Optomechanical sensing of liquids outside standard fibers using forward stimulated Brillouin scattering: supplementary material

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This document provides supplementary information to “Optomechanical sensing of liquids outside standard fibers using forward stimulated Brillouin scattering,” <http://dx.doi.org/10.1364/optica.3.000510>. An expression for the power spectral density of a probe signal that is scattered from radial acoustic modes in standard fiber is calculated theoretically. Good quantitative agreement is obtained between model and measurements. © 2016 Optical Society of America

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1. Acousto-optic excitation of radial modes

Acoustic radial modes, supported by the cylindrical shape of the cladding of a standard optical fiber, can be optically excited through electrostriction. Let $E(r, z, t)$ denote the electric field of a pump wave, as a function of time t , at some given point z along the axis of the fiber and at radius r in the transverse plane:

$$E_{pu}(r, z, t) = E_0(r) A_{pu}(z, t) \exp[j(\beta_{pu} z - \omega_{pu} t)] + \text{c.c.} \quad (\text{S1})$$

In (S1), β_{pu} and ω_{pu} are the central wave number and optical frequency of the pump wave, respectively, $A_{pu}(z, t)$ represents its slowly-varying complex amplitude, and the normalized transverse profile of the optical mode is denoted by $E_0(r)$. The transverse profile is well approximated by a Gaussian shape:

$$E_0(r) = \frac{1}{(2\pi)^{3/2} w} \exp\left(-\frac{r^2}{2w^2}\right), \quad (\text{S2})$$

where w is the Gaussian beam radius. The transverse profile of the optical mode is often stated in terms of its mode field diameter (MFD): $d = 2\sqrt{2}w$ [1].

It is assumed below that the pump wave is undepleted and unaffected by chromatic dispersion or Kerr nonlinearity. Linear losses over the short lengths of fibers under test are neglected as well. Subject to these assumptions, the temporal shape of the

pump wave at every position z is simply related to that of the input, $A_{pu,in}(t)$, at $z = 0$:

$$A_{pu}(z, t) = A_{pu,in}(t - n_{eff,0} z / c). \quad (\text{S3})$$

Here c is the speed of light in vacuum and $n_{eff,0}$ is the effective refractive index of the optical mode.

Consider an amplitude modulated pump, with an instantaneous optical power that is oscillating at radio frequency $\Omega_0 = 2\pi f_0$ with magnitude $|\tilde{P}_0|$:

$$P_{pu}(t) \equiv |A_{pu,in}(t)|^2 = |\tilde{P}_0|^2 + \frac{1}{2} [\tilde{P}_0 \exp(j\Omega_0 t) + \text{c.c.}]. \quad (\text{S4})$$

The acoustic waves driven by the pump can be described by the density fluctuations of the medium. The fluctuations can be expressed as a sum of normal modes. In this work we consider only radially symmetric modes denoted as $R_{0,m}$. The contribution of mode m to the magnitude of density fluctuation is given by:

$$\Delta\rho_m(r, z, t) = \Delta\rho_{0,m}(r) b_{m,0} \exp[j(q_0 z - \Omega_0 t)] + \text{c.c.} \quad (\text{S5})$$

Here, $\Delta\rho_{0,m}$ is the normalized transverse profile of mode $R_{0,m}$, $b_{m,0}$ denotes its complex amplitude, and q_0 is the axial component of its wave number. The acoustic wave is phase-

matched with the pump: $\Omega_0 / q_0 = c / n_{\text{eff},0}$. The acoustic amplitude $b_{m,0}$ depends on the driving frequency, and on the spatial profile of the acoustic mode. It is maximal when the driving frequency matches the resonance frequency of the mode $\Omega_{0,m} = 2\pi f_{0,m}$. The dependence of the acoustic wave magnitude on the detuning of the stimulating frequency from resonance follows a Lorentzian lineshape with a width Γ_m [2]:

$$b_{m,0} = \frac{\varepsilon_0 \gamma_e Q_{1,m}}{\Omega_0^2 - \Omega_{0,m}^2 + j\Omega_0 \Gamma_m} \tilde{P}_0. \quad (\text{S6})$$

In (S6), ε_0 is the vacuum permittivity and γ_e is the electrostrictive constant of the fiber medium. The factor $Q_{1,m}$ denotes the transverse overlap integral between the profiles of the acoustic mode and the electrostrictive driving force [2]:

$$Q_{1,m} \equiv \langle \nabla_T^2 E_0^2, \Delta \rho_{0,m} \rangle \equiv \int_0^{2\pi} \int_0^a \nabla_T^2 E_0^2 \Delta \rho_{0,m} r dr d\phi. \quad (\text{S7})$$

Here ∇_T^2 is the transverse component of the Laplacian operator, and a is the outer radius of the fiber cladding.

In our experiments we use a pulsed pump. The instantaneous optical power of the pulse $P_{pu}(t)$ may be represented in terms of its Fourier components:

$$P_{pu}(t) = \int_{-\infty}^{\infty} \tilde{P}_{pu}(\Omega) \exp(j\Omega t) d\Omega. \quad (\text{S8})$$

The linear dependence of $b_{m,0}$ on $\tilde{P}_0$ allows for the following generalization of (S5):

$$\Delta \rho_m(r, z, t) = \Delta \rho_{0,m}(r) h_m(n_{\text{eff},0} z / c - t), \quad (\text{S9})$$

where the time and axial position dependent magnitude $b_{0,m} \exp(jq_0 z - j\Omega_0 t)$ of (S5), now becomes $h_m(n_{\text{eff},0} z / c - t)$, with:

$$h_m(\xi) = \int_{-\infty}^{\infty} \tilde{h}_m(\Omega) \exp(j\Omega \xi) d\Omega, \quad (\text{S10})$$

and:

$$\tilde{h}_m(\Omega) \equiv \frac{\varepsilon_0 \gamma_e Q_{1,m} \tilde{P}_{pu}(\Omega)}{\Omega^2 - \Omega_{0,m}^2 + j\Omega \Gamma_m}. \quad (\text{S11})$$

2. Optical phase delay of a probe wave due to acoustic perturbations of radial modes

Eq. (S9) gives the expected density fluctuations for a given temporal shape of the pump pulse. The density variations $\Delta \rho_m$ are accompanied by perturbations in the refractive index Δn_m , due to the photo-elastic effect [3]:

$$\Delta n_m(r, z, t) = \frac{\gamma_e}{2n_0(r)\rho_0} \Delta \rho_m(r, z, t). \quad (\text{S12})$$

Here ρ_0 is the density of silica and $n_0(r)$ denotes the transverse profile of the refractive index in the fiber. The relation between local refractive index changes (Eq. (S12)) and the modification to the overall effective refractive index of the optical mode can be derived using perturbation theory, in the following manner. The transverse profile $F_m(r)$ of an optical mode oscillating at optical frequency ω must be a solution to the eigenvalue equation [3]:

$$\nabla_T^2 F_m + k_0^2 n_m^2(r) F = \beta_m^2 F_m, \quad (\text{S13})$$

where $k_0 = \omega / c$ and $n_m(r)$ denotes the transverse profile of refractive index which is given by a combination of $n_0(r)$ and a photo-elastic perturbation due to oscillations of $R_{0,m}$:

$$n_m(r) = n_0(r) + \Delta n_m(r) = n_0(r) + \alpha \bar{n}_m(r). \quad (\text{S14})$$

Here, $\alpha \bar{n}_m \equiv \Delta n_m$ is the acoustic disturbance of the m^{th} radial mode, with α a dimensionless scaling factor. Note that typically the photo-elastic contribution to the refractive index is many orders of magnitude smaller than n_0 [4]. With that in mind, we may approximate: $n_m^2(r) \approx n_0^2 + 2\alpha \bar{n}_m n_0$, and rewrite the eigenvalue equation using the following operator notation:

$$(\hat{H}_0 + \alpha \hat{H}_{1,m}) F_m = \beta_m^2 F_m, \quad (\text{S15})$$

where:

$$\hat{H}_0 = \nabla_T^2 + k_0^2 n_0^2(r), \quad (\text{S16})$$

$$\hat{H}_{1,m} = 2k_0^2 \bar{n}_m n_0. \quad (\text{S17})$$

The eigenvalues β_m^2 and eigenvectors F_m would be those of $\hat{H}_0$ with perturbations due to $\hat{H}_{1,m}$. Given that the perturbation is sufficiently weak, they can be written as a power series of α :

$$\beta_m^2 = \beta_m^{2(0)} + \alpha \beta_m^{2(1)} + \alpha^2 \beta_m^{2(2)} + \dots, \quad (\text{S18})$$

$$F_m = F_m^{(0)} + \alpha F_m^{(1)} + \alpha^2 F_m^{(2)} + \dots. \quad (\text{S19})$$

Using standard perturbation theory [3], we find that the zero-order terms are those of $\hat{H}_0$: $\beta_m^{2(0)} = n_{\text{eff},0}^2 k_0^2$ and $F_m^{(0)} = E_0$. The first order correction to the eigenvalue $\beta^{2(1)}$ equals:

$$\beta_m^{2(1)} = \langle F_m^{(0)}, \hat{H}_{1,m} F_m^{(0)} \rangle = 2k_0^2 \langle E_0^2, n_0 \bar{n}_m \rangle. \quad (\text{S20})$$

Neglecting higher order terms in α , and using (S12), we may relate the first-order correction to the propagation constant with the magnitude of density fluctuations associated with mode $R_{0,m}$:

$$\beta_m \approx n_{\text{eff},0} k_0 \sqrt{1 + \frac{\gamma_e}{n_{\text{eff},0}^2 \rho_0} \langle E_0^2, \Delta \rho_m \rangle}. \quad (\text{S21})$$

Noting that $n_{\text{eff}} = \beta / k_0$, we obtain the following first-order correction to the effective index of the optical mode due to $R_{0,m}$:

$$n_{\text{eff},m}(z,t) \approx n_{\text{eff},0} + \frac{\gamma_e}{2n_{\text{eff},0}\rho_0} \langle E_0^2, \Delta \rho_m \rangle. \quad (\text{S22})$$

Substituting $\Delta \rho_m$ from Eq. (S9), we obtain:

$$\Delta n_{\text{eff},m}(z,t) \equiv n_{\text{eff},m} - n_{\text{eff},0} = \frac{\gamma_e Q_{0,m}}{2n_{\text{eff},0}\rho_0} h_m(t - n_{\text{eff},0}z/c) \quad (\text{S23})$$

With:

$$Q_{0,m} \equiv \langle E_0^2, \Delta \rho_{0,m} \rangle = \int_0^{2\pi} \int_0^a E_0^2 \Delta \rho_{0,m} r dr d\phi \quad (\text{S24})$$

3. Interferometric measurements of acoustic modes

In the experiment described in the main text, a Sagnac loop interferometer was used to measure the non-reciprocal phase delay $\Delta \phi$ induced by optically-stimulated radial acoustic modes. The optical power of a probe wave at the loop output P_{det} , when the loop is balanced and biased at quadrature and $\Delta \phi \ll \pi$, may be expressed as (see main text, [2]):

$$P_{\text{det}}(t) \approx P_{\text{det,dc}} [1 + \Delta \phi(t)]. \quad (\text{S25})$$

Here $P_{\text{det,dc}}$ is a bias power level which is obtained in the absence of non-reciprocal phase delay. The non-reciprocal phase, in-turn, is determined by the photo-elastic variation in the refractive index induced by the acoustic density fluctuations of all modes:

$$\Delta \phi(t) = k_0 \int_0^L \sum_m \Delta n_{\text{eff},m}(L-z, t - n_{\text{eff},0}z/c) dz, \quad (\text{S26})$$

with L the length of the fiber. Substituting (S23) into (S26), we have:

$$\Delta \phi(t) = \frac{k_0 \gamma_e L}{2n_{\text{eff},0}\rho_0} \sum_m Q_{0,m} h_m(t - n_{\text{eff},0}L/c). \quad (\text{S27})$$

Taking the Fourier transform of the non-reciprocal phase, we obtain the following expression for the spectral components of the detected power at any $\Omega \neq 0$:

$$\tilde{P}_{\text{det}}(\Omega) = \int P_{\text{det}}(t) \exp(-j\Omega t) dt = \frac{P_{\text{det,dc}} k_0 \gamma_e L}{2n_{\text{eff},0}\rho_0} \sum_m Q_{0,m} \tilde{h}_m(\Omega) \quad (\text{S28})$$

Let us define the constant:

$$C_1 \equiv \frac{P_{\text{det,dc}} k_0 \gamma_e^2 \varepsilon_0 L}{2n_{\text{eff},0}\rho_0}, \quad (\text{S29})$$

for brevity. The power spectral density (PSD) of the detected power is given by substituting (S11) into (S28):

$$|\tilde{P}_{\text{det}}(\Omega)|^2 = C_1^2 \sum_m \frac{|Q_{0,m}|^2 |Q_{1,m}|^2 |\tilde{P}_{pu}(\Omega)|^2}{\Omega^2 [4(\Omega - \Omega_{0,m})^2 + \Gamma_m^2]} \quad (\text{S30})$$

In (S30) a negligible spectral overlap between $\tilde{h}_{m1}(\Omega)$ and $\tilde{h}_{m2}(\Omega)$ was assumed for $m1 \neq m2$. Similarly, the value of the PSD on a specific resonance $m = M$ is only affected by the contribution of $R_{0,M}$:

$$|\tilde{P}_{\text{det}}(\Omega_M)|^2 = C_1^2 \frac{|Q_{1,M} Q_{0,M}|^2}{\Omega_M^2 \Gamma_M^2} |\tilde{P}_{pu}(\Omega_M)|^2 \quad (\text{S31})$$

4. Experimental Validation

In order to validate the prediction of Eq. (S31), the PSD of a probe wave was measured at the output of a Sagnac fiber loop, in which radial acoustic modes were stimulated by a pump pulse $P_{pu}(t)$

shown in Fig. S1(a). The PSD of the pump $|\tilde{P}_{pu}(\Omega)|^2$ is presented in Fig. S1(b).

Fig. S1. (a) –Instantaneous optical power of pump pulses. (b) – PSD of pump pulses.

The overlap integrals $Q_{0,m}, Q_{1,m}$ were evaluated numerically according to (S7) and (S24). The transverse profile of the optical mode is provided in (S2), and those of the radial acoustic modes are given by [5]:

$$\Delta \rho_{0,m}(r) = \frac{J_0(\xi_m r/a)}{\sqrt{\int_0^{2\pi} \int_0^a J_0^2(\xi_m r/a) r dr d\phi}} \quad (\text{S32})$$

where ξ_m is the m^{th} order solution to the boundary conditions equations of the acoustic modes:

$$\left[1 - (V_s/V_d)^2\right] J_0(\xi) = (V_s/V_d)^2 J_2(\xi) \quad (\text{S33})$$

Here $V_{d,s}$ are the velocities of longitudinal and shear acoustic waves in silica ([6], see main text). A standard single-mode fiber with a MFD of 10.4 μm was considered in the calculation. The linewidths Γ_m were calculated as the full-widths at half maximum (FWHM) of the observed spectral lines.

Fig. S2 [Fig. 4(d) of the main text, repeated here for convenience] shows the measured and calculated relative values of $|\tilde{P}_{\text{det}}(\Omega_M)|^2$. The values are brought in Table 1 in the main text. Good agreement between analysis and experiment is obtained. The analysis was also carried out using the exact transverse profile of the optical mode $E_0(r)$, in terms of Bessel functions, instead of the form of (S2). The resulting power spectral density (not shown) is very similar to that of Fig S2.

Fig. S2. – Power spectral density values of a probe wave, sampled at the resonance frequencies of radial acoustic modes, following accumulation of photo-elastically induced non-reciprocal phased delay in a Sagnac loop configuration: calculation (green) and measurements (yellow).

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