Subsampled scanning holographic imaging (SuSHI) for fast, non-adaptive recording of three-dimensional objects: supplementary material

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S1. NONLINEAR HETERODYNE DETECTION FOR ALL-DIGITAL PHASE EXTRACTION

In conventional optical scanning holography (OSH), phase information is measured from the heterodyne beat frequency generated from a coherent light source and another frequency-shifted light source [1]. This is usually implemented with acousto-optic technology, which often results in a beat frequency at around 100 MHz. Phase extraction requires either a full-analog lock-in amplifier or a second frequency shifter to down adjust the modulation frequency to match the bandwidth of the digitizer — both are prohibitively expensive for a compressed-sensing project and adds to the complexity of the hardware. By adopting nonlinear heterodyne detection technique [2, 3], we realize direct signal acquisition with an electro-optic phase modulator driven by a sinusoidal waveform at 50 kHz frequency. Such implementation enables all-digital phase extraction at 150 kHz sampling rate.

Fig. S1 shows the setup for time-varying FZP generation, which is labelled in Fig. 1 of the main text. A Mach-Zehnder configuration is shown with a phase-modulation unit in one path, and a plano-convex lens in another path. The electro-optic phase modulator (Thorlabs EO-PM-NR-C1) is driven with an electronic signal with a sinusoidal waveform by the digital frequency synthesizer (Agilent 33220A). Note that this is in contrast to conventional approach that feeds a sawtooth signal [4, 5]; the latter approach requires an extra photodetector for balanced detection. After the beam-splitter BS1, the wave function describes a time-varying FZP pattern in the 3D coordinate system (u, v, z) , which is given by

$$U(u, v, z, t) = A(u, v, z)e^{j\omega t + j\phi(u, v, z)} + A_r(u, v)e^{j\omega t + j\beta \sin(\Omega t + \phi_d)}, \quad (\text{S1})$$

where $\omega = 2\pi c/\lambda$ corresponds to the wavelength λ of the light

Fig. S1. Mach-Zehnder interferometer for time-varying FZP generation. HWP: half wave plate; PBS: polarizing beam splitter; BS: beam splitter; EOPM: electro-optic phase modulator; BE: beam expander; L1: plano-convex lens; M: plane mirror.

source, Ω is the phase-modulation frequency, and β is the modulation depth. The quantity ϕ_d is the constant phase drift which is a function of electronic delay in the modulation control. The amplitude $A_r(u, v)$ corresponds to the intensity profile of the collimated laser beam; the depth-dependent amplitude $A(u, v, z)$ and phase $\phi(u, v, z)$ is the result of the 2D quadratic phase generated by the plano-convex lens. As discussed in the introduction of the main text, the plano-convex lens can be replaced with a custom-designed phase mask to generate advanced illumination patterns.

For the sake of simplicity, we assume the sample in Fig. 1 is a pinhole fixed at coordinates (x_0, y_0, z_0) . When the 2D galvo-scanning system moves the FZP to position (x, y) , the detected

Fig. S2. Phase extraction of the hologram with a non-linear heterodyne detection. The object is a 50 μm pinhole fixed at a predefined location (x_0, y_0, z_0) in place of the microscope slide in Fig. 1c. (a) Quadrature and (b) in-phase component of the hologram extracted from the fundamental frequency and the second harmonics respectively. (d,e) Corresponding scatter plot of all the pixels in (a) and (b). The relative scaling and rotation are compensated by principal component analysis (PCA). (c) The hologram restored by fusing the first principal components from (d) and (e), as expressed in Eq. (S5). (f) Phase map of the hologram in (c).

photocurrent is

$$\begin{aligned} i(x, y, t) &= \int_{-\infty}^{+\infty} U(x - x_0, y - y_0, z, t) U^*(x - x_0, y - y_0, z, t) dz \\ &= |A(x - x_0, y - y_0, z_0) e^{j\phi(x - x_0, y - y_0, z_0)} \\ &\quad + A_r(x - x_0, y - y_0) e^{j\beta \sin(\Omega t + \phi_d)}|. \end{aligned} \quad (\text{S2})$$

The challenge here is to extract holographic amplitude and phase from the optical beating signal with lock-in amplifier. For the sake of brevity, we define $A_0 := A(x - x_0, y - y_0, z_0) / A_r(x - x_0, y - y_0)$ and $\phi_0 := \phi(x - x_0, y - y_0, z_0)$. Applying Jacobi-Anger expansion on the phase modulation term, we have

$$\begin{aligned} i(x, y, t) &\propto C + (2J_1 A_0 \sin \phi_0) \sin(\Omega t + \phi_d) - \\ &\quad (4J_0 J_2 - 2J_1^2 + 4J_2 A_0 \cos \phi_0) \cos(2\Omega t + 2\phi_d) + \dots \\ &\approx C + (2J_1 A_0 \sin \phi_0) \sin(\Omega t + \phi_d) - \\ &\quad (4J_2 A_0 \cos \phi_0) \cos(2\Omega t + 2\phi_d), \end{aligned} \quad (\text{S3})$$

when $A_0 \gg |J_0|$. Here, C and $J_{0,1,2}$ are constants. Since only the quadrature component ($A_0 \sin \phi_0$) is preserved in the fundamental frequency, phase cannot be extracted from it alone (refer to Fig. S2a). Instead, we extract the complex-valued coefficients of both the fundamental and second harmonics $C_\Omega, C_{2\Omega}$ by digital heterodyne detection, i.e.

$$\begin{aligned} C_\Omega(x, y) &= \frac{1}{T} \int_{-T/2}^{T/2} i(x, y, t) e^{-j\Omega t} dt \\ &\propto [A_0 \sin \phi_0] \left(2J_1 e^{j\phi_d + j\pi/4} \right) \end{aligned} \quad (\text{S4a})$$

$$\begin{aligned} C_{2\Omega}(x, y) &= \frac{1}{T} \int_{-T/2}^{T/2} i(x, y, t) e^{-j2\Omega t} dt \\ &\propto [A_0 \cos \phi_0] \left(4J_2 e^{j2\phi_d + j\pi/2} \right), \end{aligned} \quad (\text{S4b})$$

Fig. S3. Objective function $f(z)$ for the digital focusing of microbeads. Inset: The filtered image $g(\bar{h} * I)$ at depth (a) 33.0 mm, (b) 42.4 mm, and (c) 50.0 mm correspondingly.

where the pixel dwell time $T \gg 1/\Omega$. The constant phase drift ϕ_d and the relative scales $J_{1,2}$ can be compensated by extracting the first component from principal component analysis (PCA) of all the hologram pixels (Fig. S2d-e), treating real part and imaginary part of the coefficients as separate measurements. The complex-valued hologram in Eq. 1 is now evaluated as

$$I(x, y) = \frac{C_{2\Omega}(x, y)}{R_{2\Omega} e^{j2\phi_d}} + j \frac{C_\Omega(x, y)}{R_\Omega e^{j\phi_d}}, \quad (\text{S5})$$

where $\{R_\Omega, R_{2\Omega}\}$ and $\{\phi_\Omega, \phi_{2\Omega}\}$ are the relative scales and phase drifts estimated by PCA, depicted as the solid straight line in Figs. S2d-e.

To verify the above equations, we acquired and digitized the heterodyne signal by raster-scanning a pinhole object in our setup. The microscope slide in Fig. 1b of the main text is replaced with the 50 μm pinhole (Thorlabs P50S) stacked onto the fluorescence reference slide (Ted Pella 2273-R); the pinhole aperture is fixed at unknown 3D coordinates (x_0, y_0, z_0) . We set the phase-modulation frequency $\Omega = 2\pi \times 50$ kHz, and the modulation depth $\beta = 0.4\pi$. The components C_Ω and $C_{2\Omega}$ are extracted by a software lock-in amplifier. Fig. S2c demonstrates a good extraction of the complex-valued point spread function of the system, i.e. the Gaussian intensity profile and quadratic phase profile (Fig. S2f) of the FZP.

S2. AUTOFOCUSING OF SUBSAMPLED HOLOGRAM WITH SPIRAL-SCANNING TRAJECTORY

We formulate depth estimation as an ℓ_1 -norm minimization problem [6]. We wish to find all local minima in the objective function

$$f(z) = \left\| g[\bar{h}(x, y, z) * I(x, y)] \right\|_1, \quad (\text{S6})$$

within a pre-defined range of depth (Fig. S3). Here $g(\cdot)$ is the nonlinear filter which suppresses out-of-focus objects and measurement noise [7, 8]. Meanwhile, $I(x, y)$ defines the subsampled hologram at which all hologram pixels are filled with zeros except where measurements are acquired along the spiral trajectory. This is analogous to the common practice of image interpolation by zero-padding the signal in Fourier domain. For the binary object in Section 3.B of the main text, the nonlinear filter $g(\cdot)$ is the soft-thresholding of the gradient of the image [7].

For microbead object in Section 3.C of the main text, the filter is defined as the real part of the normalized image

$$g(I_1) = \text{Re} \left\{ \frac{I_1}{\|I_1\|_\infty} \right\}. \quad (\text{S7})$$

S3. SOLVING EQ. (7) WITH ALTERNATING DIRECTION METHOD OF MULTIPLIERS (ADMM)

Eq. (7) is called ℓ_1 -analysis problem because the cross-correlation operation in the regularization term is non-invertible [9]. All existing algorithms that solve the ℓ_1 -analysis problem require that either the hologram measurement operation fulfills the isometry property [i.e. $\overline{h_i(x, y)} * h_i(x, y) = \delta(x, y)$] [9], or that the sparsifying operator $W(\cdot)$ is some standard function, e.g. total-variation [10]. Nevertheless, Eq. (7) can be solved by noting that the average fluorescence beads diameter $2h_\sigma$ is sufficiently small compared to the scanning field-of-view. In this case, we can approximate the FZP as

$$\begin{aligned} h(x, y) &= e^{r^2/(2W^2) + jr^2} \\ &\approx e^{r^2/(2W_1^2) + jr^2} * e^{r^2/(2\sigma^2)} \\ &= h_1(x, y) * \overline{h_\sigma(x, y)}, \end{aligned} \quad (\text{S8})$$

where $r = \sqrt{x^2 + y^2}$, $W_1 = \sqrt{W^2 - \sigma^2}$. Substituting back to Eq. (7), we have

$$\hat{s} = \arg \min_s \frac{1}{2} \|I_S - B_S(h_1 * s)\|_2^2 + \alpha \|s\|. \quad (\text{S9})$$

where $s = \overline{h_\sigma} * O$, and O is the the microbeads at depth z estimated in Section S2. Now it can be solved by either gradient projection or the augmented Lagrangian. In this paper, an alternating direction method of multipliers (ADMM) algorithm (YALL1) is adopted [11]. The approximated object can be restored as $\hat{O} = h_\sigma * \hat{s}$.

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