Quantum correlations of light and matter through environmental transitions: supplementary material

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This document provides supplementary information to "Quantum correlations of light and matter through environmental transitions," <http://dx.doi.org/10.1363/optica.3.000207>. Here we give details on several aspects of solid-state cavity quantum electrodynamic (CQED) systems. We first highlight that a pure dephasing approximation is insufficient to capture host environment induced asymmetries (and more generally environmental processes) in solid-state photonic devices. We then discuss the role of driving strength in altering the cavity reflectivity spectra and comparisons between the semiclassical, atomic, and solid-state cases. Finally, we consider the influence of a thermal environment in the Fano and quantum strong coupling regimes of CQED, both of which are important to a variety of physical implementations. © 2016 Optical Society of America

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1. PURE DEPHASING NOISE

A phenomenological approach to modelling noise processes in solid-state systems can be obtained by assuming a pure dephasing form for the dissipator, that is, the master equation

$$\frac{\partial \rho(t)}{\partial t} = -i[H_s, \rho(t)] + \frac{\gamma}{2} \mathcal{L}_\sigma[\rho(t)] + \frac{\kappa}{2} \mathcal{L}_a[\rho(t)] + \frac{\Gamma}{2} \mathcal{L}_{\sigma_z}[\rho(t)], \quad (1)$$

where the final term represents pure dephasing with rate Γ , and replaces the full environmental superoperator used in the main manuscript. Here, $\mathcal{L}_x[\rho(t)] = 2x\rho x^\dagger - \{x^\dagger x, \rho\}$ denotes a non-unitary dissipator in Lindblad form, while the system Hamiltonian, H_s , and the other rates are exactly as defined in the main manuscript.

Despite its use in the literature, the pure dephasing master equation is insufficient to fully capture noise processes in solid-state photonic systems [1]. One of the primary reasons for this is its disregard for the internal eigenstructure of the system under consideration, which as shown in the manuscript is essential to accurately model processes induced by the host environment. For example, in Fig. S1 we consider an emitter-cavity system in the same parameter regime as Fig. 2 of the main manuscript. We see that increasing the pure dephasing rate leads to a symmetric broadening of the cavity reflectivity spectra, with the semiclassical theory (now including pure dephasing) remaining valid over the full range of dephasing rates. Such a phenomenological treatment cannot, therefore, lead to the deviations from semi-

Fig. S1. Effects of pure dephasing on the cavity reflectivity spectra predicted by the pure dephasing master equation of Eq. 1 (dashed) and the semiclassical theory (points). Other parameters: $\kappa = g = 0.2 \text{ ps}^{-1}$, $\kappa_s = 0.025 \text{ ps}^{-1}$, $\gamma^{-1} = 300 \text{ ps}$.

classicality that we predict with a full microscopic treatment of the solid-state environment.

2. DRIVING DEPENDENCE AND CAVITY ASYMMETRY

The semiclassical equations derived in the main manuscript rely on the assumption of small emitter occupations, that is, $\langle \sigma_z \rangle \approx -1$. This is valid at weak driving, when the emitter remains close to its ground state, rarely scattering a photon. It stands to reason then that at strong driving, the semiclassical expressions derived previously will become invalid. For example, they are unable to capture driving dependent broadening of the cavity emission [2].

Despite this eventual failure, the semiclassical theory remains a robust description of the atomic (though of course not the solid-state) cavity reflectivity even when the driving strength increases by an order of magnitude from that considered in the main manuscript ($\eta = 0.001 \text{ ps}^{-1}$), as demonstrated in Fig. S2 (a) and (b). As the driving strength increases yet further, the line shape broadens until the normal mode splitting is no longer visible, leaving only a single peak in the cavity reflectivity spectra as shown in Fig. S2 (d) at extremely strong driving. Nevertheless, signatures of phonon processes can remain observable outside the semiclassical limit, with asymmetries present even when the driving is of the same order as the cavity loss and light-matter coupling strength, see Fig. S2 (c).

3. THE FANO AND QUANTUM STRONG COUPLING REGIMES

The sensitivity of the solid-state CQED system optical properties to the host environment is not restricted to the intermediate coupling regime described in the main manuscript. In fact, we shall demonstrate that similar effects emerge in the both the Fano and the quantum strong coupling (QSC) regimes.

The Fano regime of cavity QED occurs when $\kappa + \kappa_s \gg g \gg \gamma$, where κ and κ_s are the cavity loss and side-leakage rates respectively. This regime is characterised by a sharp peak in the cavity reflectivity at the emitter resonance (while the rest of the cavity line shape remains unchanged), which is the result of classical interference between two competing decay pathways [3]. This

Fig. S2. Cavity reflectivity for increasing driving strength derived from the atomic QOME (dashed), semiclassical theory (points), and the solid-state polaron master equation (solid). Other parameters: $\kappa = g = 0.2 \text{ ps}^{-1}$, $\kappa_s = 0.025 \text{ ps}^{-1}$, $\gamma^{-1} = 300 \text{ ps}$, $\alpha = 0.075 \text{ ps}^2$, $\Lambda = 2.2 \text{ ps}^{-1}$, and $T = 4 \text{ K}$.

Fig. S3. Upper: Asymmetry in Fano resonance profiles between emitter-cavity detuning $\omega_C - \omega_X = -0.5 \text{ ps}^{-1}$ (a), and $\omega_C - \omega_X = 0.5 \text{ ps}^{-1}$ (b). Parameters are $\eta = 0.003 \text{ ps}^{-1}$, $\kappa = 1 \text{ ps}^{-1}$, $g = 0.1 \text{ ps}^{-1}$, $\kappa_s = 0.5 \text{ ps}^{-1}$, $\gamma^{-1} = 300 \text{ ps}$, $\Lambda = 2.2 \text{ ps}^{-1}$, $\alpha = 0.1 \text{ ps}^{-1}$, and $T = 2 \text{ K}$. Lower: Thermal effects in the QSC regime at weak driving $\eta = 0.005 \text{ ps}^{-1}$ (c), and strong driving $\eta = 0.08 \text{ ps}^{-1}$ (d). Parameters are $\kappa = 0.1 \text{ ps}^{-1}$, $g = 1 \text{ ps}^{-1}$, $\kappa_s = 0$, $\gamma^{-1} = 300 \text{ ps}$, $\Lambda = 2.2 \text{ ps}^{-1}$, $\alpha = 0.025 \text{ ps}^{-1}$, and $T = 4 \text{ K}$. Here points correspond to the semiclassical theory, dashed curves to the atomic QOME, and solid curves to the solid-state polaron theory.

behaviour can be seen in line shapes given in Fig. S3 (a) and (b). Here we see excellent agreement between the semiclassical theory and the atomic QOME, showing a sharp peak in the reflectivity spectra at the emitter resonance $\Delta = \pm 0.5 \text{ ps}^{-1}$, which is symmetric for both the positively and negatively detuned cases. Though less pronounced than in the intermediate coupling regime, host environment induced asymmetries become apparent when comparing the cavity line shape obtained from the full polaron master equation for an emitter tuned above the cavity resonance ($\omega_C - \omega_X = 0.5$) and below ($\omega_C - \omega_X = -0.5$).

In the QSC limit [Fig. S3 (c) and (d)], the coupling strength g becomes the dominant energy scale, $g \gg \kappa + \kappa_s, \gamma$, which allows contributions from higher order dressed states to be resolved. Additionally, the long cavity lifetime results in host environmental influences becoming particularly significant. This is especially true at strong driving, shown in Fig. S3 (d), where in the polaron theory the asymmetric broadening is so pronounced that it prevents us from resolving any higher order contributions to the upper dressed state resonance.

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