previous section, the way we obtain the expected $g_{cw,r}^{(2)}$ is by considering the cross-correlation values without the frequency conversion ($g_{w,r}^{(2)}$) and the imperfections that the QFC process would introduce. The measured values of $g_{w,r}^{(2)}$ are shown in Fig. S1. In order to take the data in the correct conditions (e.g. without additional noise), the write photons were filtered with a Fabry-Perot cavity with similar characteristics as the one used for the read photons (see main text).

Fig. S1. Second order cross-correlation function of the write and read photon fields, without frequency conversion.

3. THEORETICAL MODEL FOR THE DATA AS A FUNCTION OF THE STORAGE TIME

Due to collective interference, spin-waves can be converted with a high efficiency into read photons emitted in a particular transition and direction. The ratio between the directional emission and the random emission (e.g. spontaneous emission) depends, among other factors, on the preservation of the spin-wave coherence. In our experiment this coherence is mainly affected by the random motion of the atoms, which induces a spin-wave dephasing that decreases the ratio between directional and random emission when the storage time increases. The effect of this dephasing on η_{ret} and $g_{cw,r}^{(2)}$ can be seen in Fig. 4 of the main text.

In order to find theoretical expressions for these quantities depending on the storage time, we start by writing them as a function of the photon detection probabilities $\eta_{ret} = p_{cw,r}/p_w$ and $g_{cw,r}^{(2)} = p_{cw,r}/(p_{cw}p_r)$. Since the ratio between the read photon directional and random emission will depend on the storage time, we can write $p_r(t) = p_r^{dir}(t) + p_r^{rand}(t)$ and $p_{cw,r}(t) = p_{cw,r}^{dir}(t) + p_{cw,r}^{rand}(t)$. The ratio of directional emission can be characterized by the intrinsic retrieval efficiency $\eta_{ret}^I(t)$, while the random emission is proportional to the total number of atoms in the $|s\rangle$ ground state (N_s), the branching ratio of the read photon atomic transition ($\zeta_g = 1/6$) and the read photon spatial mode solid angle ($\Delta\Omega_r$). Hence, we can write all the photon detection probabilities as:

$$p_{cw} = p\eta_{cw} \quad (\text{S3})$$

$$p_r(t) = p\eta_{ret}^I(t)\eta_r + N_s[1 - \eta_{ret}^I(t)]\frac{\Delta\Omega_r}{4\pi}\zeta_g\eta_r \quad (\text{S4})$$

$$p_{cw,r}(t) = p\eta_{ret}^I(t)\eta_{cw}\eta_r + p\eta_{cw}N_s[1 - \eta_{ret}^I(t)]\frac{\Delta\Omega_r}{4\pi}\zeta_g\eta_r \quad (\text{S5})$$

where p is the probability to create a spin-wave together with a write photon in the coupled spatial mode and $\eta_{cw(r)}$ are the write (read) photon total detection efficiencies. In the previous equations we have taken into account non-unity detection efficiencies, but we have not considered any noise. The reason is that we experimentally measured that the probability to detect a noise count for the data in Fig. 4 is much lower than the probability to detect a real photon. The noise detection probabilities for the write and read photon modes are

$$p_{Nw} = 2.30 \cdot 10^{-5} \text{ and } p_{Nr} = 7.8 \cdot 10^{-5}, \text{ respectively.}$$

Since write photons and excitations in state $|s\rangle$ are created in pairs, N_s can be obtained from p and the write photon spatial mode solid angle $\Delta\Omega_w$, with the expression $N_s = p4\pi/\Delta\Omega_w$. In our experiment the solid angle of the write and read photonic modes are the same ($\Delta\Omega_w = \Delta\Omega_r$).

The storage time dependence of the directional emission will be represented by $\eta_{ret}^I(t)$ which is proportional to the overlap between the state of the dephased spin-wave with the unperturbed one: $\eta_{ret}^I(t) \propto |\langle\Psi(0)|\Psi(t)\rangle|^2$. As explained in literature [1, 2], in the presence of random atomic motion dephasing, the spin-wave state evolves with time as

$$|\Psi(t)\rangle = \frac{1}{N} \sum_j e^{i(\mathbf{k}_w - \mathbf{k}_w)(\mathbf{x}_j + \mathbf{v}_j t)} |g_1 \dots s_j \dots g_N\rangle \quad (\text{S6})$$

where $\mathbf{k}_w$ is the wave vector of the write pulse (write photon), N is the total number of atoms, $\mathbf{x}_j$ is the position of atom j at the moment of the write process and $\mathbf{v}_j$ its velocity. Assuming a Maxwell-Boltzmann distribution for the atomic velocities, the temporal dependence of the intrinsic retrieval efficiency becomes $\eta_{ret}^I(t) = \eta_{ret}^I e^{-t^2/\tau^2}$ with $\tau = \sqrt{m/(k_B T \Delta k^2)}$, where m is the atomic mass, k_B is the Boltzmann constant, T is the atomic temperature and $\Delta k = |\mathbf{k}_w - \mathbf{k}_w|$ [2].

Introducing the previous expressions in equations (S3)-(S5), we can finally write the theoretical formulas used for the fits in Fig. 4 of the main text.

$$\eta_{ret} = \eta_r \left[\eta_{ret}^I e^{-t^2/\tau^2} (1 - p\zeta_g) + p\zeta_g \right] \quad (\text{S7})$$

$$g_{cw,r}^{(2)} = 1 + \frac{\eta_{ret}^I e^{-t^2/\tau^2} (1 - p)}{p \left[\eta_{ret}^I e^{-t^2/\tau^2} (1 - \zeta_g) + \zeta_g \right]} \quad (\text{S8})$$

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