Single-pixel optical camera for video rate ultrasonic imaging: supplementary material

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Published 6 January 2016

This document provides supplementary information to “Single-pixel optical camera for video rate ultrasonic imaging,” <http://dx.doi.org/10.1364/optica.3.000026>. We give more details of the experimental setup, including details of the coherent illumination of the DMD, and describe the scrambled Hadamard patterns and reconstruction algorithm used for the compressed sensing. Four movie visualization files are also included. © 2016 Optical Society of America

<http://dx.doi.org/10.1364/optica.3.000026.s001>

Experimental Setup

The experimental setup is illustrated in Fig 3(a). The FPI sensor (glass substrate; 40 μ m parylene spacer; aluminium coatings, one fully reflective, one partially reflective) was illuminated by a 20mm diameter expanded interrogation beam (Santec TSL-510 tunable laser source connected to a IPG Laser GmbH Erbium Fiber Amplifier EAD-4-L). The reflected beam from the sensor (see below) was then redirected to the digital micromirror device, or DMD, (Vialux V-7000vis DLP, TI 0.7" XGA, 1024x768 micromirror array, pitch 13.68 μ m, maximum binary frame rate 22 kHz, on-board memory for 43,690 binary patterns) by the polarised beam splitter (Thorlabs CM1-PBS254). In the experiments described here, the patterns were chosen so that in all cases 50% of the micromirrors are ON, so the power level remains constant. The active area on the DMD was chosen to be 640x640 pixels ($\sim$ 8.7x8.7 mm²) and positioned to match the most uniformly sensitive region (at a single interrogation wavelength) on the FPI sensor. The DMD output beam was collected by a lens (focal length 25.4mm) and focused into a customised InGaAs photodiode (G8376-03). The photodiode output was connected to an oscilloscope (NI PCI-5114 digitiser) for data acquisition. The scope sampling rate and bandwidth were set at 50MHz and 20MHz in all of the experiments. A LABVIEW program was written to control the whole system. The focused ultrasound transducer (Panametrics V319, 15MHz, 0.5" diameter, focal length 0.75"), used to provide the dynamically changing acoustic field, was driven by either a pulser (Olympus 5072PR) with a maximum pulse repetition rate of 5kHz, or a power amplifier (Precision Acoustics,

PA25-150) with the signal provided by a signal generator (Agilent 33210A).

Coherent Illumination of Digital Micromirror Device

To coherent incident light, a DMD acts as a two-dimensional diffraction grating due to the periodic structure of its micromirror array [1]. The directions of the diffracted orders are determined by the micromirror pitch, the optical wavelength, and the incident angle (Fig. S1(a)). However, the light scattered into each order is determined by the Fraunhofer envelope which peaks in a direction specular to the micromirror tilt angle, which therefore depends on just the incident angle and the angles of the micromirrors (Figs. S1(b) and S1(c)). Therefore, by choosing the direction of the incident light or the optical wavelength suitably, the peak of the Fraunhofer envelope can be aligned with just one order – known as a ‘blazed’ condition (Fig. S1(b)). Fig S1(d) gives the combinations of incidence angle and wavelength necessary for a blazed condition for diffraction orders 1-4. As the optical wavelength was chosen to maximise the sensitivity of the Fabry Prot sensor, it was necessary, in this case, to adjust the incidence angle to ensure close to a blazed condition was achieved. Fig. S1(d) shows that with $\lambda \approx 1580$ nm, an incidence angle of $\approx 26^\circ$ (in the plane perpendicular to the micromirror tilt axis) satisfies this for the 2nd diffraction order. The light scattered in this direction can then be collected into the photodiode (Fig. S1(b)). This calculation assumes that the light is incident in the plane perpendicular to the micromirror tilt axes. However, when considering the illumination direction, an additional consideration is how the pattern of the

Fig. S1. Coherent illumination of the micromirror array. (a) A flat reflective grating will scatter incident light into a set of diffraction orders, with the majority of the light scattered into the specular direction. (b) For the DMD, the tilted micromirrors scatter the majority of the light into the direction of a specular reflection from the micromirrors. The incidence angle was chosen so that the majority of the light was scattered into one order ('blazed'). (c) The proportion of the light scattered into the photodiode depends on the number of micromirrors in the 'ON' position. The remainder of the light is shared among other modes and is not detected. (d) The incidence angles and wavelengths required to achieve a blazed condition for diffraction orders 1-4. At the working wavelength of about 1580 nm, shown by the dashed line, the 2nd diffraction order can be blazed with an incidence angle of about 26°.

DMD is projected onto the FP sensor. By arranging for the light to be incident in the plane perpendicular to one of the sides of the micromirrors, the square micromirrors project onto square pixels on the FP sensor, rather than sheared, kite-shaped pixels. As the DMD mirrors can be at one of two angles ($\pm 12^\circ$, ON/OFF, depending on the state of the underlying static random access memory cell), the collected power will vary with how many mirrors are in the ON position (Fig. S1(c)).

Binary Hadamard Matrix

Hadamard patterns take positive and negative values. However, each micromirror on the DMD can either reflect light to the photodiode (1) or not (0), so rather than implementing $(-1, 1)$ patterns, binary (0,1) patterns were used instead. A simple transformation takes such an experimental Hadamard matrix $H_j^{(0,1)}$ into the Hadamard matrix: $H_j = (2 H_j^{(0,1)} - \mathbf{1} \mathbf{1}^T) / \sqrt{2^j}$. Therefore, with the measured data $\mathbf{w}^{(t)} = H_j^{(0,1)} \mathbf{p}^{(t)}$, in order to use the orthogonal, self-inverse Hadamard transform H_j , $\mathbf{w}^{(t)}$ has to be transformed as follows

$$\begin{aligned} H_j \mathbf{p}^{(t)} &= \frac{1}{\sqrt{2^j}} (2 H_j^{(0,1)} - \mathbf{1} \mathbf{1}^T) \mathbf{p}^{(t)} \\ &= \frac{1}{\sqrt{2^j}} (2 \mathbf{w}^{(t)} - \mathbf{1} \mathbf{w}^{(t)}(1)). \end{aligned}$$

Here $\mathbf{w}^{(t)}(1)$ corresponds to the measurement acquired with the "all-1s" pattern which is also the first row of $H_j^{(0,1)}$, hence $\mathbf{w}^{(t)}(1) = \mathbf{1}^T \mathbf{p}^{(t)}$. Each row of $H_j^{(0,1)}$, i.e. each pattern on the DMD, comprises an equal number of 0 and 1 entries except for the first, in which all entries are 1. In order to keep the optical power incident on the photodiode the same for each pattern, the first row of $H_j^{(0,1)}$ was replaced with a vector whose first half contains all 0s and the second half all 1s. As the opposite vector (0 becomes 1 and vice versa) occurs in row $(2^j/2 + 1)$ in the Hadamard matrix, the data corresponding to the "all-1s" pattern can be reconstructed by adding the data from the modified first row pattern and the $(2^j/2 + 1)$ th row pattern. Scrambling, i.e. permutation of the columns and rows, does not interfere with the described transformations. (Recently, one-dimensional patterned detection of US using Hadamard patterns was demonstrated in laser ultrasonics using copper patterning masks [2].)

Scrambled Hadamard Matrix

The measurement matrix Φ was chosen as a scrambled Hadamard matrix [3]. A $2^j \times 2^j$ Hadamard matrix can be recursively defined as

$$H_j = \frac{1}{\sqrt{2}} \begin{pmatrix} H_{j-1} & H_{j-1} \\ H_{j-1} & -H_{j-1} \end{pmatrix}, \quad j > 0$$

and $H_0 = 1$. A scrambled Hadamard matrix is formed by permuting the columns and rows of H_j , to give $H_j^s = P_r H_j P_c$, where P_c and P_r are the column and row permutation matrices, respectively. Each row of the matrix H_j^s represents a measurement pattern, so for compressed sensing, where $M < N = 2^j$, only M rows out of N are selected. The selection of M rows yields an underdetermined system matrix with the desirable properties, similar to those of a random Bernoulli matrix, of nearly optimal performance (number of measurements for a perfect reconstruction is close to the theoretical bound) and near universality (can be used with many different sparse bases). The scrambled Hadamard transform, $u = H_j^s v = P_r H_j P_c v$, amounts to applying the column permutation matrix, P_c , to the vector v , then applying the Hadamard transform to the permuted vector and subsequently permuting the rows. Thus the cost of the scrambled Hadamard transform is of the same order as that of the Hadamard transform. Similarly, and noting that the Hadamard transform is orthogonal and self-inverse, $H_j = H_j^T = H_j^{-1}$, the inverse scrambled Hadamard transform, $v = H_j^{s-1} u = P_c^{-1} H_j^{-1} P_r^{-1} u = P_c^T H_j P_r^T u$, is equivalent to applying

inverse row permutation matrix, P_r^T , to u , then applying the Hadamard transform to the permuted vector and subsequently applying the inverse column permutation to the vector. Both the forward and inverse operations can be realized with a fast algorithm, $O(2^j \log(2^j))$.

Data Capture

Two modes of operation were used: *off-line*, in which the frames were reconstructed after the measurements for all frames had been captured, and *on-the-fly*, in which the data from each frame was reconstructed and displayed before the data for the next frame was measured. In *off-line* mode, for patterns consisting of N pixels, N scrambled Hadamard patterns were used in a repeating sequence, so that any consecutive N patterns would constitute a complete scrambled Hadamard matrix, and any consecutive subset would constitute a random selection from the scrambled Hadamard patterns. The frame rate F was determined by the degree of compression used and is given by $F = R/CN$ where R is the pattern refresh rate (limited either by the DMD or the US pulser) and C is the compression rate.

Reconstructing Pixelwise Data

When the measurements were made with no compression, ie. using a full set of measurement patterns, the fast Hadamard transform was used for reconstructing the pixel-wise data from the measured coefficients. When using compressive sampling only a subset of the measurements was available and the technique used to reconstruct the pixelwise data depended on the mode of operation. In *on-the-fly* mode the fast Hadamard transform was used with the missing data replaced by zeros; this is not optimal but it is fast. In *off-line* mode, however, a sparse recovery approach was used. The acoustic pressure values on the sensor at time t , $\mathbf{p}^{(t)}$, were found by optimising the fit to the data whilst subject to a constraint enforcing sparsity in the basis Ψ , $\mathbf{p}^{(t)} = \Psi \mathbf{a}^{(t)}$:

$$\mathbf{a}^{(t)} = \arg \min_{\mathbf{a}^{(t)}} \left\{ \frac{1}{2} \left\| \Phi \Psi \mathbf{a}^{(t)} - \mathbf{w}^{(t)} \right\|_2^2 + \alpha \left\| \mathbf{a}^{(t)} \right\|_1 \right\}$$

where α is a regularisation parameter. Here Ψ was chosen to be the identity basis. Other choices such as curvelets or wavelets could prove to be more optimal in other applications. This minimisation problem was solved using the SALSA algorithm [4].

Visualizations

Four movies are included to better demonstrate the examples described in the main text of the paper.

Visualization 1 – Movie giving an example of dynamic, real time, ultrasound field mapping using a patterned detection scheme (no compression, 100% patterns). All the videos show the squared acoustic pressure, which is proportional to the acoustic intensity, interpolated onto a 64x64 pixel image. See Fig. 3.

Visualization 2 – Movie giving an example of dynamic, real time, ultrasound field mapping using a patterned detection scheme, 12.5% patterns, reconstructed on-the-fly using a zero-padded fast Hadamard transform). See Fig. 3.

Visualization 3 – Movie. For details see caption to Fig. 4.

Visualization 4 – Movie. For details see caption to Fig. 5.

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