Generation of intensity controlled two-dimensional shape preserving beams in plasmonic lossy media: supplementary material

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24 December 2015

This document provides supplementary information to “Generation of intensity controlled two-dimensional shape preserving beams in plasmonic lossy media,” <http://dx.doi.org/10.1364/optica.3.000015>. We elaborate here on several issues related to the information presented in the manuscript. © 2015 Optical Society of America

<http://dx.doi.org/10.1364/optica.3.000015.s001>

Mathematical derivation of the propagation characteristics of shape-preserving beams in lossy media.

Fig. S1. Geometrical analysis of the Cosine beam as an interference pattern between two plane-waves.

The electric field in point z is a superposition of the fields E_1 and E_2 . Under the geometric approximation, this field has an amplitude contribution of $|E_1(0, y)|e^{-k''d}$ from the upward propagating wave and amplitude contribution of $|E_2(0, -y)|e^{-k''d}$ from the downward propagating wave, where d is the distance between points $(0, \pm y)$ and $(z, 0)$ as can be seen in Figure S1, and k'' is the imaginary part of k_{sp} . Since both fields start with the same phase and propagate the same distance, they will constructively interfere at point $(z, 0)$:

$$|E(z, y = 0)| = [|E_1(z = 0, y)| + |E_2(z = 0, -y)|]e^{-k''d}$$

We aim for a central lobe to have the arbitrary intensity profile proportional to $A^2(z)$, therefore we write the condition:

$$|E(z, y = 0)| = A(z)$$

where $A(z)$ is a nonnegative arbitrary function. In addition, the condition for beam symmetry with respect to y axis is $|E_1(0, y)| = |E_2(0, -y)|$. Combining both conditions gives:

$$|E(z, y = 0)| = 2|E_1(z = 0, y)|e^{-k''d} = A(z)$$

Using the identities $\frac{y}{d} = \sin\theta$ and $\frac{z}{d} = \tan\theta$, we get:

$$|E_1(z = 0, y)| = \frac{1}{2}A\left(\frac{y}{\tan(\theta)}\right)e^{k''y/\sin(\theta)}$$

Under the geometric approximation, E_1 is a semi-plane wave propagating in direction $(k_y, k_z) = (\sin\theta, \cos\theta)$, and therefore has the form:

$$E_1(z, y) = |E_1(z, y)|e^{ik'(\sin\theta y + \cos\theta z)}$$

Hence the transverse distribution of the input field E_1 at $z=0$ is:

$$E_1(0, y) = A\left(\frac{y}{\tan(\theta)}\right)e^{k''y/\sin(\theta)}e^{ik'y\sin\theta}$$

Therefore the total field on the input transverse line $(0, y)$ is:

$$\begin{aligned} E(z = 0, y) &= E_1(0, y) + E_1(0, -y) \\ &= \frac{1}{2}A\left(\frac{y}{\tan(\theta)}\right)e^{k''y/\sin(\theta)}e^{ik'y\sin\theta} \\ &\quad + \frac{1}{2}A\left(\frac{-y}{\tan(\theta)}\right)e^{-k''y/\sin(\theta)}e^{-ik'y\sin\theta} \end{aligned}$$

The approximation above holds assuming E_1 and E_2 can be regarded as plane waves. This is true as long as the amplitude $\frac{1}{2}A\left(\frac{y}{\tan(\theta)}\right)e^{k''y/\sin(\theta)}$ varies slowly with respect to the wavelength. A frequency of period Λ in the modulation function would mean E_1 will have a component in direction $\theta + d\theta$, where $\sin(d\theta) = \pm \frac{\lambda}{\Lambda}$. We therefore expect the approximation to hold as long as $\theta \gg \frac{\lambda}{\Lambda}$.

Spatial behavior of infinite SPBs in lossy media. As mentioned in the manuscript, although in the analysis we focus on the central lobe of the plasmonic CSG beam, in the case of infinite SPBs in lossy media the entire beam will exhibit the same characteristics of the central lobe. This means that once $A(z)$ is chosen, all the lobes of the beam will exhibit the chosen longitudinal intensity behavior, up to a normalization constant. To demonstrate this we simulate in Fig. S2 a fixed intensity enhanced CSG plasmonic beam, whose aperture is much larger than the $80\mu\text{m} \times 80\mu\text{m}$ presented area, which to a good amount can be approximated as infinite. It can be seen that all the lobes within this area now present a fixed intensity characteristic (although with different relative intensity owing to the issue mentioned above), rather than primarily the central lobe, as presented for the smaller aperture CSG.

Fig. S2. Simulation of a constant intensity enhanced CSG plasmonic beam whose aperture is larger than the $80\mu\text{m}$ presented.

Encoding two-dimensional plasmonic near-field holograms.

As mentioned in the paper, the generation and accurate control of the plasmonic wavefront is crucial to achieve the desired unique SPBs. This is done by a realization of two-dimensional plasmonic nearfield holograms, whose encoding method is based on computer-generated-holograms [1]. These two-dimensional plasmonic holograms are encoded in a binary fashion by the following equation [2]:

$$t(z, y) = \frac{h_0}{2} \left[1 + \text{sign} \left[\cos \left(\frac{2\pi}{\Lambda} z + \phi(y) \right) - \cos(\sin^{-1}[A(y)]) \right] \right]$$

where $A(z, y)$, $\phi(z, y)$ are the amplitude and phase of the desired beam, Λ is the period of modulation, and h_0 is the height of the hologram. In the encoding scheme above the desired phase is imposed directly on the carrier frequency and the desired amplitude modulates the duty-cycle of the hologram. For the complete details please refer to [2].

Behavior of SPB-like plasmonic beam. As stated in the paper, there are multi-lobed beams that are not an exact SPB solutions of Helmholtz's equation, but still exhibit SPB properties over a finite distance, and therefore, can be manipulated by our approach. This was already shown for the regular plasmonic LCG beam [3], but not for a LCG with some arbitrary intensity distribution $A(z)$. In this case, $A(z)$ is defined to be zero for negative values of z . The expression for E then becomes:

$$E(z = 0, y > 0) = \frac{1}{2}A\left(\frac{y}{\tan(\theta)}\right)e^{k''y/\sin(\theta)}e^{ik'y\sin\theta}$$

and

$$E(z = 0, y < 0) = \frac{1}{2}A\left(\frac{-y}{\tan(\theta)}\right)e^{-k''y/\sin(\theta)}e^{-ik'y\sin\theta}$$

or alternatively, for every y :

$$E(z = 0, y) = \frac{1}{2}A\left(\frac{|y|}{\tan(\theta)}\right)e^{k''|y|/\sin(\theta)}e^{ik'|y|\sin\theta}$$

Note that for $A(z > 0) = 1$ and $k'' = 0$ we get the local cosine Gauss expression: $e^{-jk'|y|\sin\theta}$, multiplied by Gaussian. Since this expression is not a solution of Helmholtz's equation, it can be seen in Fig. S3(a) that the infinite plasmonic LCG beam is not shape preserving. However, even though it is not an exact SPB solution, as noted in [3] and [4], it will still exhibit the properties of being non-diffracting over a certain range. To assure this claim is valid, we present a previous experimental measurement of the self-healing property of the plasmonic LCG beam, presented in Fig. S3(b), taken from [2]. This validates the non-diffracting property of the beam since non-diffracting beams exhibit the "self-healing" property.

Fig. S3. (a) Simulation of a plasmonic LCG showing its non-diffracting property is limited to a certain range. (b) Experimental measurement of the self-healing property of the plasmonic LCG beam (taken from [2]).

Extended plasmonic propagation length of LCG beam. The previous generation of a plasmonic LCG beam, performed by Lin et al [3], at wavelength of 740nm , $\theta=10^\circ$, $W_0=8\mu\text{m}$, and its central lobe was reported to be non-diffracting over a distance of $80\mu\text{m}$. For a comparison, we show that applying our approach with the experimental conditions mentioned above, yields a plasmonic LCG beam whose central lobe is confined and do not diffract over a propagation length of almost three times that distance (Fig. S4).

Fig. S4. Comparison between a regular and enhanced plasmonic LCG beam, generated for the conditioned found at [3], showing an enlargement of almost three times the propagation length.

Finite size holograms and there effect on the generated beam.

As mentioned in the text, the holograms are of finite nature, meaning that they are built out of finite size features, which will create diffraction and interfere with the generated beam. This effect is similar to the general edge effect discussed in the paper, but refers locally to the holograms features. The difference between the output analytical beam and the output from simulated hologram, for the linearly growing plasmonic SPB beam, is presented in Figure S5, (a) and (b), respectively. This diffraction effect also slightly affects the designed trajectory of the beam, and a small difference between the analytical trajectories (Fig. S5(c), green line) and the experimental trajectory (Fig. S5(c), blue/red curves) is seen. However, it is clearly seen that the simulation of the output beam from the hologram (Fig. S5(c), purple curve) is in very good agreement with the experimental result, meaning that the trajectory can be easily corrected if needed.

Fig. S5. The difference between (a) the output analytical beam and (b) the output from the simulated hologram, for the linearly growing plasmonic SPB beam. (c) Although a small difference between the analytical trajectories (green line) and the experimental trajectory (blue/red curves) can be seen, a very good agreement between the simulation of the output beam from the

hologram (purple curve) and the experimental result (blue/red curves) is clearly seen.

Beam propagation using Green's function approach. The propagation dynamics of SPPs can be described by the 2D scalar Helmholtz Equation, while assuming that these are already transverse magnetic (TM) waves. Thus, the propagation dynamics can be described by the 2D Green's function of the 2D Helmholtz Equation.

The field at any given point $g(y, z)$ is a convolution of the sources distribution $f(y, z)$ with the Green function G :

$$g(y, z) = \iint f(y', z') G(y - y', z - z') dy' dz'$$

For the 2D case, $G_{2D} = -\frac{i}{4} H_0^{(2)}(kr)$, where $H_0^{(2)}(kr)$ is the

Hankel function.

This calculation can be used in order to predict the propagation dynamics of any 1D plasmonic wavefront, and specifically those of the enhanced SPB's.

Intensity evolution of the wavepacket. In order to elucidate the nature of the wave packet, we performed several calculations based on the second moment of the beam's intensity, for both the fixed intensity and the linearly growing intensity Cosine-Gauss beam.

We start by analyzing the evolution of the second moment for the fixed intensity beam, whose initial intensity profile and 2D distribution are the following:

Fig. S6. (a) Initial intensity profile at $z=0$, and (b) and 2D distribution of the fixed intensity beam

It can be seen from Fig. S6(b) that the three inner lobes are undistorted throughout the propagation distance. We therefore calculate the second moment of that area along the propagation coordinate and compare it to that of the full beam.

Fig. S7. Square root of the second moment calculation for the central three lobes and the complete beam, along the propagation coordinate.

It is seen from the calculation in Fig. S7 that the second moment of the inner lobes is the same throughout the propagation distance, meaning that these are indeed not disturbed, while that of the full beam gradually decreases due to energy transfer from the side lobes inwards, as seen in Fig. S6(b).

In order to assess the evolution of the intensity distribution inside the beam, we calculate the intensity residing within these three lobes relative to that in the full beam:

Fig. S8. Evolution of the relative intensity within the central three lobes along propagation.

The portion of the intensity residing in this area can be evaluated to an average of around 10% of the full intensity, and is growing with propagation due to the transfer of energy from the side lobes into the inner lobes.

Next we perform the same analysis on the linearly growing beam. It is seen from Fig. S9 that the second moment of the central lobes only slightly decreases, while that of the full beam decreases more significantly, meaning less energy within the undistorted lobes.

Fig. S9. Square root of the second moment calculations and relative intensity of the three main lobes for the linearly growing beam.

For a choice of a very sharp slope of the linearly growing beam this decrease will continue, and less energy will reside in the inner lobes compare to the outer lobes (Fig. S10). In addition, the initial second moment of the full beam is extended due to the higher intensity of the side lobes, and that of the inner lobes decreases further meaning that this area as well begins to be distorted.

Fig. S10. Square root of the second moment calculations and intensity evolution of the linearly growing beam, with a sharp slope.

We note that in all of the current discussion, the Gaussian envelope was kept at a fix width, and as mentioned in the manuscript, extending it will increase the inner area which is undistorted. However, this would also mean that even more energy will reside in the side lobes, further decreasing the portion of energy residing in the inner lobes. For convenience, comparison between all of the results is presented in Fig. S11.

Fig. S11. Comparison between the evolutions of the different beams.

Negligible diffraction effect. In order to assess the beam's diffraction we analyze its envelope properties near the edges of the aperture, where the field changes is rapidly. Since the beam is composed out of two tilted plane waves, each is growing exponentially towards the edge, and is then truncated by the Gaussian envelope, hence it will have the following transverse intensity distribution:

Fig. S12. Envelope function for a single plane-wave of the fixed intensity beam.

Therefore, the geometrical approximation holds as long as the beam divergence is negligible. We calculate for the envelope in Fig. S12 its second moment and from that we can deduce its Rayleigh range. For the fixed intensity beam this calculation yields a Rayleigh range of $\sim 235\mu\text{m}$, while for the linearly growing beam it yields $\sim 150\mu\text{m}$. This means that in all of our measurements, spanned across only $80\mu\text{m}$ of propagation, the diffraction is negligible and the geometrical approximation holds. We note that the Rayleigh range will continue to decrease with the addition of energy at the edges, making this effect more pronounced. This, however, can be corrected by a more gradual termination of the envelope at the edges of the aperture.

Generation of 1D plasmonic Bessel SPBs. Similarly to the LCG beam presented in the paper, the plasmonic 1D Bessel beam also exhibits a larger localization of energy in its main lobe (Fig. S13(a)). In addition it will exhibit all the properties of an SPB within a certain range. Fig. S13 presents the experimental measurements of both regular and enhanced plasmonic 1D Bessel SPB. It can be seen that while the central lobe of the regular 1D Bessel beam almost completely decays after 80 μ m of propagation, that of the enhanced 1D Bessel beam can propagate for longer distances with a fixed intensity.

Fig. S13. Experimental measurements of (a) a regular and (b) enhanced 1D plasmonic Bessel SPBs.

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