Dispersion-enabled quantum state control in integrated photonics: supplementary material

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This document provides supplementary information to "Dispersion-enabled quantum state control in integrated photonics," <http://dx.doi.org/10.1364/optica.3.000243>, concerning the coupler model, Schmidt number calculation, and two-photon probabilities. It also provides an illustrative example of a practical dispersive coupler that can be used to implement the dispersion-unlocked functionalities we have described. © 2016 Optical Society of America
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1. PARAMETERIZATION OF COUPLER RESPONSE

Our analysis assumes that the waveguides are single-mode, identical, and rectangular (i.e. non-tapered) as illustrated in Figure 1(a) in the main text, although more sophisticated design geometries are possible [1]. Spatial mode overlap [2, 3] between the waveguides leads to $\eta(\lambda) = \cos^2(\kappa(\lambda)L)$ in terms of a coupling strength $\kappa(\lambda)$ over an interaction length L . As defined, $\eta(\lambda)$ represents the probability that a photon exits from the same waveguide it enters from (i.e. $\eta(\lambda) = 1$ means no power is transferred).

It is useful to parameterize the coupler's response to the two-photon state in terms of generic dimensionless variables that can be mapped to any combination of coupler and state properties. The dimensionless product $M\Lambda$, where $\Lambda = |\lambda_{02} - \lambda_{01}|$ is the photon pair non-degeneracy and $M = d\kappa(\lambda)L/d\lambda$ is the first-order coupler dispersion, gives the absolute difference in $\kappa(\lambda)L$ between the photon central wavelengths. For discussing spectral dependencies, the product $M\Delta\lambda$ similarly gives the absolute difference in $\kappa(\lambda)L$ across the FWHM of the marginal spectra.

A convenient parameter space for navigating the coupler response can be created from $\eta(\lambda_{00})$ and $M\Lambda$ if the reference wavelength λ_{00} is taken to be the average of the photon central wavelengths λ_{01} and λ_{02} . For photon pairs with a tunable non-degeneracy, such as those generated through spontaneous nonlinear interactions [4–6], λ_{00} can be set as the photon pair degeneracy wavelength, since λ_{01} and λ_{02} tend to remain approximately equidistant from the degeneracy point for Λ of up to hundreds of nanometers (e.g. see the pair generation tuning curves of Fig. 2 in Ref. [7]). All possible coupler responses to the quantum state then occur within the bounds $\eta(\lambda_{00}) \in [0, 1]$ and $M\Lambda \in [0, \pi]$. Behaviours for $M\Lambda > \pi$ can be mapped back to

the interval $M\Lambda \in [0, \pi]$.

Figure 1(b) in the main text shows how $\Delta\eta$ varies within these bounds. There are four coordinates where the coupler responds as a 50:50 beamsplitter with $\eta(\lambda_{01}) = \eta(\lambda_{02}) = 0.5$, and one central coordinate where it responds as a WD with $\Delta\eta = 1$. These provide a reference for tracking transitions between beamsplitter and WD behaviour. The special condition $\eta(\lambda_{01}) + \eta(\lambda_{02}) = 1$ occurs along the lines $\eta(\lambda_{00}) = 0$ and $M\Lambda = \pi/2$, where the splitting ratios at λ_{01} and λ_{02} are anti-symmetric about $\eta = 0.5$. We note that if the assumptions of linear $\kappa(\lambda)$ or $\lambda_{00} = |\lambda_{01} + \lambda_{02}|/2$ break down, the parameter space shown in Figure 1(b) becomes skewed with respect to the horizontal axis.

2. DISPERSIVE COUPLER EXAMPLE

Here we provide a realistic example of a directional coupler with sufficient dispersion to achieve the capabilities described in our article. Our aim is to affirm that high dispersion is obtainable under feasible conditions, using a numerically simulated device. The design is intentionally simplistic to show this can be accomplished without much deviation from conventional coupler designs. More optimal approaches will then be discussed.

We consider the manipulation of photon pairs degenerate at 1550 nm in the telecom band, having a maximum tunable non-degeneracy of at least $\Lambda = 50$ nm. Such states can be generated through waveguide-based SPDC (e.g. see Ref. [7]). We seek a coupler that can reach the operating point $M\Lambda = \pi/2$ within this tunable range.

The design of the coupler is shown in Figure S1(a) and is straightforward to fabricate. Figure S1(b) shows its coupling strength in the vicinity of 1550 nm, which is linear and described by $\kappa(\lambda) = 2.2055 \times 10^{10}\lambda - 2.0245 \times 10^4 \text{ m}^{-1}$. For 50:50 splitting

at the degeneracy point, the smallest suitable interaction length is $L = 56.3 \mu\text{m}$. From the definition of M , this yields $M\Lambda = 0.0621 \approx \pi/50$ at the maximum non-degeneracy of $\Lambda = 50 \text{ nm}$, which is below our target. However, since M scales linearly with L , we can multiply the dispersion by choosing a larger value of L that still gives 50:50 splitting at degeneracy. An interaction length of $L = 1521 \mu\text{m}$ achieves this and gives $M\Lambda = 1.07 \times \pi/2$ for our design, meeting our objective.

Fig. S1. A simple dispersive coupler. (a) Cross-section of coupler within its coupling length L . The design is based on silicon nitride waveguides. (b) Simulated coupling strength κ for this design, obtained from commercial FEA software.

While the approach above shows that the dispersion can be made arbitrarily large by increasing the device length, this comes at the price of increasing its footprint and insertion losses. Typical losses at 1550 nm for this silicon-nitride waveguide geometry are around 3dB/cm, and hence roughly 10% of the photons would be lost in the design we described. However, this serves merely as an illustrative example. More sophisticated coupler designs exhibiting appreciable dispersion have been studied in the past. Some examples are grating assisted couplers [8, 9] and couplers implemented in asymmetric vertical structures [10], including Bragg Reflection waveguides [11]. These can provide more compact and efficient ways of achieving the necessary dispersion.

3. CALCULATION OF SPECTRAL ENTANGLEMENT

The spectral entanglement of a state is completely described by its biphoton amplitude (BPA), which is defined in the Appendix of the main text. For a given BPA, the Schmidt Number is calculated from $SN = 1 / [\sum_n p_n^2]$, where the p_n are the eigenvalues of the matrix [12, 13]

$$\rho_{\omega\omega'} = \int d\omega'' \phi(\omega, \omega'') \phi^*(\omega', \omega''), \quad (1)$$

and are normalized according to $\sum_n p_n = 1$. To quantify the entanglement of anti-bunched states at the coupler output, we associate the labels 1 and 2 with output paths A and B respectively, and post-select for terms containing $\hat{b}^{A\dagger}(\omega_1) \hat{b}^{B\dagger}(\omega_2) |\text{vac}\rangle$. The associated BPA is proportional to

$$\Xi^{AB}(\omega_1, \omega_2) = \Phi^{A \rightarrow AB}(\omega_1, \omega_2) + \Phi^{B \rightarrow AB}(\omega_1, \omega_2), \quad (2)$$

which replaces $\phi(\omega_1, \omega_2)$ in Equation (1). For the non-path-entangled input state $|\psi\rangle_A$, we set $\Phi^{B \rightarrow AB}(\omega_1, \omega_2)$ to zero.

4. BANDWIDTH DEPENDENCE OF FIGURE 4

For completeness, we refer to Figure 4 in the main text and briefly describe how it changes with bandwidth. When the product $M\Delta\lambda$ increases but spectral entanglement remains low (i.e. $SN \approx 1$), Figs. 4(a)-(f) all begin to flatten. In Fig. 4(a),

the classical contribution P_S^C at all coordinates approaches a value of 0.5; the interference contributions P_S^I and visibility V_S in Figs. 4(b)-(d) all approach zero; correspondingly the total separation probability P_S approaches 0.5 in Figs. 4(e)-(f). In comparison, when the photons are highly frequency-entangled, increases to $M\Delta\lambda$ do not flatten the surfaces uniformly in this way. Instead, for Figs. 4(a)-(d) it causes the surfaces to ‘smear’ along the $M\Lambda$ axis, with the effect of averaging the values along this axis. Figs. 4(e)-(f) are exceptions: for large values of SN , P_S at $\theta = 0$ remains relatively unchanged from its values at small bandwidths; however, P_S at $\theta = \pi$ instead flattens to approach values of 0.5. These differ because the smearing of Figs. 4(a)-(d) along the $M\Lambda$ axis alters the symmetry in how the P_S^C and P_S^I contributions sum between the two cases.

5. BUNCHED PROBABILITIES AND VISIBILITY

In addition to the separated (anti-bunched) probabilities P_S^C , P_S^I , and P_S , there is naturally a complementary set of bunched probabilities P_B^C , P_B^I , and P_B , corresponding to outcomes where the photons exit together from the same output port. For anti-coalescence, these are related as follows: $P_S + P_B = 1$; $P_S^C + P_B^C = 1$; and $|P_S^I| = |P_B^I|$. It is likewise possible to define a bunched-outcome interference visibility $V_B = |P_B^I|/P_B^C$, which behaves differently from V_S .

The behaviour of these visibilities also depends on whether we are implementing coalescence (i.e. with photons beginning in different waveguides) or anti-coalescence (i.e. with photons beginning in the same waveguide). For simplicity, consider the familiar case where the coupler is non-dispersive and hence η is a fixed value. For coalescence such as in the HOM effect, $V_S = 2\eta(1-\eta)/[\eta^2 + (1-\eta)^2]$, while $V_B = 1$ and is independent of η because the classical and non-classical contributions to P_B scale identically. These behaviours are reversed for anti-coalescence. We also note that without dispersion, both visibilities must be equal to unity for perfect coalescence or anti-coalescence to occur. However, with dispersion, this requirement is lifted.

REFERENCES

1. A. Takagi, K. Jinguji, and M. Kawachi, “Wavelength characteristics of (2x2) optical channel-type directional couplers with symmetric or nonsymmetric coupling structures,” *J. Lightwave Technol.* **10**, 735 (1992).
2. A. Yariv, “Coupled-mode theory for guided-wave optics,” *IEEE J. Quant. Electron.* **9**, 919 (1973).
3. H. F. Taylor and A. Yariv, “Guided wave optics,” *Proc. IEEE* **62**, 1044 (1974).
4. M. H. Rubin, D. N. Klyshko, Y. H. Shih, and A. V. Sergienko, “Theory of two-photon entanglement in type-II optical parametric down-conversion,” *Phys. Rev. A* **50**, 5122 (1994).
5. Z. Yang, M. Liscidini, and J. E. Sipe, “Spontaneous parametric down-conversion in waveguides: A backward heisenberg picture approach,” *Phys. Rev. A* **77**, 033808 (2008).
6. J. Chen, X. Li, and P. Kumar, “Two-photon-state generation via four-wave mixing in optical fibers,” *Phys. Rev. A* **72**, 033801 (2005).
7. R. T. Horn, P. Kolenderski, D. Kang, P. Abolghasem, C. Scarcella, A. D. Frera, A. Tosi, L. G. Helt, S. V. Zhukovskiy, J. E. Sipe, G. Weihs, A. S. Helmy, and T. Jennewein, “Inherent polarization entanglement generated from a monolithic semiconductor chip,” *Sci. Rep.* **3**, 2314 (2013).

8. D. Marcuse, "Directional couplers made of nonidentical asymmetric slabs. part ii: Grating-assisted couplers," *J. Lightwave Technol.* **5**, 268 (1987).
9. G. Griffel and A. Yariv, "Frequency response and tunability of grating-assisted directional couplers," *IEEE J. Quant. Electron.* **27**, 1115 (1991).
10. U. Peschel, T. Peschel, and F. Lederer, "A compact device for highly efficient dispersion compensation in fiber transmission," *Appl. Phys. Lett.* **67**, 2111 (1995).
11. B. R. West and A. S. Helmy, "Dispersion tailoring of the quarter-wave bragg reflection waveguide," *Opt. Express* **14**, 4073 (2006).
12. S. Parker, S. Bose, and M. B. Plenio, "Entanglement quantification and purification in continuous-variable systems," *Phys. Rev. A* **61**, 032305 (2000).
13. T. S. Humble and W. P. Grice, "Effects of spectral entanglement in polarization-entanglement swapping and type-i fusion gates," *Phys. Rev. A* **77**, 022312 (2008).