Scalable performance in solid-state single-photon sources: supplementary material

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1. AREAS IN TIME-CORRELATION HISTOGRAMS

Here we deduce the area distribution of the time-correlation measurements described in the main text. For simplicity, we first consider two (fully-distinguishable) single-photons distributed in time-bins $\{t_1, t_2\}$, entering an unbalanced Mach-Zehnder interferometer composed of a first 50:50 beamsplitter and a second beamsplitter with reflectance $\mathcal{R}$ (transmittance $\mathcal{T}=1-\mathcal{R}$). Our task is to find all possible output distributions leading to a coincidence detection between events separated in time by Δt . There are two timescales relevant in such coincidence measurements: the difference in occupied time-bins $\delta t=|t_2-t_1|$, and the temporal delay inside the unbalanced interferometer Δ . By inspecting this reduced scenario, we can find that there are 8 events leading to a coincidence detection, as depicted in Fig. S1. This results in local patterns of peak areas $A_{\Delta t}$ given by: $A_{-\delta t-\Delta}=\mathcal{R}^2$, $A_{-\delta t}=2\mathcal{R}\mathcal{T}$, and $A_{-\delta t+\Delta}=\mathcal{T}^2$, the local pattern around $-\delta t$; and $A_{\delta t-\Delta}=\mathcal{R}^2$, $A_{\delta t}=2\mathcal{R}\mathcal{T}$, and $A_{\delta t+\Delta}=\mathcal{T}^2$, the local pattern around δt . From this, we find simple rules for the time-correlation measurement of an array of single-photons distributed in arbitrary time-bins $\{t_i\}$ passing through a Δ -unbalanced Mach-Zehnder:

rule 1: Find all possible temporal delays δt relating each pair of photons within the given time-bin distribution.

rule 2: Around each $\pm\delta t$, assign the relative frequency of events $\{\mathcal{R}^2, 2\mathcal{R}\mathcal{T}, \mathcal{T}^2\}$ at temporal delays $\Delta t=\{\pm\delta t-\Delta, \pm\delta t, \pm\delta t+\Delta\}$.

We note that these two simple rules describe different interesting histograms relevant in the literature. For instance, by simply identifying the involved parameters, one can find histograms of $g^{(2)}(\Delta t)$ measurements of arbitrary $|n\rangle$ Fock states by considering n single-photons occupying the same time-bin, resulting in distributions agreeing with $g^{(2)}(0)=1-1/n$, or the well known 5-peak structures in two-photon interference experiments involving pairs of photons separated by $\Delta\tau_e < 12.5$ ns repeated every 12.5 ns.

Now, the experiment described in the main text is the particular case of an infinitely long stream of single-photons separated by a fixed $\delta t=12.5$ ns, and passing through an unbalanced interferometer with $\Delta=\Delta\tau_e$. Under this consideration, and following *rule 1* and *rule 2*, we derive the distribution of areas $A_{\Delta t}$, given by: $A_k=N$, $A_{-\Delta\tau_e}=N(1-\mathcal{R}^2)$, $A_{\Delta\tau_e}=N(1-\mathcal{T}^2)$, and

Fig. S1. Two consecutive single-photons separated by δt passing through a Δ -unbalanced Mach-Zehnder interferometer. 8 outcome distributions, occurring with a given relative frequency, lead to a coincidence signal between events separated in time by Δt . The relative delay Δt is positive if a detector in the upper output fires first, and it is negative in the opposite case.

$A_0 = N((\mathcal{R}^2 + \mathcal{T}^2) - 2\mathcal{R}\mathcal{T}V)$, with $k = \pm 12.5 \text{ ns}, \pm 25 \text{ ns}, \dots$, excluding peaks at $\pm \Delta\tau_e$, and N an integration constant. The visibility term V in A_0 appears from noticing (in virtue of *rule 1* and *rule 2*) that the area at $\Delta t = 0$ for fully-distinguishable photons is $A_0^{V=0} = N(\mathcal{R}^2 + \mathcal{T}^2)$, and then one simply uses the well-known relation $V = (1 - A_0/A_0^{V=0})(\mathcal{R}^2 + \mathcal{T}^2)/(2\mathcal{R}\mathcal{T})$, with A_0 relating the coincidence rate at zero delay of photons with non-zero V indistinguishability.

2. VISIBILITY POWER-DEPENDENCE

Following the main text, the interference visibility V of two photons separated in time by $\Delta\tau_e$ exhibits a linear-dependence in the pump power P . For a given $\Delta\tau_e$, we measure V at various values of P , up to three saturation powers $P = 3P_0$, and fit the data to $\bar{V} = V_{\Delta\tau_e}^{\max} + m_{\Delta\tau_e}P$. Figure S2 shows the power-dependence of V for $\Delta\tau_e = 12.5 \text{ ns}$, $\Delta\tau_e = 50 \text{ ns}$, and $\Delta\tau_e = 400 \text{ ns}$. The fitted parameters are $V_{12.5\text{ns}}^{\max} = (70.3 \pm 0.3)\%$, $m_{12.5\text{ns}} = -(6.1 \pm 0.2)\%$ at short timescales; $V_{50\text{ns}}^{\max} = (65.0 \pm 0.3)\%$, $m_{50\text{ns}} = -(4.4 \pm 0.2)\%$ at moderate timescales; and $V_{400\text{ns}}^{\max} = (60.8 \pm 0.3)\%$, $m_{400\text{ns}} = -(3.6 \pm 0.2)\%$ at the longest timescales explored in this work.

3. VISIBILITIES OF TEMPORALLY-DISTANT PHOTONS

The interference visibility of two photons from two sources a and b reads [1]:

$$V = \left(\frac{\gamma_a \gamma_b}{\gamma_a + \gamma_b} \right) \frac{(\gamma_a + \gamma_b + \gamma_a^* + \gamma_b^*)}{[(\gamma_a + \gamma_b + \gamma_a^* + \gamma_b^*)/2]^2 + \delta\omega^2}, \quad (1)$$

where the γ_i are the radiative rates, γ_i^* the pure dephasing rates, and $\delta\omega$ the frequency detuning between the two sources. If the interfering photons are emitted by the same quantum dot, we assume that $\gamma_a = \gamma_b = \gamma$ and $\gamma_a^* = \gamma_b^* = \gamma^*$ are constant, but only the frequency $\omega = \omega_0 + \delta\omega(t)$ varies over time (i.e. spectral wandering) around a central value ω_0 . This model makes sense here as the timescale over which ω varies is much larger than

Fig. S2. Power-dependence of V for $\Delta\tau_e = 12.5 \text{ ns}$ (orange), $\Delta\tau_e = 50 \text{ ns}$ (purple), and $\Delta\tau_e = 400 \text{ ns}$ (brown). Curves are fits to $\bar{V} = V_{\Delta\tau_e}^{\max} + m_{\Delta\tau_e}P$. V is above 50% (the classical limit) at all powers and timescales here explored.

the radiative lifetime. Then Eq. (1) reduces to:

$$V = \left\langle \frac{V(0)}{1 + \delta\omega_r^2} \right\rangle, \quad (2)$$

where we have used $V(0) = \gamma/(\gamma + \gamma^*)$ the "intrinsic" degree of indistinguishability, and $\delta\omega_r = \delta\omega/(\gamma + \gamma^*)$ the ratio between the frequency detuning and the spectral linewidth $\gamma + \gamma^*$.

One can define a time correlation function for the frequency fluctuations as

$$F(\Delta\tau_e) = \langle \delta\omega(t) \delta\omega(t + \Delta\tau_e) \rangle = \langle \delta\omega^2 \rangle f(\Delta\tau_e), \quad (3)$$

then, the frequency difference as a function of the delay $\Delta\tau_e$ can be expressed as

$$\begin{aligned} \langle \delta\omega^2(\Delta\tau_e) \rangle &= \langle (\delta\omega(t + \Delta\tau_e) - \delta\omega(t))^2 \rangle \\ &= 2 \langle \delta\omega^2 \rangle (1 - f(\Delta\tau_e)). \end{aligned} \quad (4)$$

Fig. S3. Method to extract the raw and corrected interference visibilities. a) Interference histogram of two photons separated by $\Delta\tau_e=12.2$ ns. b) Subset of data involved in the evaluation of V . c) Integrated counts from data in b). d) Measured background in between peaks.

A common assumption is to assume an exponential correlation function

$$f(\Delta\tau_e) = e^{-\Delta\tau_e/\tau_c}, \quad (5)$$

with τ_c a characteristic wandering timescale. Which is expected for a Markovian dynamics of the environment. An additional input which is required is the distribution for $\delta\omega$. Generally one assumes a Gaussian distribution, but for simplicity, and without loss of generality, we take a two-value distribution $\delta\omega = \pm\sqrt{\langle\delta\omega^2\rangle}$, so that:

$$\begin{aligned} V(\Delta\tau_e) &= \left\langle \frac{V(0)}{1 + \delta\omega^2(\Delta\tau_e)} \right\rangle \\ &= \frac{V(0)}{1 + \langle\delta\omega^2(\Delta\tau_e)\rangle} \\ &= \frac{V(0)}{1 + 2\delta\omega^2(1 - e^{-\Delta\tau_e/\tau_c})} \end{aligned} \quad (6)$$

4. EXTRACTION OF VISIBILITY UNDER RESONANT EXCITATION

Here we describe the methods to extract the raw and corrected two-photon interference visibilities under strictly-resonant excitation and π -pulse preparation, see Fig. S3. Figure S3a shows the interference histogram of two photons separated by $\Delta\tau_e=12.2$ ns, from which a visibility is extracted via $V = (\mathcal{R}^2 + \mathcal{T}^2 - A_0/A) / (2\mathcal{R}\mathcal{T})$, where A_0 is the area of the peak around $\Delta t=0$, and A is taken as the average area of 14 adjacent peaks (excluding the peak at $\Delta\tau_e$). These areas are taken as the integrated counts within a temporal window of 2 ns (considerably longer than the subnanosecond lifetimes) around $\Delta t = k \times 12.2$ ns, with $k=0, 2, 3, \dots, 15$, see Fig. S3b. The resulting integrated areas are shown in Fig. S3c, from which we extract a raw $V_{12.2\text{ns}}^\pi = (89.0 \pm 1.5)\%$. As described in the

Fig. S4. Indistinguishability vs temporal distance. Blue squares are corrected indistinguishabilities taken with *Device 2*, and red stars are the corrected values taken with *Device 3*. Black squares are raw values from *Device 2*, and gray stars are raw values from *Device 3*.

main text, the remaining non-vanishing area at $\Delta t=0$ is indeed quite small and it is on the order of experimental noise. We take into account this noise by integrating coincidence counts within a 2 ns window but now located in between peaks, that is at $\Delta t = (m + 1/2) \times 12.2$ ns, with $m=1, 2, \dots, 14$, see Fig. S3d. After subtracting the average of these background counts to the areas in Fig. S3c, we obtained the corrected visibility $V_{12.2\text{ns}}^\pi = (95.0 \pm 1.0)\%$. These same methods were employed for all measurements under strictly-resonant excitation. Figure S4 shows both raw and corrected visibilities for two devices (*Device 2* and *3*) extracted with this method.

Measurements under quasi-resonant excitation, as described in the main text, exhibit a noise level $< 1\%$, and therefore no noise-correction was employed.

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