Gouy phase shift of a tightly focused radially polarized beam: supplementary material

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1. THEORETICAL BACKGROUND

Fig. S1. Illustration of a radially polarized beam as superposition of two, mutually orthogonal polarized Hermite-Gaussian beams.

A radially polarized beam can be considered as the superposition of two, mutually orthogonally polarized Hermite-Gaussian (HG) beams as illustrated in Fig. S1. These are general solutions of the Helmholtz equation in paraxial approximation. The time independent part of the electric field at a point (x, y, z) of such HG beams propagating along the z -axis has the general form [1]

$$E_{mn}(x, y, z) = E_0 \frac{w_0}{w(z)} H_m \left(\frac{\sqrt{2}x}{w(z)} \right) H_n \left(\frac{\sqrt{2}y}{w(z)} \right) \cdot \exp \left[-\frac{(x^2 + y^2)}{w^2(z)} + \frac{ik(x^2 + y^2)}{2R(z)} \right] \cdot \exp \left[i \left(kz - (m + n + 1) \arctan \frac{z}{z_0} \right) \right], \quad (\text{S1})$$

where H_m and H_n are Hermite polynomials, $k = \omega/c$ is the wavenumber, with the angular frequency ω and the speed of light in vacuum c , E_0 is the amplitude and w_0 the beam width in the focal plane $z = 0$. Note that the term $(m + n + 1) \arctan \frac{z}{z_0}$ is a general expression for the Gouy phase shift. The Rayleigh range z_0 , the beam waist $w(z)$, and the radius of curvature $R(z)$ are given by

$$z_0 = \frac{\pi w_0^2}{\lambda}, \quad (\text{S2})$$

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0} \right)^2}, \quad (\text{S3})$$

$$R(z) = z \left(1 + \frac{z_0^2}{z^2} \right). \quad (\text{S4})$$

Thus, the electric field vectors of the HG₀₁ and the HG₁₀ mode are given by

$$\tilde{E}_{10}(x, y, z) = E_0 \frac{w_0}{w(z)} x \hat{x} \exp \left[-\frac{(x^2 + y^2)}{w^2(z)} + \frac{ik(x^2 + y^2)}{2R(z)} \right] \cdot \exp \left[ikz - i2 \arctan \frac{z}{z_0} \right], \quad (\text{S5})$$

$$\tilde{E}_{01}(x, y, z) = E_0 \frac{w_0}{w(z)} y \hat{y} \exp \left[-\frac{(x^2 + y^2)}{w^2(z)} + \frac{ik(x^2 + y^2)}{2R(z)} \right] \cdot \exp \left[ikz - i2 \arctan \frac{z}{z_0} \right], \quad (\text{S6})$$

with $\hat{x}$ and $\hat{y}$ being the unit vectors in the x and y directions. The total electric field of a radially polarized beam, $E_{rad}(x, y, z, t)$, is given by the superposition

$$E_{rad}(x, y, z, t) = [\tilde{E}_{10} + \tilde{E}_{01}] \exp(-i\omega t), \quad (\text{S7})$$

resulting in the following expression for the electric field of a radially polarized beam

$$E_{rad}(\rho, z, t) = E_0 \frac{w_0}{w^2(z)} \hat{\rho} l_0(\rho, z) \cdot \exp \left[ikz - i2 \arctan \frac{z}{z_0} \right] \exp[-i\omega t], \quad (\text{S8})$$

with $\rho = (x^2 + y^2)^{1/2}$ and $\hat{\rho} = (x, y, 0)/\rho$ as a unit vector pointing in the radial direction. The radial amplitude function $l_0(\rho, z)$ is given by

$$l_0(\rho, z) = \rho \exp \left[ik \frac{\rho^2}{2R(z)} \right] \exp \left[-\frac{\rho^2}{w^2(z)} \right]. \quad (\text{S9})$$

According to Eq. (8) the Gouy phase shift of a radially polarized beam propagating from $z = -\infty$ to $z = +\infty$ is 2π . In contrast, the well-known Gaussian beam, e.g. for linearly polarized radiation, is obtained for $m, n=0$ in Eq. (1) resulting in

$$E(\rho, z, t) = E_0 \frac{w_0}{w(z)} \exp \left[\frac{-\rho^2}{w^2(z)} + ik \frac{\rho^2}{2R(z)} \right] \cdot \exp \left[ikz - i \arctan \frac{z}{z_0} \right] \exp[-i\omega t], \quad (\text{S10})$$

which yields the typical Gouy phase shift for a Gaussian beam of π . A phase shift of π results in polarization reversal of a linearly polarized beam which, as pointed out earlier, can be directly measured by THz time domain spectroscopy (THz-TDS) [2]. Unfortunately, such a direct measurement is not possible with a radially polarized beam, since the phase experiences a shift by 2π effectively leaving the polarization unchanged (no polarization reversal). Consequently, the phase of a wave has to be traced along a given path, rather than just determining its state before and after passing through the focus. Here we present a procedure, which allows to experimentally trace the phase of a focussed THz beam along the beam's entire path, as described in the following section.

2. GOUY PHASE SHIFT OF A LINEARLY POLARIZED THZ BEAM

Fig. S2. Field maps in the frequency domain of the transverse component of the linearly polarized beam at (a) 1.0 THz and (b) 1.35 THz.

We also apply our method to the extensively studied and well-known case of a focussed linearly polarized beam. In order to generate linearly polarized THz pulses the radial emitter has been replaced by a conventional photoconductive strip-line antenna, while the focussing mirror optics and the detection system was left unchanged. Figure S2 shows the measured field maps of the transverse field component of the y-polarized electric field plotted at two selected frequencies, showing the characteristic circular wavefronts of a focussed Gaussian beam. Within the Rayleigh range around the focus the strongest field strengths are observed.

Again the field has been projected onto the beam axis by the described procedure and is plotted in Fig. S3(a) together with a fit according to Eq. (8) of the main article. As demonstrated in Fig. S3(b) the wave experiences a Gouy phase shift of π after it has propagated through the focus so that it would be out-of-phase with a corresponding unfocussed plane wave. The spatial evolution of the phase is shown in Fig. S4 for 1.35 THz (a) and for various frequencies (b). The theoretically expected total Gouy phase shift of π (dashed curve in Fig. S4(a) with $n = 1$ and $z_0 = 641\mu\text{m}$) which has also been confirmed in previous studies of linearly polarized beams [2, 3] is in excellent agreement with our measurement.

REFERENCES

1. Q. Zhan, "Cylindrical vector beams: from mathematical concepts to applications," *Adv. Opt. Photon.* **1**, 1–57 (2009).
2. A. B. Ruffin, J. V. Rudd, J. F. Whitaker, S. Feng, and H. G. Winful, "Direct observation of the Gouy phase shift with single-cycle terahertz pulses," *Phys. Rev. Lett.* **83**, 3410–3413 (1999).
3. R. W. McGowan, R. A. Cheville, and D. Grischkowsky, "Direct observation of the Gouy phase shift in THz impulse ranging," *Applied Physics Letters* **76**, 670–672 (2000).

Fig. S3. Transverse component of the linearly polarized beam: (a) projected and integrated field data and fit and (b) comparison of fit with and without Gouy phase term.

Fig. S4. Transverse component of the linearly polarized beam: (a) Gouy phase shift at 1.35 THz and (b) compilation of several other frequencies.