Ultracompact metasurface in-line polarimeter: supplementary material

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This document provides supplementary information to "Ultracompact metasurface in-line polarimeter," <http://dx.doi.org/10.1364/optica.3.000042>. Figure S1 shows the channel output data recorded using the NIR camera. Figures S2-S5 show data of polarization measurements at wavelengths of 1500 nm, 1530 nm, and 1565 nm, and measurements at 1550 nm including intensity measurements. Sections 1-6 provide more in-depth discussions of some aspects of this work regarding the design and behavior of the antenna array (Sections 1-5) and the calibration measurements (Section 6). © 2016 Optical Society of America
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Figure S1: Channel output data. Images obtained of the channel outputs (visible as bright spots) for different input polarizations which correspond to the cardinal directions on the Poincare sphere. For this particular measurement, a reference image (REF) containing the scattered light remaining after optical filtering was subtracted to emphasize the channel output in the images (this was not necessary for the measurement shown in Fig. 4, where we improved the optical filter). The trapezoidal shadow of a piece of aluminum foil that was used to block the directly transmitted light is visible in the reference image. Each incident polarization gives rise to a unique combination of channel intensities.

Figure S2: State of polarization measurements at 1500nm. Comparison of state of polarization measurements made with the commercial polarimeter (Thorlabs, blue) and the antenna array (orange) at a wavelength of 1500 nm.

Figure S3: State of polarization measurements at 1530nm. Comparison of state of polarization measurements made with the commercial polarimeter (Thorlabs, blue) and the antenna array (orange) at a wavelength of 1530 nm.

Figure S4: State of polarization measurements at 1565nm. Comparison of state of polarization measurements made with the commercial polarimeter (Thorlabs, blue) and the antenna array (orange) at a wavelength of 1565 nm.

Figure S5: Intensity measurements. Given that the light propagating in the fiber has constant, fully polarized DOP, the polarimeter can be used to measure intensity, changes in which correspond to a homogenous increase or decrease of the intensity on all channels. In the dataset shown, both intensity and polarization were varied and referenced using the commercial polarimeter at an incident wavelength of $\lambda = 1550 \text{ nm}$. The data taken with the antenna array polarimeter is shown in orange and the data taken with the commercial polarimeter (Thorlabs) is shown in blue.

Supplement 1: Cylindrical wave emission by a row of sub-wavelength spaced antennas.

We analyze the field emitted by a long row of sub-wavelength spaced dipoles that is parallel to the x-axis. In figure S1, the x-polarized electric field emitted by such a row is shown. Coupling between the dipoles is neglected.

Figure S6: Cylindrical wave emitted by a long row of dipoles. Shown is the x-polarized component of the electric field emitted by a long row of dipoles parallel to the x-axis, located at $y=0, z=0$. The wavelength of the emitted wave shown here is normalized to $\lambda = 1$. The spacing of the individual antennas is 0.25.

To a very good approximation, the electric far-field scattered by a long row of sub-wavelength spaced antennas is a cylindrical wave of the form:

$$\mathbf{E} = \frac{E_0 e^{i\mathbf{k} \cdot \mathbf{r}}}{\sqrt{|\mathbf{r}|}} \hat{\mathbf{x}}$$

Where E_0 is the complex amplitude, $\mathbf{r}$ is the radial distance vector from the x-axis to the observation point, and $\hat{\mathbf{x}}$ is a unit vector along the x-axis. $\mathbf{k}$ is the wavevector of the cylindrical wave, with magnitude $|\mathbf{k}| = 2\pi/\lambda$. The polarization of the scattered field is *independent* of the polarization of the wave that excites the antennas (it is always along the axis of the row of antennas).

A. The electric field is primarily polarized along the axis of the row (x).

Figure S7: Polarization components of the field emitted by a row of subwavelength spaced dipoles. x (blue), y (orange) and z (yellow) polarized components of the electric field in the direction perpendicular to a row situated at $y=0, z=0$ (refer to Fig S6). The field is predominantly polarized along the axis of the row (x).

B. The intensity decay perpendicular to the axis of the column follows $1/r$.

Figure S8: Intensity decay of the field emitted by a row of subwavelength spaced dipoles. Following the cylindrical wave approximation, the intensity $|E_x|^2$ of the E_x component of the electric field decays as the inverse of the distance. Consequently $1/|E_x|^2$ (blue) increases linearly away from the row. Straight lines are shown in comparison (orange). Here the field is shown for a row of 60 antennas. The finite length of the row results in the small ripples that can be seen further away from the column.

Supplement 2: Electric field radiated by a single pair of rows of sub-wavelength spaced antennas.

Figure S9: Polarization-dependent directional scattering by two parallel rows of subwavelength spaced dipoles. The intensity of the scattered field of a pair of rows in the y-z plane for right-handed elliptically polarized incident radiation, showing directional scattering in the y-direction. The wavelength is normalized to $\lambda = 1$.

Let an electromagnetic plane wave be incident normally upon the row-pair along the z-axis, and have a polarization described by an angle ψ and a phase δ , and an electric field amplitude of E_0 :

$$\mathbf{E}_{\text{in}} = E_0 \begin{bmatrix} \cos\psi \\ \sin\psi e^{i\delta} \end{bmatrix} \quad (\text{S1})$$

Assume that the antennas within each row have an angle of $\pm 45^\circ$ with respect to the row axis, so that they couple with equal efficiency c to orthogonal polarization components of the incident field, where the coupling efficiency is some complex constant such that $|c|^2 \leq 1$. Both rows are parallel to the x-axis. The first row is located at $y=0, z=0$ and scatters a cylindrical wave:

$$\mathbf{E}_1 = \frac{cE_0 e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{|\mathbf{r}|}} \cos\psi \hat{\mathbf{x}} \quad (\text{S2})$$

The second row is located at $y = S, z = 0$ and emits a field:

$$\mathbf{E}_2 = \frac{cE_0 e^{i\mathbf{k}\cdot(\mathbf{r}-\hat{\mathbf{x}}S)}}{\sqrt{|\mathbf{r}-\hat{\mathbf{x}}S|}} \sin\psi e^{i\delta} \hat{\mathbf{x}} \quad (\text{S3})$$

The total scattered field is then $\mathbf{E}_{\text{sc}} = \mathbf{E}_1 + \mathbf{E}_2$, i.e.:

$$\mathbf{E}_{\text{sc}} = \frac{cE_0 e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{|\mathbf{r}|}} \left[\cos\psi + \sin\psi e^{i(\delta - |\mathbf{k}|S\cos\theta)} \left(1 + \frac{1}{2} \frac{S}{|\mathbf{r}|} \cos\theta + O\left(S^2/|\mathbf{r}|^2\right) \right) \right] \quad (\text{S3})$$

For $S/r \ll 1$, the intensity of the scattered field observed at a point $\mathbf{r}$ is then given by:

$$|\mathbf{E}_{\text{sc}}|^2 = |\mathbf{E}_1 + \mathbf{E}_2|^2 = \frac{|cE_0|^2}{|\mathbf{r}|} \left[1 + 2 \sin\psi \sin\psi \cos(|\mathbf{k}|S\cos\theta - \delta) \right] \quad (\text{S3})$$

Supplement 3: Electric field emitted by multiple pairs of antenna rows, spaced apart by a constant distance D .

In order to increase the cross-section of the polarimeter, several pairs of rows can be spaced in parallel at a distance that corresponds to an integer multiple of the wavelength. For $2N+1$ row pairs spaced apart by D , the scattered field is:

$$\mathbf{E}_{\text{sc},2N+1} = \sum_{n=-N}^N \left[\frac{cE_0 e^{i\mathbf{k}\cdot(\mathbf{r}-\hat{\mathbf{x}}nD)}}{\sqrt{|\mathbf{r}-\hat{\mathbf{x}}nD|}} \cos\psi + \frac{cE_0 e^{i\mathbf{k}\cdot(\mathbf{r}-\hat{\mathbf{x}}(nD+S))}}{\sqrt{|\mathbf{r}-\hat{\mathbf{x}}(nD+S)|}} \sin\psi e^{i\delta} \right] \quad (\text{S4})$$

Which, assuming that $S(|\mathbf{r}-\hat{\mathbf{x}}nD|) \ll 1$ and $ND/|\mathbf{r}| \ll 1$ (i.e., assuming that we are observing from a point far away from the polarimeter), can be rewritten as:

$$\begin{aligned} \mathbf{E}_{\text{sc},2N+1} &= \frac{cE_0 e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{|\mathbf{r}|}} \left[\cos\psi + \sin\psi e^{i(\delta - |\mathbf{k}|S\cos\theta)} \right] \left[\sum_{n=-N}^N e^{-i|\mathbf{k}|Dn\cos\theta} \right] \\ &= \mathbf{E}_{\text{sc}} f_N(|\mathbf{k}|D\cos\theta) \end{aligned} \quad (\text{S5})$$

Where $f_N(|\mathbf{k}|D\cos\theta) = \left[\sum_{n=-N}^N e^{-i|\mathbf{k}|Dn\cos\theta} \right]$ depends on N

(i.e. the number of row-pairs), as well as the ratio of the spacing to the wavelength (D/λ), but not on polarization. Additional row-pairs therefore *do not affect the polarization response* of the array, but they do affect the directionality of the scattered field, which is given by $|\mathbf{E}_{\text{sc},2N+1}|^2 = |\mathbf{E}_{\text{sc}}|^2 |f_N(|\mathbf{k}|D\cos\theta)|^2$.

Fig S10 shows $|f_N(|\mathbf{k}|D\cos\theta)|^2$ for $|\mathbf{k}|D = 2$ for $N = 0, 1, 5, 50$, corresponding to 1, 3, 11, and 51 pairs of rows. As expected, for a single row-pair $|f_N|^2$ results in no modulation of

the scattered field. For $N > 0$, diffraction orders emerge, which become sharper but do not change angle with increasing N .

Figure S10: Diffraction orders of an array with 2λ spacing as a function of the number of unit cells. Diffraction orders of an array of $N=0$ (blue), $N=1$ (orange), $N=5$ (yellow) and $N=50$ (purple) row-pairs ($S = (1 + 1/8)\lambda$) spaced two wavelengths apart, showing a progressive sharpening, but no change in direction, of the diffraction orders with increasing N .

In our experiment, we wish to sample the radiation field in-plane with the array, so that we choose D such that $|\mathbf{k}|D = 2\pi m$, where m is an integer (in our case 2, this is the case also in Fig S10). This condition is only satisfied for one wavelength λ_0 . We designed the polarimeter to work best at a wavelength of 1550nm , which, in BCB with index $n_{\text{BCB}} \approx 1.5$, corresponds to $D = 2\lambda_0 \approx 2000\text{nm}$. Fig S11 shows the change of $|f_N|^2$ for $N = 5$ and the C-band wavelengths between 1530-1565nm. Note that the wavevector $|\mathbf{k}| = 2\pi n_{\text{BCB}}/\lambda$, where λ is the free-space wavelength. Changing the wavelength greatly affects the directionality of the radiation field of the array in the plane of the array. It can therefore be advantageous to place the antenna array in a waveguide geometry to maximize the efficiency of the detection process over a range of wavelengths. Wavelength dependence can also be alleviated for example by placing the detectors such that they detect the first-order diffraction lobes (at $\theta \approx 60^\circ$ and $\theta \approx 120^\circ$, or $\theta \approx -60^\circ$ and $\theta \approx -120^\circ$), which are much less sensitive to wavelength fluctuations (Fig S12). However, the advantage of co-planar detection is lost in this case, and detection would have to be made at a distance at which the lobes have sufficiently separated (i.e. in the far field). As mentioned in the main text, in particular the latter is highly undesirable as greatly increases system footprint, along with the associated stability issues such as alignment problems [7,31,37].

Figure S11: Wavelength dependence of the diffraction pattern. Normalized modulation $|f_N|^2 / (2N+1)^2$ of the radiated intensity due to the presence of multiple pairs of rows, where $D = 2000$ nm and the free-space wavelength is varied across the C-band from 1530 nm to 1565 nm. The intensity of the in-plane scattered field (where we placed the detectors in the present experiment) is very sensitive to the wavelength, but has acceptable intensity across the C-band.

Figure S12: Wavelength dependence of the diffraction pattern (II). Normalized modulation $|f_N|^2 / (2N+1)^2$ of the radiated intensity due to the presence of multiple pairs of rows, where $D = 2000$ nm and the free-space wavelength is varied from 1400 nm to 1700 nm. The first-order diffraction lobes at $\theta \approx \pm 60^\circ$ and $\theta \approx \pm 120^\circ$ are relatively insensitive to this wavelength variation.

Supplement 4: Device matrix design

As mentioned in the main text, polarization measurements generally require several power measurements that correspond to different polarization components of the signal. Assuming a linear relationship, the set of power measurements $\mathbf{P}$ is related to the

incident Stokes vector via a matrix transformation $\mathbf{P} = \mathbf{M}\mathbf{S}$. That is:

$$\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{pmatrix} = \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n,1} & m_{n,2} & m_{n,3} & m_{n,4} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (\text{S6})$$

Each power measurement P_i then corresponds to a projective measurement of $\mathbf{S}$ along a row-vector of $\mathbf{M}$ that is $P_i = (\mathbf{m}_{i,1}, \mathbf{m}_{i,2}, \mathbf{m}_{i,3}, \mathbf{m}_{i,4}) \cdot \mathbf{S} = \mathbf{D}_i \cdot \mathbf{S}$. We refer to these row-vectors as ‘device vectors’⁸, which encode the polarization-selective properties of one channel of the polarimeter. The magnitude of the device vectors corresponds to the maximal coupling efficiency to a particular channel, which depends in particular on the scattering efficiency of the antenna array. As will become clear below, the current four-channel antenna array design does not allow full flexibility over the polarization states that may be selected, and consequently we cannot address the polarization states which correspond to the absolute optimum choice with respect to sensitivity to the full Stokes vector. The optimal choice of the polarization components that should be sampled in order to minimize measurement error is a general property of polarimeters, and is treated extensively in the literature [37]. For a four-channel device, for example, the optimal device vectors should form a regular tetrahedron on the Poincare sphere. The choice of the device vectors we make here yields good resolution of the state of polarization, i.e. it is obvious that any vector on the surface of the Poincare sphere will yield a distinct combination of intensities on the four channels. However, since the device vectors as designed all lie in plane, they in principle do not yield information about both intensity and degree of polarization. This is because the Stokes description of radiation describes the state of the wave as a point in a four-dimensional conical vector space. The description of polarization as a point on a sphere corresponds to the three-dimensional projection of this vector space on a plane perpendicular to the $\mathbf{S}_0$ axis (the axis of the cone). In order to be sensitive to this fourth dimension, the four device vectors would have to span a volume between them (as opposed to lying in plane, as is the case for our current device). As is, changes in DOP cannot be unambiguously distinguished from changes in SOP. Identical output intensity on all channels (as designed) could either correspond to unpolarized light or to the SOP that has equal projection along all four device vectors – when the device vectors span a volume between them, this symmetry is broken. However, given that we are operating with light from a narrowband laser source propagating in a single mode fiber, we assume the DOP to be 1 (which was also confirmed through measurement). In this case a homogenous increase or decrease of the channel intensities unambiguously corresponds to a change in intensity, and each combination of channel intensities unambiguously corresponds to a particular SOP. A very well written reference for understanding the 4D geometry of the full Stokes vector space is given by J. S. Tyo [37]. Having established the cylindrical wave approximation for a row of sub-wavelength spaced antennas in the previous section, we proceed to show how two parallel rows cause an asymmetric scattered field with sensitivity to polarization helicity.

5. Device matrix implementation with two sets of parallel rows

Equation (S7) shows that, for a normally incident plane wave, the polarization dependence of the field scattered by a pair of rows is independent of the number of row-pairs. It is already contained in the expression for the scattered field of a single row-pair (Equation S5), namely as:

$$A(\psi, \delta; \theta) = \cos\psi + \sin\psi e^{i(\delta - |\mathbf{k}|S\cos\theta)} \quad (\text{S7})$$

We have already decided to collect the scattered field left and right, in-plane with the array (i.e. $\theta = 0, \pi$), for which equation (8) becomes:

$$\begin{aligned} A_R(\psi, \delta; \theta = 0) &= \cos\psi + \sin\psi e^{i(\delta - |\mathbf{k}|S)} \\ A_L(\psi, \delta; \theta = \pi) &= \cos\psi + \sin\psi e^{i(\delta + |\mathbf{k}|S)} \end{aligned} \quad (\text{S8})$$

The device vector will correspond to the polarization (given by $\psi \in (0, \pi/2)$, $\delta \in (-\pi, \pi)$ c.f. equation (1)) for which:

$$|A_{R,L}|^2 = 1 + 2\cos\psi \sin\psi \cos(\delta \mp |\mathbf{k}|S) \quad (\text{S9})$$

is maximal. This fixes $\psi = \pi/4$, while the phase δ for which $|A_{R,L}|^2$ is maximal can be adjusted via changing S . $\delta = \pm\pi/4$ corresponds to elliptical polarizations, which can be regarded as “half way” between circularly and linearly polarized light, and a corresponding device vector is equally able to measure helicity and axis of linear polarization (Fig S8). Equation S10 is then maximal for S such that $|\mathbf{k}|S = \pi/4 + m\pi$, where m is an integer, corresponding to $S = (1/8 + m)\lambda_0$, where λ_0 is the operating wavelength in the material in which the polarimeter is embedded.

Figure S13: Polarization ellipses of the directionally scattered polarization states. Polarization Ellipses for $\psi = \pi/4$, and (a) $\delta = \pi/4$, resulting in right-handed helicity and (b) $\delta = -\pi/4$, resulting in left-handed helicity. Note that E_1 and E_2 are parallel to the antennas, not the rows.

Using Equation S10, we can deduce that the device vectors for a row-pair separated by $S = (1/8 + m)\lambda_0$ are:

$$\begin{aligned} \mathbf{D}_R &= c^2 \left[1, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), 0, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) \right] \\ \mathbf{D}_L &= c^2 \left[1, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), 0, \frac{1}{2} \left(-\frac{1}{\sqrt{2}} \right) \right] \end{aligned} \quad (\text{S10})$$

with respect to the axes of the rows.

For two arrays of $S = (1/8 + m)\lambda_0$ row-pairs, rotated by 45° with respect to each other, we will therefore anticipate device vectors close to:

$$\begin{aligned} \mathbf{D}_1 &= c^2 \left[1, 0, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) \right] \\ \mathbf{D}_2 &= c^2 \left[1, 0, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), \frac{1}{2} \left(-\frac{1}{\sqrt{2}} \right) \right] \\ \mathbf{D}_3 &= c^2 \left[1, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), 0, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) \right] \\ \mathbf{D}_4 &= c^2 \left[1, \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right), 0, \frac{1}{2} \left(-\frac{1}{\sqrt{2}} \right) \right] \end{aligned} \quad (\text{S11})$$

Where $\mathbf{D}_1$ and $\mathbf{D}_2$, as well as $\mathbf{D}_3$ and $\mathbf{D}_4$ each correspond to measurements on opposite sides of the polarimeter. As mentioned in the main text, it is important to keep in mind that these device vectors do not correspond exactly to the device vectors realized in the final implementation. The latter are subject to a host of effects that will distort the polarization response. This includes for example deviations from the approximation of the antennas as non-interacting, identical dipoles, including that the scattered field by each of the antennas is not further rescattered by other antennas. Reflections at the cladding or substrate interfaces is another reason, as well as potentially inhomogeneous detection efficiencies of the scattered light from the outcoupling gratings.

Examining Eq. S11 reveals that the polarization components scattered to opposite sides of the structure always the same angle of linear polarization given by ψ and opposite helicity given by the sign of δ . This represents the aforementioned constraint on the choice of device vectors scattered towards opposite sides of the structure. The value of ψ and δ that are selected can be chosen by changing the orientation of the row pair, and the row-spacing S respectively. However, with a given polarization state scattered towards the left side, the polarization state scattered towards the right side always corresponds to the same polarization state with conjugate helicity, or its mirror reflection about the $s_1 - s_2$ plane on the Poincaré sphere. With two row pairs, it is hence impossible to design for device vectors that do not lie in a plane. This is, however, possible with a three-pair design which we defer to future work (Fig. S14a). As mentioned in the main text, using additional output channels can also be used to reduce statistical error on polarization measurements. A design with 8 outputs is shown in Fig S9b. The device vectors of the 6 and 8 output designs are shown in Fig S10. Given that we conducted our experiment with fully polarized light, the simpler 4-vector design was adequate.

Figure S14: Exemplary antenna array designs with more than four outputs. (a) Antenna array with six output channels, corresponding to three overlapping row-pairs rotated at 60° angles with respect to each other. The antennas are shown in red. (b) Antenna array with eight output channels, corresponding to four overlapping row-pairs rotated at 45° degree angles with respect to each other.

Figure S15: Directionally scattered polarization components of the 6- and 8- output designs. Device vectors of the 6 output design (a) and the 8 output design (b).

Supplement 5: Device matrix calibration

To determine the device matrix, a set of at least four sufficiently different, known polarization states are used to illuminate the polarimeter. As mentioned above, the power P measured on the i th channel under illumination by a particular polarization $\mathbf{S}$ is given by the dot product of that channel's device vector $\mathbf{D}_i$ and the incident Stokes vector $\mathbf{P} = \mathbf{S} \cdot \mathbf{D}_i$ (note Stokes vectors are real vectors, so $\mathbf{S} \cdot \mathbf{D}_i = \mathbf{D}_i \cdot \mathbf{S}$). Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3 \dots \mathbf{a}_n$ be the Stokes vectors of known polarization states used for the calibration of the polarimeter. Then the corresponding power measurements $\mathbf{P}$ on the i th channel are related to that channel's device vector $\mathbf{D}_i$ via a matrix $\mathbf{A}$ that contains the Stokes vectors used for calibration as rows:

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{pmatrix} = \begin{pmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_n^T \end{pmatrix} \mathbf{D}_i = \mathbf{A} \mathbf{D}_i \quad (\text{S12})$$

The device vector is then found by inverting the matrix $\mathbf{A}$, that is: $\mathbf{D}_i = \mathbf{A}^{-1} \mathbf{P}$, where the left generalized inverse $\mathbf{A}^{-1} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$ can be used when $n > 4$. The appropriate choice of the calibration polarizations $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3 \dots \mathbf{a}_n$ is such

that the conditioning number of the corresponding matrix $\mathbf{A}$ is minimized. This minimizes the vulnerability of the measurement of $\mathbf{D}_i$ to measurement error. Assuming that the noise of the power measurements is Gaussian, using more than 4 calibration polarizations to measure the device vectors reduces the least squares error [23]. Once all device vectors are determined in this way, the device matrix is given by:

$$\mathbf{M} = \begin{pmatrix} \mathbf{D}_1^T \\ \mathbf{D}_2^T \\ \vdots \\ \mathbf{D}_n^T \end{pmatrix} \quad (\text{S13})$$

For the polarimeter experiment shown in the main text, the calibration was performed by gathering channel data for a large number of different polarization states distributed as homogeneously as possible over the Poincaré sphere, which were referenced using the commercial Polarimeter. Subsequently, the combination of four polarization states which minimized the conditioning number of $\mathbf{A}$ was selected to deduce the polarization-response of the four channels.

Supplement 6: Nanoantenna Response

The wavelength response of the nanoantennas was simulated with a commercial finite-difference time-domain (FDTD) solver (Lumerical) to better understand their resonant properties. The actual response of the array is affected also by Fabry-Perot type resonances in the substrate-BCB stack as well as possible interactions between the nano antennas.

Figure S16: FDTD simulation results for the scattering cross section of a 250x50x20nm gold antenna in BCB as a function of wavelength. The resonance lies close to the operating wavelength of the polarimeter, but does not coincide.