Stochastic optical sensing: supplementary material

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This document provides supplementary information to “Stochastic optical sensing,” <http://dx.doi.org/10.1364/optica.3.000058>. In this document we provide detailed information about the mathematical model, the experimental setup, and the procedure utilized. Also included is a complete description of optical situations considered in the main text and a detailed demonstration of the procedure repeatability. © 2016 Optical Society of America

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1. Propagation and limited size of detector effect

To simplify the exposure of main concept, the Eq. (1) in the text does not include the effect of propagation towards the detector. In this section we will show that average scattered intensity at axial distance ζ is equivalent to average scattered intensity right after the target. Consistent with the main text, we consider a targeted scattering potential with complex field scattering coefficient $v(\rho)$ and intensity scattering coefficient of $V(\rho) = |v(\rho)|^2$. The illumination field at the target plane is shown with $\mathcal{E}_s(\rho, 0)$ where the zero denotes hypothetical reference of axial direction on the target plane. As stated in the main text, the scattered field from the targeted potential will propagate at distance ζ with a Green's function $G(\rho, 0)$ and will become

$$\begin{aligned} \mathcal{E}(\rho, \zeta) &= \int v(\rho') \mathcal{E}_s(\rho', 0) G(\rho - \rho', \zeta) d\rho' \\ &= v(\rho) \mathcal{E}_s(\rho, 0) * G(\rho, \zeta) \end{aligned} \quad (\text{S1})$$

where $*$ denotes the convolution operator. The intensity distribution in the detection plane is $\mathcal{I}(\rho, \zeta) = \mathcal{E}(\rho, \zeta) \mathcal{E}^*(\rho, \zeta)$ where $\mathcal{E}^*(\rho, \zeta)$ is the conjugate version of the scattered field. The total recorded intensity is

$$\begin{aligned} \mathcal{I}(\zeta) &= \int \mathcal{I}(\rho, \zeta) d\rho \\ &= \int (v(\rho) \mathcal{E}_s(\rho, 0) * G(\rho, \zeta)) \cdot (v^*(\rho) \mathcal{E}_s^*(\rho, 0) \\ &\quad * G^*(\rho, \zeta)) d\rho \\ &= \int v(\rho) \mathcal{E}_s(\rho, 0) \cdot (v^*(\rho) \mathcal{E}_s^*(\rho, 0) * G(-\rho, \zeta) \\ &\quad * G^*(\rho, \zeta)) d\rho \\ &= \int (G(-\rho, \zeta) * G^*(\rho, \zeta)) \cdot (v(\rho) \mathcal{E}_s(\rho, 0) \\ &\quad * v^*(-\rho) \mathcal{E}_s^*(-\rho, 0)) d\rho \end{aligned} \quad (\text{S2})$$

Using the property of convolution and multiplication in the frequency domain, by calculating Fourier transform over the 2 dimensional Cartesian domain, one can show that

$$\begin{aligned} \mathcal{I}(\zeta) &= \int |\mathcal{F}_\rho\{G(\rho, \zeta)\}|^2 \cdot |\mathcal{F}_\rho\{v(\rho) \mathcal{E}_s(\rho, 0)\}|^2 d\mathbf{k} \\ &= \int |G(\rho, \zeta)|^2 |v(\rho) \mathcal{E}_s(\rho, 0)|^2 d\rho \\ &= C(\zeta) \int V(\rho) S(\rho, 0) d\rho \end{aligned} \quad (\text{S3})$$

where $\mathcal{F}_\rho\{\cdot\}$ denotes the two dimensional Fourier transform operator in transverse plane and $C(\zeta)$ is a constant that only scales the recorded intensity. If the propagation is lossless or if the loss is the same for all the realizations, $C(\zeta)$ will be the same for all the realizations. In either case, the average scattered light right after the target, is proportional to the detected average intensity for any axial distance ζ

$$\mathcal{I} \propto \int V(\rho) S(\rho, 0) d\rho \quad (\text{S4})$$

which is equivalent to the model of the sensing scenario shown in Eq. (1) in the main text. This formulation is based on the assumption of infinite size of integration window. Of course, if the detector size becomes too small the signal itself is lost as is the case with any other sensing approach relying on far-field measurements. However, as we rely on statistics of scattered light, limited detection window is not expected to affect the results significantly.

Here we consider the targets shown in figure 3(a) and figure 5(a) in the manuscript which are a sparse and a non-sparse target respectively. The SNR is 2dB to mimic a highly noisy condition with unity magnification between target and detector. To consider the effect of propagation we have used well known concept of point spread function (PSF) and 4 different Gaussian PSF with FWHM of $\approx 0.45\Delta$, 0.7Δ , 0.95Δ and 1.2Δ . Figure S1 presents the Fano factor spectra for different PSF. Figure S1(a) shows that the two peaks in the spectra remain distinguishable after propagation, when the detector size is kept constant. The Figure S1(b) illustrates that limited data collection does not affect the Fano factor spectrum even for a non-sparse target.

Fig. S1 Effect of diffraction on characterizing scattering potential length scale without considering propagation (—), and with Gaussian Green's function with FWHM of (---)0.45 Δ , (---•)0.7 Δ , (•••)0.95 Δ , and (•□•)1.2 Δ for target shown in figure 3 (a) figure 5 (a) in the text. Inset shows difference between different conditions around the second peak in the Fano spectrum.

As clearly seen, the propagation does not affect the results significantly. Propagation works as a perturbation out that does not affect statistical properties of the integrated scattered light intensity. This perturbation changes the spatial distribution of the light but preserves the energy and as a result, the statistics of the recorded intensity which characterize the intensities in different measurements (not at different locations) will not change. Of course, in these demonstration the area covered by light is increased by 25-70% for unity magnification and there is some loss of information about the target, which, however, does not affect the statistics.

2. Reconstruction of statistical point properties of the targeted potential

As it is discussed in the main text, in the case of δ -correlated illumination, one can reconstruct statistical moments of the scattering potential shown in Eq. (2) and (3). These statistical moments describe point properties of the targeted potential. Here in this section, we derive Eq. (2) and (3), from the sensing problem model Eq. (1)

$$I(\xi) = \int (S(\rho; \xi) + n(\rho; \xi))P(\rho; \xi)V(\rho) + R(\rho; \xi)d\rho \\ = \int \sigma(\rho; \xi)V(\rho) + R(\rho)d\rho, \quad (S5)$$

where $\sigma(\rho; \xi)$ is effective projection of perturbed illumination on the targeted scattering potential. Fluctuations of the intensity over its ensemble average is given by

$$\tilde{I}(\xi) = I(\xi) - \langle I(\xi) \rangle_\xi = \int \tilde{\sigma}(\rho; \xi)V(\rho) + \tilde{R}(\rho)d\rho. \quad (S6)$$

One can define k^{th} order moment of the intensity fluctuations by

$$\tilde{m}_I^{(k)} = \left\langle \left(\tilde{I}(\xi) \right)^k \right\rangle_\xi \\ = \left\langle \left(\int \tilde{\sigma}(\rho; \xi)V(\rho) + \tilde{R}(\rho)d\rho \right)^k \right\rangle_\xi, \quad (S7)$$

that for the case of δ -correlated illumination can be written as

$$\tilde{m}_I^{(k)} = \left\langle \left(\tilde{I}(\xi) \right)^k \right\rangle_\xi = \left\langle \left(\int (\tilde{\sigma}(\rho; \xi)V(\rho) + \tilde{R}(\rho)) d\rho \right)^k \right\rangle_\xi \\ = \left\langle \int \sum_{n=0}^k \frac{k!}{n!(k-n)!} \tilde{\sigma}^n(\rho; \xi) V^n(\rho) \tilde{R}^{k-n}(\rho) d\rho \right\rangle_\xi \quad (S8)$$

Considering each term in the summation in Eq. (S8)

$$\left\langle \int \frac{k!}{n!(k-n)!} \tilde{\sigma}^n(\rho; \xi) V^n(\rho) \tilde{R}^{k-n}(\rho) d\rho \right\rangle_\xi \\ = \begin{cases} 0 & k - \text{odd} \\ \text{nonzero} & k - \text{even} \end{cases}. \quad (S9)$$

As a result, one can simplify Eq. (S8) and rewrite it as

$$\tilde{m}_I^{(k)} = \sum_{n=0}^{k/2} \frac{k!}{2n!(k-2n)!} \tilde{m}_\sigma^{(2n)} M_V^{(2n)} \tilde{m}_R^{(k-2n)}. \quad (S10)$$

By reformatting this summation, one can evaluate recursively the moments

$$M_V^{(k)} = \frac{\tilde{m}_I^{(k)}}{\tilde{m}_\sigma^{(k)}} - \sum_{n=0}^{(k-2)/2} \frac{k!}{2n!(k-2n)!} \frac{\tilde{m}_R^{(k-2n)}}{\tilde{m}_\sigma^{(2n)}} M_V^{(2n)} \quad (S11)$$

defining the spatial distribution of the scattering potential. In the practical case where $k = 2$, one can easily use Eq. (S5) and (S11) to calculate Fano factor associated with the targeted scattering potential

$$M_V^{(2)} = \frac{\tilde{m}_I^{(2)}}{\tilde{m}_\sigma^{(2)}} - \frac{\tilde{m}_R^{(2)}}{\tilde{m}_\sigma^{(2)}} \quad (S12)$$

$$M_V^{(1)} = \frac{\langle I(\xi) \rangle_\xi - \langle R(\rho; \xi) \rangle_\xi}{\langle \sigma(\rho; \xi) \rangle_\xi} = \frac{m_I^{(1)} - m_R^{(1)}}{m_\sigma^{(1)}} \quad (S13)$$

$$F_V = \frac{M_V^{(2)}}{M_V^{(1)}} = F_I \frac{m_\sigma^{(1)} m_I^{(1)}}{\tilde{m}_\sigma^{(2)} (m_I^{(1)} - m_R^{(1)})} - \frac{\tilde{m}_R^{(2)} m_\sigma^{(1)}}{\tilde{m}_\sigma^{(2)} (m_I^{(1)} - m_R^{(1)})} \quad (S14)$$

Eq. (S11) shows that, in principle, one can reconstruction the probability distribution function of targeted scattering potential by reconstructing all the statistical moments. However, in practice one must consider overall procedural noise for reconstruction of higher order moments of the targeted scattering potential.

In the case that illumination intensity is not δ -correlated, Eq. (S7) for $k = 2$ can be rewritten

$$\tilde{m}_I^{(2)} = \iint \langle \tilde{\sigma}(\rho; \xi) \tilde{\sigma}(\rho'; \xi) V(\rho; l_V) V(\rho'; l_V) \rangle_\xi d\rho d\rho' \\ + \tilde{m}_R^{(2)} \quad (S15) \\ = \iint \langle \psi(\rho; \xi, l_s) \psi(\rho + \rho'; \xi, l_s) \rangle_\xi d\rho d\rho' + \tilde{m}_R^{(2)},$$

where the function $\psi(\rho; \xi, l_s) = \tilde{\sigma}(\rho; \xi, l_s) V(\rho; l_V)$ denotes the effective intensity fluctuations of the scattered light. Eq. (S15) shows the nonlinear transformation discussed in the main text in Eq. (4).

3. Significance of a maximum in Fano spectrum

Without loss of generality, in noiseless conditions, realization ξ of illumination, the recorded intensity is

$$I(\xi) = \int S(\rho; \xi, l_s) V(\rho; l_V) d\rho \quad (S16)$$

where $S(\rho; \xi, l_s)$ represents the illumination pattern with characteristic length l_s and $V(\rho; l_V)$ represents the targeted scattering potential with characteristic length l_V . For simplicity, a scattering potential consisting of N randomly distributed circular objects can be represented as

$$V(\rho; l_V) = \sum_{n=1}^N \delta(\rho - \rho_n) * e(\rho; l_V) = C_V(\rho) * e(\rho; l_V), \quad (S17)$$

Where $*$ denotes the operation of convolution and δ denotes delta function. The function $C_V(\rho)$ denotes the locations of the N elementary objects, each of them represented by $e(\rho; l_V)$. To describe an unknown random scattering potential, the locations of delta functions are modeled by a random variable uniformly distributed to suppress any *a priori* information about the target. In the same way, the illumination pattern can be described as

$$S(\rho; \xi, l_s) = \sum_{m=1}^M \delta(\rho - \rho_m; \xi) * e(\rho; l_s) \\ = C_S(\rho; \xi) * e(\rho; l_s). \quad (S18)$$

To cover uniformly the entire field of view, the locations of random dots is also described by a uniform distribution. It follows that:

$$\begin{aligned}\tilde{I}(\xi) &= I(\xi) - \langle I(\xi) \rangle_\xi \\ &= I(\xi) - \left\langle \int S(\boldsymbol{\rho}; \xi, l_s) V(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho} \right\rangle_\xi \\ &= \int (S(\boldsymbol{\rho}; \xi, l_s) - \langle S(\boldsymbol{\rho}; \xi, l_s) \rangle_\xi) V(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho} \\ &= \int \tilde{S}(\boldsymbol{\rho}; \xi, l_s) V(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho},\end{aligned}\quad (\text{S19})$$

where $\tilde{S}(\boldsymbol{\rho}; \xi, l_s)$ is the zero mean illumination pattern defined by

$$\begin{aligned}\tilde{S}(\boldsymbol{\rho}; \xi, l_s) &= \sum_{m=1}^M \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_m; \xi) * \tilde{e}(\boldsymbol{\rho}; l_s) \\ &= C_s(\boldsymbol{\rho}; \xi) * \tilde{e}(\boldsymbol{\rho}; l_s),\end{aligned}\quad (\text{S20})$$

The function $\tilde{e}(\boldsymbol{\rho}; l_s)$ represents the variation of the elementary unit of illumination pattern with respect to the average illumination intensity and, therefore, $\int \tilde{e}(\boldsymbol{\rho}; l_s) d\boldsymbol{\rho} = 0$. As a result, the intensity fluctuation becomes

$$\begin{aligned}\tilde{I}(\xi) &= \int (C_s(\boldsymbol{\rho}; \xi) * \tilde{e}(\boldsymbol{\rho}; l_s)) \cdot (C_v(\boldsymbol{\rho}) * e(\boldsymbol{\rho}; l_v)) d\boldsymbol{\rho} \\ &= \int C_s(\boldsymbol{\rho}; \xi) \cdot (C_v(\boldsymbol{\rho}) * (\tilde{e}(\boldsymbol{\rho}; l_s) * e(\boldsymbol{\rho}; l_v))) d\boldsymbol{\rho} \\ &= \int C_s(\boldsymbol{\rho}; \xi) \cdot (C_v(\boldsymbol{\rho}) * h(\boldsymbol{\rho}; l_s, l_v)) d\boldsymbol{\rho}\end{aligned}\quad (\text{S21})$$

The function $h(\boldsymbol{\rho}; l_s, l_v) = \tilde{e}(\boldsymbol{\rho}; l_s) * e(\boldsymbol{\rho}; l_v)$ represents the variation of the convolution between the elementary units describing the scattering potential and the illumination pattern, or in other words, $\int h(\boldsymbol{\rho}; l_s, l_v) d\boldsymbol{\rho} = 0$. With this notation the intensity fluctuation becomes

$$\begin{aligned}\tilde{I}(\xi) &= \int C_s(\boldsymbol{\rho}; \xi) * \sum_{n=1}^N h(\boldsymbol{\rho} - \boldsymbol{\rho}_n; l_s, l_v) d\boldsymbol{\rho} \\ &= \sum_{m=1}^M \sum_{n=1}^N h(\boldsymbol{\rho}_m - \boldsymbol{\rho}_n; l_s, l_v, \xi) = \sum_{p=1}^P h(R_p; l_s, l_v, \xi)\end{aligned}\quad (\text{S22})$$

where $P = MN$. Note that $\boldsymbol{\rho}_m$ changes from one realization to the other one, and R_p is a random variable with non-uniform PDF.

The intensity fluctuation $\tilde{I}(\xi)$ attains its maximum when each term in the summation in Eq. (S7) is maximized. No matter what the PDF of R is, $h(R; l_s, l_v, \xi)$ is maximum for $R = 0$. As a result, the intensity fluctuation increases by increasing $h(0; l_s, l_v)$. One can easily show that:

$$h(0; l_s, l_v) = \int \tilde{e}(\boldsymbol{\rho}; l_s) e(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho} = \int \tilde{e}(\boldsymbol{\rho}; l_s) \tilde{e}(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho} \quad (\text{S23})$$

where $\int \tilde{e}(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho} = 0$. Using Schwarz's Inequality

$$(\int \tilde{e}(\boldsymbol{\rho}; l_s) \tilde{e}(\boldsymbol{\rho}; l_v) d\boldsymbol{\rho})^2 \leq \int \tilde{e}(\boldsymbol{\rho}; l_s)^2 d\boldsymbol{\rho} \int \tilde{e}(\boldsymbol{\rho}; l_v)^2 d\boldsymbol{\rho} \quad (\text{S24})$$

with equality if and only if $l_s = l_v$. In other words, variation of intensity fluctuation becomes maximum if $l_s = l_v$.

A similar reasoning can be followed for an arbitrary scattering potential with non-monotonic correlation function which can always be approximately represented as

$$V(\boldsymbol{\rho}; l_v) \approx \sum_{k=1}^K \sum_{n=1}^N a_{n,k} \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_n) * e(\boldsymbol{\rho}; l_{v,k}) \quad (\text{S25})$$

where $a_{n,k}$ are scattering coefficients and $l_{v,k}$ is one of the K characteristic lengths of the non-monotonic scattering potential.

4. Procedure repeatability

In the main manuscript and Supplementary Material, we have shown that a random scattering potential is characterized by statistically analyzing fluctuations of scattered light in response to a random illumination, from the potential. As shown in Eq. (5) in the main text, if perturbations sources are stationary, acquired parameters describing the information will not change by

repeating the experiment. This is the advantage of working with statistical parameters.

Here we considered two different cases. In the first example we consider the target shown in figure 3(a) in the manuscript and in the second case we consider the results shown in Figure 5 in the text. In the first example the sensing procedure is reliable if the two peaks are still distinguishable and in the second example, the error bar should not decrease the visibility of the changes in the Fano factor spectrum. To make error bars more visible, they are plotted for $\pm 3\sigma$ instead of $\pm \sigma$ of variation of the spectrum while repeating the sensing procedure. To consider effect of noise, these examples are demonstrated with $\text{SNR} \approx 2\text{dB}$.

Figure S2. Demonstration of repeatability of the sensing procedure for target shown in figure 3 (a) figure 5 (b) in the main text. In figure (b) continuous line and dashed line are corresponding to the symmetric and asymmetric case respectively. Error bars show $\pm 3\sigma$ of variation of the spectrum over repetition.

As evident from the figure S2, the variation in Fano spectrum is very small. Although the error bars are plotted for $\pm 3\sigma$, they are significantly smaller than the variation of the spectrum itself. These examples show that even in highly perturbed condition, working with statistical parameters makes the characterization of a scattering potential more reliable. This is because the sources of perturbation are stationary stochastic processes that are statistically independent from the non-stationary random sampling imposed by the illumination. This property makes them statistically distinguishable.

5. Optical setup

An experimental setup was designed to demonstrate the proposed stochastic sensing technique and the result is shown in figure 3(d) in the text. Schematic of the setup is shown in figure S3. For this purpose a polarized CW laser beam (Doubled Nd:Yag, $\lambda=532$ nm) is expanded with a home built telescopic beam expander and then polarized to 45° using a half wave plate followed by a polarizer. This beam illuminates a liquid crystal spatial light modulator (SLM) operating in reflection mode with 512×512 pixels, pitch size of $15\mu\text{m} \times 15\mu\text{m}$ (Boulder Nonlinear System Model P512-0532). The SLM impose a spatially varying polarization onto a laser beam which is converted to an intensity modulation by a polarizer after spatially filtering reflected light from the SLM in its Fourier plane to remove higher diffraction orders created by the pixilation of the SLM.

Relay optics are used to image the amplitude modulated light onto the back focal plane of a 60x microscope objective (Edmund Optics DIN 60X, $\text{NA}=0.86$). The magnification is adjusted through this relay optics system. Reflected light from the target is collected by a silicon integrator detector (PULNix TM-1040, with peak quantum efficiency of 38%) and generated voltage is recorded using NI PCI-6280 National Instrument data acquisition device. The entire process is controlled by LabVIEW software developed specifically for this purpose. The experimental SNR is calculated by

the ratio of power of reflected light from the target to power of background light when the target path is blocked.

Figure S3. Detailed Schematic of experimental setup. An expanded polarized CW laser $\lambda=532\text{nm}$ is polarized to 45° and then modulated by phase only spatial light modulator (SLM). After the first lens $f=125\text{mm}$, the Iris is blocked higher diffraction order created by the pixilation of the SLM. Phase modulated light is converted to an amplitude modulated light by a polarizer. Reflected beam from the target is directed toward the integrator detector by a beam splitter cube (BS).

6. Auto-correlation function of non-sparse random targets used in main text

As evident in the results included in Figure 5(c) in the main text, changes in the auto-correlation function of non-sparse targeted potential shown in figure 5(a) and (b) in the text, could be sensed by increasing tail of Fano factor spectrum due to introducing a new characteristic length. In this section we will compare auto-correlation function of the two targets to see how it has been changed by elongating some of the objects along different directions. Auto-correlation functions for symmetric and asymmetric cases are shown in figure S4(a) and (b) respectively.

As it is clear from figure S4(c), auto-correlation function has changed slightly in the side lobes. Sensing this small change can be challenging due to the environmental noise. However, this change is reliably measured as it is shown in figure 5(c) and S2(b).

Figure S4. Comparison of correlation function of the non-sparse random shape target in (a) Symmetric and (b) Asymmetric phase. (c) Comparison of average of central portion of the correlation

function along vertical axis for (green solid line) symmetric and (red dotted line) asymmetric case.

7. High-speed experimental setup

The scattering potential is considered to be stationary in this paper. As a result, to maintain this valid during the biological sample examination, the experimental setup was modified to work fast enough. Specifically, if one wish to repeat the experiment, the setup has to work much faster than the evolution time constant of the target. The result shown in figure 6(b) in the text, is the average of 100 measurements on live H9c2 cells. For this experiment, the experimental setup was modified to take advantage of digital micro mirror device (DMD) as a fast intensity light modulator. For this purpose we used a 0.7" XGA VIS, TI DLP Discovery D4100 kit DMD to spatially modulate light generated by a CW laser with $\lambda=532\text{nm}$. The total reflection from the target is recorded with a Hamamatsu H6180-01 photomultiplier tube (PMT) connected to a data acquisition counter device PCI 6602 by National Instrument, and the sensing procedure is controlled with a LabVIEW software developed specifically for this purpose. Schematic of this setup is shown in figure S5.

Figure S5. Detailed Schematic of experimental setup. An expanded polarized CW laser $\lambda=532\text{nm}$ intensity modulated by DMD. After the first lens $f=100\text{mm}$, the Iris is blocked higher diffraction order created by the pixilation of the SLM. Reflected beam from the target is directed toward the photomultiplier tube (PMT) a beam splitter cube (BS).

8. Biological Sample Preparation

H9c2 cells are cultured in a 35mm glass bottom petri dish by MatTek Corporation with class 0 cover slip in the bottom. Cells are grown in cell aqueous media (DMEH+10%FPS+1x p/s), and then they are left in the incubator over night to have enough time to stick to the cover slip. In the next day, petri dish is used as sample in the setup and structured light, is illuminated from the bottom. Cells are examined after the experiment and they were still healthy and alive.