Computational temporal ghost imaging: supplementary material

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This document provides supplementary information to “Computational temporal ghost imaging,” <http://dx.doi.org/10.1364/optica.3.000698>. We present in detail the theoretical model used to reconstruct a temporal signal embedded in a single shot spatially multiplexed measurement. Some numerical simulations and additional experimental results are presented. © 2016 Optical Society of America

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1. INTRODUCTION

This supplementary material is organized as follows. Section 2 describes the theoretical model and presents some numerical simulations where the influence of the model parameters on the accuracy of measurement of the time object is studied. Then, section 3 presents the experiments conducted to validate our theoretical model and quantifying performances of our device.

2. THEORETICAL MODEL AND NUMERICAL SIMULATIONS

A. Theoretical model

Let us consider a two dimensional binary random pattern X of $N \times N$ pixels verifying statistical properties of a Bernoulli distribution: a pixel of the pattern takes the value 1 with probability of p and the value 0 with probability of $1 - p$. In that case, the average value $\langle X \rangle$ and the variance σ_X^2 of a Bernoulli random variable are, respectively:

$$\begin{cases} \langle X \rangle = p \\ \sigma_X^2 = p(1 - p) \end{cases} \quad (S1)$$

Now, let us consider a stack of K non correlated binary random patterns X_k of mean p and of variance $p(1 - p)$. These binary patterns can be displayed successively and recorded with a long exposure time device such as the level of a pixel of coordinate

(i, j) of the recorded image S , corresponds to the sum of the successive levels of the same pixel in each patterns of the stack. Now assume that the energy transmission of the imaging system varies during the exposure time of the device. Then, the level of a pixel S_{ij} of the recorded image can be expressed as:

$$S_{ij} = \sum_{k=1}^K T(k) X_{k,ij} \quad (S2)$$

where $T(k)$ is the value of the transmission coefficient when the pattern X_k is displayed. Although the imaging device has no temporal resolution, the value of $T(k_0)$ can be estimated by calculating the correlation coefficient C between the image S and the k_0 pattern X_{k_0} as follows:

$$\begin{aligned} C(k_0) &= \sum_{i=1}^N \sum_{j=1}^N \{S_{ij} - \bar{S}\} \{X_{k_0,ij} - \bar{X}_{k_0}\} \\ &= \sum_{i=1}^N \sum_{j=1}^N \left\{ \sum_{k=1}^K T(k) (X_{k,ij} - \bar{X}_k) \right\} \{X_{k_0,ij} - \bar{X}_{k_0}\} \\ &= \sum_{i=1}^N \sum_{j=1}^N \left\{ \sum_{k=1, k \neq k_0}^K T(k) (X_{k,ij} - \bar{X}_k) (X_{k_0,ij} - \bar{X}_{k_0}) \right. \\ &\quad \left. + T(k_0) (X_{k_0,ij} - \bar{X}_{k_0})^2 \right\} \end{aligned} \quad (S3)$$

where $\bar{S}$ and $\bar{X}_k$ are the arithmetic mean levels of the related images. Because the covariance between two non correlated Bernoulli distributions is null, the mean of the term of the third

line of Eq. S3 is null but its fluctuations contribute to the final uncertainty. Then, the estimated value and variance of the transmission coefficient $T(k_0)$ are estimated by (a hat means "estimator of"):

$$\begin{cases} \hat{T}(k_0) = \frac{C(k_0)}{N^2(p-p^2)} \\ \hat{\sigma}_{T(k_0)}^2 = \frac{\sigma_{C(k_0)}^2}{N^4(p-p^2)^2} \end{cases} \quad (\text{S4})$$

As the pixels of a random pattern are independent, the variance of the total correlation coefficient $C(k_0)$ is equal to the variance of the correlation coefficient of any pixel multiplied by the total number of pixels:

$$\sigma_{C(k_0)}^2 = N^2 \text{Var} \left[\left\{ S_{ij} - \bar{S} \right\} \left\{ X_{k_0,ij} - \bar{X}_{k_0} \right\} \right], \quad (\text{S5})$$

Considering independence property and zero mean of the variable $X_{k,ij} - \bar{X}_k$, Eq. S5 gives:

$$\begin{aligned} \sigma_{C(k_0)}^2 = & N^2 \sum_{k=1, k \neq k_0}^K T^2(k)(p-p^2)^2 \\ & + N^2 T^2(k_0) \text{Var} \left[X_{k_0,ij}^2 - 2X_{k_0,ij}\bar{X}_{k_0} + X_{k_0,ij} - \bar{X}_{k_0}^2 \right] \end{aligned} \quad (\text{S6})$$

If the number of pixels is large enough, the arithmetic mean $\bar{X}_k$ is accurately approximated by the probability p . Then, Eq. S6 becomes:

$$\begin{aligned} \sigma_{C(k_0)}^2 = & N^2 \sum_{k=1, k \neq k_0}^K T^2(k)(p-p^2)^2 \\ & + N^2 T^2(k_0)(p-5p+8p^3-4p^4) \end{aligned} \quad (\text{S7})$$

Then, the variance of the transmission coefficient can be expressed as a function of the symmetrized probability $q = p - 0.5$ as:

$$\hat{\sigma}_{T(k_0)}^2 = \frac{1}{N^2} \left\{ \sum_{k=1, k \neq k_0}^K T^2(k) + T^2(k_0) \frac{4q^2}{0.25 - q^2} \right\} \quad (\text{S8})$$

Eq. S8 clearly shows that the uncertainty in the measurement of the transmission coefficient will be minimized when random patterns with a large number of pixels and with $p = 0.5$ are considered.

Finally, for $p = 0.5$ the signal to noise ratio (SNR) of the measurement of a transmission coefficient $0 \leq T(k_0) \leq 1$ is given by:

$$\text{SNR}[T(k_0)] = \frac{T(k_0)}{\sigma_{T(k_0)}} = \frac{NT(k_0)}{\sqrt{\sum_{k=1, k \neq k_0}^K T^2(k)}} \geq \frac{N}{\sqrt{K-1}} T(k_0) \quad (\text{S9})$$

where K is the number of random patterns.

B. Numerical simulations

Binary random patterns verifying statistical properties of a Bernoulli distribution are easily obtained with a pseudo-random number generator. Fig. S1a shows an example of random pattern with 128×128 pixels where $p = 0.5$. Fig. S1b shows the sum of 20 independent random patterns attenuated uniformly with a coefficient $T = 0.5$. We verified that the mean level and the variance of an individual pattern are in agreement with Eq. S1

and that the mean of the covariance between two independent patterns is null.

In order to validate the theoretical model presented in section A, numerical simulations are performed and the statistical properties of the images are measured as a function of the probability p , the attenuation T and the number of pixels N .

Fig. S1. Random patterns of 128×128 pixels with $p = 0.5$: (a) single random pattern, (b) sum of 20 random patterns attenuated with a factor $T = 0.5$.

First, the transmission coefficient is set as a constant. Stacks of 20 independent random patterns with 128×128 pixels are generated with a probability p varying in the range $[0.1, 0.9]$. For each value of the probability p , random patterns of the stack are uniformly multiplied by the same attenuation coefficient and summed. Then, using Eq. S3, the correlation coefficients C are calculated for each pattern of the stack. From these calculations, means, variances and SNR on measurements of the transmission coefficients are estimated with Eq. S4.

Fig. S2 shows, for the attenuation coefficients $T=0.25, 0.5$ and 0.75 , the mean values (Fig. S2a), variances (Fig. S2b) and SNR (Fig. S2c) on the measurements of T as a function of the probability p . Dotted curves represent the theoretical variations of the related variables. The error bars give the uncertainties of the calculations.

Then, for the probabilities $p=0.1, 0.25$ and 0.5 , a new set of numerical simulations are performed where the number of pixels varies and the attenuation coefficient is constant: $T = 0.5$. Fig. S3 shows, for the different values of p , the mean values (Fig. S3a), variances (Fig. S3b) and SNR (Fig. S3c) on the measurements of T as a function of the number of pixels N . The dotted curves represent the theoretical variations of the related variables. The error bars give the uncertainties of calculations.

From these numerical results, several observations can be made. Firstly, numerical simulations validate the theoretical model with a very good agreement. Secondly, they confirm that the accuracy in the measurement of the transmission coefficient T mainly depends on the total number of independent pixels of the random patterns. Indeed, because using an imaging device to record random patterns with $N \times N$ pixels leads to a spatial multiplexing of the measurement, a single shot measurement with N^2 pixels is equivalent to measure the transmission coefficient N^2 times with a single pixel detector. We can also point out that the accuracy of measurements is optimised when the probability p is equal to 0.5 (Eq. S8, Fig. S2c and Fig. S3c). However, because of the numerical fluctuations, numerical simulations show that SNR is almost constant when p is in the range $[0.1, 0.9]$.

Fig. S2. For different values of the attenuation coefficient T : (a) mean values of the measured transmission coefficients T as a function of the probability p . The dotted horizontal lines give the effective levels of the attenuation coefficients T . (b) Variance on the measurements of T as a function of p . The dotted curves are obtained with Eq.S8. (c) Signal to noise ratio as a function of p . The black dotted curve corresponds to the theoretical values of the SNR . The error bars represent the uncertainties of the measured variables.

Fig. S3. For different values of the probability p : (a) mean values, (b) variances, (c) SNR on measurements of T as a function of the number of pixels N . The dotted curves correspond to the theoretical values of the related measurements. The error bars represent the uncertainties of the measured variables.

3. EXPERIMENTAL RESULTS: VALIDATION OF THE THEORETICAL MODEL

Fig. S4 depicts our computational temporal ghost imaging system. First, $N \times N$ pixels random binary patterns, verifying statistical properties of a Bernoulli distribution (Eq. S1), are generated with a commercial software and displayed on a computer LCD (Liquid Crystal Display) screen. Images of the patterns are acquired with a compact CMOS USB2.0 camera (IDS UI-1640C, 1280×1024 pixels). Because light emitted by the LCD screen is linearly polarised at 45° , the energy transmission of the imaging system can be easily controlled during the exposure time of the camera by the means of a liquid crystal variable wave plate (WP, Thorlabs LCC1113-A) placed before a linear polarizer (P) in front of the camera objective. Indeed, with an internal or an external temporal signal that drives the WP controller which controls retardation of the WP, the energy transmission of the imaging system is controlled. Random patterns are displayed with black (0) and white (1) levels and images are recorded with the 8 bits gray scale mode of the camera.

At first, each pattern of the stack is recorded individually with the same exposure time. The retardation of the WP and the direction of the polarizer axis are adjusted to maximise the energy transmission of the imaging system that will be considered later as an energy transmission level of 100% ($T = 1$). Then,

Fig. S4. Experimental setup : (P) linear polarizer (WP) liquid crystal variable wave plate. Images of random patterns displayed on a LCD screen are acquired with a CMOS camera. Temporal variation of the imaging system transmission is obtained thanks to an internal or an external signal that drives the WP controller which controls retardation of the WP placed in front of the polarizer.

the mean and variance of each image are measured. Since displayed patterns are binary, the sharp edges of the original pixels are blurred in the recorded images because of the modulation transfer function (MTF) of the objective. Consequently, as the recorded images are encoded over 255 gray levels, they should no longer verify the initial statistical properties.

Fig. S5. (a) Typical recorded image of a 256×256 binary random pattern with $p = 0.5$ addressed on the LCD screen. The white dotted square represents the region of interest (ROI) where the statistical properties of the image are calculated. (b) zoom of the selected ROI. (c) Histogram of the pixel values in the ROI. The vertical dotted line represents the mean level in the ROI.

Fig. S5a shows a typical image X_I , recorded with the CMOS camera, of a random binary pattern X of 256×256 pixels (where $p = 0.5$) displayed with the computer LCD screen. We can observe that the field of the recorded image is limited by the 1cm diameter aperture of the WP. Because this introduces some deterministic fluctuations that degrade statistical properties of the recorded image, we select a region of interest (ROI) of 700×700 pixels, where the levels of the lighted pixels are almost constant. This ROI is delimited by the white dotted square. In the selected ROI (Fig. S5b) statistical calculations are performed for a stack of 20 images of independent random patterns. Fig. S5c shows the histogram of the pixels values in the selected ROI where the vertical dotted line represents the mean level: $\langle X_I \rangle = 17.5 \pm 0.5$. The measured variance is $\sigma_{X_I}^2 = 124 \pm 2$. As this histogram clearly shows that the recorded image is not binary, the measured statistical properties are different from those of a binary random pattern. Indeed, for binary images where $p = 0.5$ and where the low and high levels are respectively $X_{I_{\min}} = 5$ and $X_{I_{\max}} = 30$ (Fig. S5c), the mean level is $\langle X_I \rangle = pX_{I_{\max}} + (1-p)X_{I_{\min}} = 17.5$ in good agreement with the measured mean. Contrariwise, the variance of the binary image should be $\sigma_{X_I}^2 = (p - p^2)(X_{I_{\max}} - X_{I_{\min}})^2 \simeq 156$.

Fortunately, this discrepancy between the expected variance and its measured value does not affect the measurement of the transmission coefficient. Actually Eq. S4 remains valid if the variance of the initial random pattern, $p - p^2$, is replaced by the measured variance of the recorded images and if the hypothesis of the independence of images is still true. Indeed, a non-zero residual covariance will lead to a biased measurement of the transmission coefficient. Because it remains some deterministic spatial intensity fluctuations in recorded images, the covariances are not negligible. When the recorded images are carefully observed, it is noticeable that pixels of the LCD screen are more or less resolved when the focus of the objective is ad-

justed. Because these pixels are not contiguous, when the frame of the LCD screen is resolved, it provides a non null correlation level between two images of two independent random patterns. Several experiments were conducted to optimize the focus and the numerical aperture of the objective that minimize these covariances. Finally, the residual covariances are eliminated by numerical spatial filtering of the recorded images.

Fig. S6. For random patterns, with $p = 0.5$ and 32×32 to 512×512 pixels, addressed on the LCD screen: ROI of the acquired images of single patterns (first row) and ROI of the acquired images when 20 random patterns of a stack are displayed successively on the LCD screen during the long exposure time of the camera and when $T = 1$ (second row).

Once residual covariances become negligible, we investigate experimentally the accuracy on the measurement of the transmission coefficient as a function of the number of pixels N . Stacks of 20 random patterns are generated with $p = 0.5$ and $N = [32, 64, 128, 256, 512]$. The transmission coefficient of the imaging system is set as a constant ($T = 1$) during the long exposure time of the camera. The mean, variances and SNR on measurements of the transmission coefficient are calculated.

Fig. S6 depicts typical pictures of ROIs of 700×700 pixels selected inside the acquired images (Fig. S5a) when random patterns with 32×32 to 512×512 pixels are addressed on the LCD screen. The pictures of the first row correspond to images of a single random pattern and the pictures of the second row correspond to images obtained when the camera is used with a long exposure time and when random patterns are addressed successively on the LCD screen.

Because of the magnification of the objective, the image of an independent pixel of the LCD screen covers several pixels of the camera, resulting in a smaller number of independent pixels in a ROI. For example, with an original random pattern of 32×32 pixels, the image of an independent pixel covers an area of 35×35 pixels of the camera. Consequently, inside a selected ROI of 700×700 pixels, there is only $N_{\text{eff}}^2 = \left(\frac{700}{35}\right)^2$ independent pixels. The mean size of the image of a pixel of the displayed random pattern is deduced from the area of the half maximum of the autocorrelation peak of a pattern's image.

Fig. S7 shows the experimental measurements of the mean values and of the SNR of the transmission coefficient as a function of the number of independent pixels N_{eff} for a stack with 20 random patterns. In Fig. S7a, error bars are deduced from the variances on the measurements of T for several realizations. The horizontal black dotted line corresponds to the effective transmission coefficient of the imaging system ($T = 1$) and the dotted curves represent the expected uncertainty levels $\pm 2 \sqrt{\frac{K-1}{N_{\text{eff}}^2}}$ de-

Fig. S7. Mean values of T and SNR on measurements of the transmission coefficient T as a function of the number of effective pixels N_{eff} . The dotted curves represent the expected values for the uncertainties (a) and for the SNR (b).

duced from Eq.S8. In Fig. S7b, error bars are obtained with several realizations of SNR measurement. The dotted curves represent the expected signal to noise ratio : $SNR = \sqrt{\frac{N_{eff}^2}{K-1}}$.

These results clearly show, in good agreement with the theoretical model, that the accuracy of measurements increases when the number of independent pixels increases. Otherwise, additional experiments have been performed in order to valid that the SNR is maximised when $p = 0.5$.