Interdimensional optical isospectrality inspired by graph networks: supplementary material

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This document provides supplementary information to “Interdimensional optical isospectrality inspired by graph networks,” <http://dx.doi.org/10.1364/optica.3.000836>. Additional details are given about the effects of higher-order couplings, the calculation of absorption spectra, and the multimode coupling between 1D and 2D structures.

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S1. Effects of higher-order couplings

Here we analyze the effect of higher-order couplings [1] in 3D real structures. Figure S1 shows the graph representations (Fig. S1a-c) and level statistics (Fig. S1d-f) of the ordered structure in Fig. 3a in the main manuscript, for the inclusion of nearest-neighbor (NN) coupling (Fig. S1a,d), NN and next-nearest-neighbor (NNN) couplings (Fig. S1b,e), and all orders of couplings (Fig. S1c,f). Despite their smaller values, higher-order couplings have a strong impact on the level statistics (Fig. S1d-f). Higher-order couplings also reduce the level degeneracy due to the broken symmetry from different values for each order of coupling, in contrast to the case of hypothetical graph systems with identical multilevel edges which derive the level degeneracy (Fig. 2a in the main manuscript).

Fig. S1. Effects of higher-order coupling on the level statistics of the ordered structure. (a-c) Graphs and (d-f) level statistics for the inclusion of (a,d) NN coupling, (b,e) NN and NNN couplings, and (c,f) all orders of couplings for the ordered structure in Fig. 3a.

Figure S2 represents the case of the disordered structure in Fig. 3b in the main manuscript. Figure S2a,d includes only the couplings smaller than that between elements with the distance r_0 ,

while Fig. S2b,e includes the couplings smaller than that between elements with the distance $2^{1/2} \cdot r_0$ (all orders of couplings for Fig. S2c,f). As seen, the change of the level statistics according to the considered orders of couplings is apparent.

Fig. S2. Effects of higher-order coupling on the level statistics of the disordered structure. (a-c) Graphs and (d-f) level statistics for the inclusion of (a,d) NN coupling, (b,e) NN and NNN couplings, and (c,f) all orders of couplings for the disordered structure in Fig. 3b.

S2. Calculation of absorption spectra

To obtain absorption spectra of the 3D structure and its isospectral 1D partner, we assume lossy elements ($\text{Im}\{\rho_0\} > 0$) of the 3D one. Each element is assumed to have the intrinsic Q -factor of $Q = 0.5 \cdot \text{Re}\{\rho_0\} / \text{Im}\{\rho_0\} = 500$. From the eigenvalue equation $H\psi = E\psi$, we can obtain the set of complex eigenvalues E_j ($j = 1, 2, \dots, N$). The corresponding absorption spectrum [2] $A(\rho)$ neglecting the interference between eigenmodes then becomes

$$A(\rho) = \sum_{j=1}^N \frac{1}{[\rho - \text{Re}\{E_j\}]^2 + [\text{Im}\{E_j\}]^2}.$$

The absorption spectrum of the 1D partner potential can also be obtained from the eigenvalue equation $H_1\psi = E\psi$ where $H_1 = MHM^T$. Figure 4c in the main manuscript shows the normalized absorption spectra of the 3D structure (lines) and its isospectral 1D partner (symbols).

S3. Inter-dimensional multimode coupling

The isospectrality in multimode structures provide the global phase matching condition [3,4]: the simultaneous matching of eigenvalues, allowing the power transfer for all of multimodes. As an application example, here we demonstrate the multimode coupling between different dimensions (2D-to-1D) for the first time, by utilizing the global phase matching condition from the inter-dimensional isospectrality. Figure S3a shows the proposed structure. From the original 2D structure composed of identical optical atoms (blue dashed circle), we achieve the 1D isospectral partner structure (red dashed ellipse), by following the transformation $H_1 = MHM^T$ in the main manuscript (Fig. S3b for the self-energy and coupling strength distribution of the 1D structure; Fig. S3c for the identical level statistics of both structures). We note that 2D and 1D structures are coupled through a single atom of each structure, with the coupling coefficient κ_d . To preserve the global phase matching condition, the strength of κ_d is smaller than internal couplings in 2D and 1D structures as $\kappa_d < \kappa_{ij}$.

Fig. S3. The design of the inter-dimensional multimode coupling. (a) A schematic for 2D and 1D structures. For the optical atoms in the 2D structure, we use the same parameters as those in Fig. 3 in the main manuscript: $\kappa = 0.236 \cdot \exp(-0.451 \cdot r)$, $r_o = 5$, $\rho_o = 1$, and $\Delta_{xyz} = 2 \cdot \text{unif}(-1, 1)$. The inter-dimensional coupling $\kappa_d = 0.01$. The colors of the elements represent the value of the modification of local resonances $\Delta\rho = \rho - \rho_o$. (b) The distributions of $\Delta\rho$ and κ in the 1D structure. (c) The level statistics of both structures. Red eigenstates for Fig. S4.

Figure S4 shows the coupling between 2D and 1D structures. As an example, the 3rd and 9th eigenstates are excited in the 2D structure. Due to the matching of each eigenvalue, full couplings between different dimensions are successfully achieved for both eigenstates (3rd state: Fig. S4a, 9th state: Fig. S4b). Figure S4c-S4e (S4f-S4h) presents the field distribution for the excitation of the 3rd (9th) eigenstate in the 2D structure. We also note that because the effective coupling between structures is dependent on each modal profile, the beat length is different for each mode.

Fig. S4. Inter-dimensional coupling between 2D and 1D structures. Waveguide optical atoms are assumed, for the implementation of the directional coupling during the propagation. (a,b) Total power flow in each structure for the propagation length L_{prop} . (c-h) Field distributions in both structures at the propagation lengths depicted in (a,b). (a,c-e) 3rd state, (b,f-h) 9th state.

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