Ultrafast pulse-amplitude modulation with a femtojoule silicon photonic modulator: supplementary material

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1. POWER CONSUMPTION

The power consumption of the modulator is evaluated using the assumption that the energy is dissipated on rising transitions and that the energy dissipated per rising transition of magnitude V is given CV^2 , where C is the capacitance of the RC element, here the p-n junction.

A. Measurement of the p-n Junction Capacitance

The high-speed behavior of the modulator is examined using a small-signal circuit representation, as per Fig. S1. The values of the components of the circuit, which are the metal pads C_{pad} , p-n junction resistance R_{pn} and capacitance C_{pn} , and parasitic capacitance C_{par} , and resistance R_{par} , including the current paths through the oxide (cladding and BOX), and vias, and potentially doped silicon substrate. The values are obtained by fitting with the measured electrical-electrical (EE) S_{11} and S_{21} scattering parameters. The measurement has been done using non-terminated GSG microprobe, calibrated using the short-open-load-thru (SOLT) method. The procedure we use is thus similar to the one found in [1], with the notable addition that, for each frequency, we measure 4 different quantities (for 5 unknowns), the phase and amplitude of both S_{11} and S_{21} , for which excellent theoretical fitting is achieved, as shown in Fig. S2. Therefore, values of the components are extracted with a high level of confidence.

Using the methodology described in [2], we can estimate the value of C_{pn} when a reverse-bias V is applied. The model is first calibrated using DC and small-signal measurements. Then, the reduction of C_{pn} from its equilibrium value up to a given bias can be simulated. Hence, the capacitance of the p-n junction is

Fig. S1. Circuit representation, between port A and B, of the modulator with extracted component values under equilibrium. The dashed lines denotes the configuration of the VNA ports for S parameters measurement.

estimated around $C_{p-n}(V = -5.5) = 9.4$ fF under operating conditions, *i.e.* $V = -5.5$ V. Therefore, under operating conditions, we calculate the electrical bandwidth of the p-n junction to be ~ 32 GHz.

B. Power Consumption : Modulation

The knowledge of the values of the circuit components allows for the computation of the power consumed on-chip, as shown in Fig. S3a and Fig. S3b. The calculation yields, under operating conditions ($V = -5.5$ V), the total energy consumed on the silicon chip, which is ~ 12 fJ/bit and ~ 4 fJ/bit for PAM-4 at 40 GBaud and PAM-8 at 15 GBaud, respectively. However, under

Fig. S2. a), b) Measured and fitted EE scattering parameters.

Fig. S3. a), b) On-chip power consumption of PAM-4 and PAM-8 modulation formats, respectively. We have assumed that the driving circuit is ideal and can thereby be represented by an ideal source and a $50\ \Omega$ resistor, *i.e.* a Thevenin equivalent. The figures also contain the power consumed in the p-n diode itself (Power in R_{pn}), which is regarded as the effective modulation energy. The curves noted by $V = 0$ V indicate that the computations have been made using the measured capacitance $C_{pn}=17.9$ fF, as per Fig. S1, whereas $V = -5.5$ V indicates that the computations have been made using the simulated capacitance under operating conditions, $C_{pn}(V = -5.5)=9.4$ fF.

the same operating conditions as aforementioned, careful circuit analysis shows that the energy dissipated in the p-n junction, *i.e.* the *effective modulation energy*, is ~ 6.5 fJ/bit ~ 1 fJ/bit for PAM-4 and PAM-8 modulation formats, respectively.

The parasitics can be further reduced for higher efficiency by using smaller pads and substrate removal [3].

C. Power Consumption : Tuning

The resistance of the heater is measured to be $1800\ \Omega$ and its thermal efficiency is experimentally measured to be $33\ \mu\text{W}/\text{GHz}$. However, the picture is completed only if one takes into account the data rate of the modulator since the power consumption on thermal tuning is independent of the modulation speed, as shown in Fig. S4. For example, a shift of 10°C would request roughly 130 fJ/bit at 80 Gb/s, which is comparable to the results at 10 Gb/s [4] and 25 Gb/s [1], but at a much higher data rate. The 10°C span represents the thermal compensation required for a WDM system at 100 GHz channel spacing, given the thermo-optic response of the silicon, *i.e.* $10\ \text{GHz}/^\circ\text{C}$.

It is noteworthy that the efficiency of the device could readily

be improved by using an optimized heater design, such as using a metal layer to implement the heater for a potentially larger p-n junction phase shifter. In addition, substrate removal and oxide deep trench can be used to further increase the efficiency, by which 35-time enhancement has been reported [5]. Hence, an optimized design of the heater and thermal isolation could bring down the power consumption below 5 fJ/bit at 80 Gb/s for a 10°C temperature variation. This ultra-low power consumption would be comparable to the 2 fJ/bit tuning efficiency achieved by the electro-optic effect [6]. However, the thermo-optic tuning can cover a much wider range without increasing the capacitance of the modulator, effectively keeping intact the modulation bandwidth and performance. This is beneficial compared to the electro-optic tuning.

2. NON-LINEAR PHENOMENA

The optical intensity that exists inside the optical cavity is of importance due to non-linear effects that might take place. Despite the complex physics, the upper-bound of the change in effective index is estimated as follows.

Fig. S4. Power consumption, in femtojoules per bit, as a function of the temperature shift (horizontal axis) and bit rate (vertical axis). Each section spans 50 femtojoules, starting at 0 on the left. A thermo-optic coefficient of $1.87 \cdot 10^{-4} \text{ K}^{-1}$ is assumed.

We use a laser with an output power of 10.5 dBm. Due to the IL of the SGC, *i.e.* IL=6.5 dB, there is 4 dBm of optical power inside the bus waveguide before the MRM. Assuming the power is distributed uniformly on the waveguide cross-section of 220 nm by 500 nm, the optical intensity is calculated as 22.85 MW/m^2 . Now, with a cross-coupling coefficient κ of 0.18, extracted from the measured DC spectra, and a loss-less point coupler, the optical intensity inside the ring is found to be 657 MW/m^2 , or $\sim 18.6 \text{ dBm}$, under the continuous wave assumption [2, 7]. To obtain this result, we use the measured transmission coefficient T , which is defined as the ratio of the modulated wave B over the incoming wave A , *i.e.* $T = B/A$. Then, the wave amplitude U that initiate a cycle across the cavity is given by

$$U = \frac{\sigma B - A}{j\kappa} \quad (\text{S1})$$

where σ is the straight-through coupling coefficient. Additionally, since we research an upper-bound, we have considered that the laser is tuned to the resonant wavelength and that the MRM is under equilibrium.

It turns out that this optical intensity corresponds to a change in effective index of roughly 10^{-4} [8], which is comparable to the effective index change due to the modulation. However, the MRM is operated at a high detuning, $\Delta f \sim 10 \text{ GHz}$. The estimated optical power inside the cavity at the detuning point is $\sim 13.2 \text{ dBm}$. Thus, neglecting the effect of the modulation, the change of effective index is found to be of order 10^{-5} [8, 9]. Hence, the input power used is close to the maximum value before non-linear effects begin to impair the modulation quality. Precisely, we noticed degradation of the signal integrity when using 14 dBm of laser power and above. These findings are coherent to what is reported in [10].

3. CHIRP

By their very nature, MRMs modulate both the phase $\phi(t)$ and amplitude $|T(t)|^2$ simultaneously, as shown in Fig. S5a. It shows

the results of simulations for a 40 Gb/s OOK signal, $V_{p-p} = 4 \text{ V}$ at $\Delta f = 10 \text{ GHz}$ using a DC bias of -5 V , hereafter called the operating conditions. We see that the modulated signal emanating from a MRM is chirped, that is, its instantaneous frequency changes as a function of time, as shown in Fig. S5b. Generally speaking, chirp is of great concern because it can potentially lead to significant signal degradation, although it is worth remembering that typical pulses inevitably acquire chirp during propagation in a dispersive medium. Further, reliable description of the chirp requires the use of a large-signal model [2].

As shown in Fig. S5b, the instantaneous frequency changes in a linear fashion with time, indicating that the phase of the signal has a quadratic time dependence. We computed the frequency chirp $\delta\nu$, here defined as

$$\delta\nu(t) \equiv -\frac{1}{2\pi} \frac{\partial \phi(t)}{\partial t} \quad (\text{S2})$$

for the simulated signal and found that the maximal value found in a 0-1 transition is $\delta\nu \sim 2.27 \text{ GHz}$. The effect of the chirp can be obtained by approximating the pulse shape as a Gaussian, given the high bit rates here considered.

Under the Gaussian approximation, the full-width at half maximum (FWHM) is measured at $T_{FWHM} = 75.4 \text{ ps}$. Then, the chirp parameter C is readily obtained [12]

$$C = \frac{\pi T_{FWHM}^2}{2 \ln(2)} \frac{\partial(\delta\nu)}{\partial t}. \quad (\text{S3})$$

In our case, the maximal value of $\partial\nu$ for a 0-1 transition arises in a time window of 21 ps, giving us $C \simeq 1.4$. We consider, under the operating conditions, that the SSMF (Corning SMF-28) dispersion parameter D is $15.11 \text{ ps/(nm}\cdot\text{km)}$, giving a group velocity dispersion (GVD), of β_2 of $-1.88 \cdot 10^{-26} \text{ s}^2/\text{m}$. Consequently, we have $\beta_2 C < 0$, which indicates that pulse compression occurs.

Defining the full width at $1/e$ as $T_0 = T_{FWHM} / (2\sqrt{\ln(2)})$, it becomes straightforward to evaluate the full width at $1/e$ as a function of C and z , the propagation distance [12]

$$T_1(C, z) = T_0 \left[\left(1 + \frac{C\beta_2 z}{T_0^2} \right)^2 + \left(\frac{\beta_2 z}{T_0^2} \right)^2 \right]^{\frac{1}{2}}. \quad (\text{S4})$$

Hence, after 5 km of SSMF, we find, under the assumption of Gaussian pulse shape, that 6% pulse compression occurs for the chirped signal. For comparison, 0.1% pulse dilatation would occur if the signal was chirp-free. The chirping pattern of the MRM under an OOK signal represents a good estimation of the chirp encountered in PAM modulation scheme.

Here, we use a simple model to evaluate the power penalty $\delta\mathcal{P}_c$ induced by the chirp for OOK signal [13, 14]

$$\delta\mathcal{P}_c = -10 \log(1 + BzD\delta\lambda_c) \quad (\text{S5})$$

where B is the bit rate and $\delta\lambda_c$ is the wavelength excursion due to the chirp. Thus, using the values above, considering $z = 5 \text{ km}$ and $B = 40 \text{ Gb/s}$, we find $\delta\mathcal{P}_c \simeq -0.23 \text{ dB}$, which indicates a slight gain due to the chirp. Operating the MRM at the opposite frequency detuning would give us a penalty of 0.24 dB.

Consequently, we can assert that, although present, the chirp is not a significant contributor to the signal degradation, in this specific case. More careful description is however needed when complex systems such as WDM and coherent systems based on MRMs are considered.

Fig. S5. a) Simulated amplitude and phase of the modulated wave for OOK signal at 40 Gb/s, driven by a PRBS, under operating conditions, *i.e.* $V_{p-p} = 4$ V, $\Delta f = 10$ GHz, using a DC bias of -5 V. b) Reassigned Gabor spectrogram of the simulated signal [11], using a dynamical range of 40 dB. The color grade shows the optical power from the lowest value in dark blue to the highest value in red. The inset shows a zoom of a 0-1-0 transition, similar to the ones occurring at $t \simeq 50$ ps and $t \simeq 125$ ps. The simulation is done using the model of [2].

Fig. S6. Simulated transfer coefficient, *i.e.* power at the output over the power at the input, as a function of the driving signal amplitude, under the DC assumption, for various detuning. Vertical and horizontal lines show how non-equally spaced voltage levels give equally spaced optical power levels.

4. EYE DIAGRAM

Examples of optical eye diagrams, before equalization, are shown in Fig. S7. The OOK eyes are open and exhibit decent Q-factors, 12.6, 8.0 and 3.7 for 11.5, 25 and 55 Gb/s, respectively, Fig. S7a–S7c. PAM-4 signals are shown in Fig. S7d–S7g. Finally, PAM-8 optical eye diagrams are provided through Fig. S7h and Fig. S7i.

A. Linearization

It is well known that the transfer function of MRM is non-linear [7]. Hence, linearization of the driving signal must be done in order to obtain equally spaced level in the optical output, for optimized BER. Although dynamic equalization is complex and depends on multiple factors such as on the frequency detuning, bit rate, driving signal and bias, and non-linear gain of the RF amplifier, a good approximation can be obtained by using DC considerations.

Table S1. Linearization of the optical outputs for 3 different detuning, considering PAM-4 modulation format. The values are in unit of volts.

Rising Transitions	ΔV_{lin}	$\Delta V_{\Delta f=0}$	$\Delta V_{\Delta f=5}$	$\Delta V_{\Delta f=10}$
1→2	1.17	1.28	0.62	0.55
1→3	2.33	2.26	1.60	1.51
1→4	3.50	3.50	3.50	3.50
2→3	1.17	0.98	0.98	0.97
2→4	2.33	2.22	2.88	2.95
3→4	1.17	1.24	1.9	1.99
ΔE_b	–	-2.8%	+3.2%	+4.6%

As shown in Fig. S6, the transmitted power follows a non-linear relation as a function of the applied voltage. This non-linear relation is useful in order to establish the non-equally spaced electrical levels that will produce equally spaced optical levels. Therefore, the linearization procedure induces variation of the power consumption, due to variation of the electrical levels. To that extent, the voltages that produce equally spaced optical levels $\Delta V_{\Delta f=0, 5, 10}$, in a PAM-4 modulation scheme, are provided in Table S1 for frequency detuning of 0, 5, and 10 GHz. The last line provides the variations of the consumed power ΔE_b between non-equally spaced electrical levels and the reference ΔV_{lin} , in which electrical levels are equally spaced. We can see that the electrical levels must be adjusted to produce equally spaced optical outputs. This does not induce significant change in power consumption, as per Table S1, *i.e.* lower than 5% for detuning of to 10 GHz.. The same principle and results apply to PAM-8 modulation format.

B. Probability Density Function

The signal integrity can be further assessed by looking at the probability density functions (PDF) of the collected data. Examples of PDFs of the received signals at the decision instant are provided in Fig. S8, in which Gaussian fits are also plotted. Good agreement between the Gaussian fits and measurements

Fig. S7. a), b), c) OOK optical eye diagrams at 11.5, 25 and 55 Gb/s having a Q-factor of 12.6, 8.0 and 3.7, respectively. d), e), f), g) PAM-4 optical eye diagrams at 16.5 (33), 23 (46), 32 (64) and 40 (80) GBaud (Gb/s), respectively. h), i) PAM-8 optical eye diagrams at 11.5 (34.5) and 15 (45) GBaud (Gb/s), respectively. High-speed eye diagrams, *i.e.* c), i) and g), are all produced at high frequency detuning, $\Delta f \sim 11$ GHz whereas eye diagrams at lower speed, *i.e.* a), b), d), e), h), i) are produced around $\Delta f \sim 6$ GHz.

is obtained, validating the Gaussian representation. It further indicates that the bandwidth of the MRM is sufficient to produce signals with low inter-symbol interference (ISI), and indicates low-degradation due to non-linear phenomena. Thus, they are coherent with the observed optical eye diagrams, measured BER, and aforementioned estimations of signal impairments.

Interestingly, the PDFs of the OOK and PAM-4 signals exhibit slightly wider PDF at lower amplitude. This is explained by considering the dynamical behavior of MRM. In fact, the optical bandwidth, and by extension, the total bandwidth, are a function of the frequency detuning. By looking at the measured EO S_{21} scattering parameters, we see that the bandwidth is proportional to the frequency detuning. Therefore, the MRM takes more time to respond as the transmitted amplitude decreases. This is also discernible in Fig. S7 for OOK eye diagrams at 11.5 and 25 Gb/s.

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Fig. S8. Normalized probability density functions (PDF) of the samples acquired at the RTO with their Gaussian approximations. Each curve represents the probability density function (PDF) at the instant that correspond to the optimal eye opening. We used 8 millions time samples in the computation of each PDF. a) PAM-4, 32 GBaud. b) PAM-4, 32 GBaud, equalized. c) PAM-4, 40 GBaud, equalized. d) OOK, 30 Gb/s. e) PAM-8, 15 GBaud. f) PAM-8, 15 GBaud, equalized. All of the PDFs represent back-to-back cases. We used 7 dBm of received power for the PAM-4 and PAM-8, and 3.5 dBm of received power for the OOK.

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