Nanophotonic cavity optomechanics with propagating acoustic waves at frequencies up to 12 GHz: supplementary material

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This document provides supplementary information to “Nanophotonic cavity optomechanics with surface acoustic waves at frequencies up to 12 GHz,” <http://dx.doi.org/10.1364/optica.2.000826>. The design of the photonic crystal nanocavity is discussed in more details. Additional representative simulation results of the surface acoustic wave (SAW) interdigital transducer (IDT) eigenmodes are provided. The method used to calculate the power of the SAW excited in the experiment is described. The optomechanical coupling coefficient pertaining to propagating phonons is defined. The optomechanical modulation in the sideband resolved regime is theoretically analyzed. Finally, the device parameters for the sideband resolved optomechanical experiments, especially the optomechanical coupling coefficient, are determined and summarized. © 2015 Optical Society of America

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1. PHOTONIC CRYSTAL NANOCAVITY DESIGN

In order to reduce loss of propagating acoustic wave, the photonic crystal nanocavity is shallow etched, leaving an AlN slab. Therefore, the radiative loss channels for shallow-etched photonic crystal nanocavity include radiation into the air, the substrate and the slab mode supported by the surrounding remaining AlN film. To achieve high optical quality factor, the radiative loss should be minimized by maximizing the effective index contrast between the fundamental nanocavity mode (n_{c0}) and the AlN slab mode (n_{slab}), because n_{slab} is

higher than the refractive indices of the substrate (n_{sub}) and the air.

To achieve strong optomechanical interaction, the overlap between the electric field distribution of the fundamental nanocavity mode and the mechanical strain field of the propagating SAW modes should be maximized. This indicates that the nanocavity should support dielectric modes [1] and n_{c0} should be maximized so that electric field is well confined inside the AlN structures.

The optimized nanocavity designs (subject to fabrication considerations) and coupling schemes are summarized as the follows.

Fig. S1. Photonic crystal nanocavity fundamental dielectric mode E_y profile. The vertical dashed line is the center of the nanocavity. The E_y profile is symmetric with respect to the center.

Table S1. Summary of the SAW IDT eigenmodes.

	IDT Period = 3 μm			IDT Period = 0.45 μm			
Rayleigh Mode Number	R1	R2	R3	R1	R6	R9	R14
Eigenfrequency (GHz)	1.22	1.48	1.74	8.39	9.20	10.11	11.76
Out of Plane Displacement Field Profile
Top Layer: AlN (330 nm)
Middle Layer: SiO ₂ (3 μm)
Bottom Layer: Si

The cavity and the coupling waveguide are etched 200 nm deep into a 330 nm thick AlN film on top of a 3 μm thick silicon dioxide layer. The etched AlN sidewall is tilted by 15°, so all the dimensions here on refer to those at half the etching depth. The cavity lattice constant and width are 500 nm (corresponding to $n_{\text{eff}} \approx 1.54$) and 800 nm, respectively. The rectangular hole in each unit cell is 600 nm long (in the transverse direction of the cavity), while the hole width varies gradually from 242 nm at the cavity center to 108 nm across 90 unit cells away from the cavity center, and then remains 108 nm from there to the two ends of the cavity [1]. The coupling waveguide is 800 nm wide. For the cavity side-coupled to the waveguide, the total cavity length is 120 μm (240 unit cells) and the minimum coupling gap is 1.6 μm , resulting in high loaded and intrinsic quality factors suitable for sideband resolved optomechanical experiments. For the cavity embedded inside the waveguide (i.e. end coupled), the total cavity length is reduced to 100 μm (200 unit cells) to facilitate the coupling to the waveguide, resulting in enhanced on-resonance transmission and moderate loaded quality factor suitable for sideband unresolved optomechanical transduction. The simulated mode profiles are shown in Fig. S1.

2. SAW IDT EIGENMODE CALCULATION

We employed finite element method simulation to calculate the eigenmodes of the SAW IDT. The calculation was done using the piezoplane strain model of the commercial FEM software, COMSOL Multiphysics. Due to the periodicity of the IDTs, only a unit cell, containing a pair of fingers, was simulated. A periodic boundary condition was imposed on the ends of the unit cell. The bottom boundary was set fixed, while the top boundary was allowed to displace freely. The calculation used experimentally reported material parameters [2]. The structure was meshed using appropriate parameters to increase the accuracy of the simulation. This calculation was done for the 3 μm and 0.45 μm IDTs that were implemented in the experiment. The calculation showed that, in the AlN/SiO₂/Si multilayers, the 3 μm IDT can excite three Rayleigh modes, whereas the 0.45 μm IDT can excite as many as sixteen Rayleigh modes. The eigenfrequencies of these modes agree well with the measured S_{11}

spectra of the IDTs (see Fig. 2(c)). The simulation results are summarized in Table S1.

3. CALCULATION OF THE SAW POWER

Fig. S2. Calculation of the SAW power from S_{11} spectrum. The resonance dip at 12.1 GHz is the R14 mode of the 0.45 μm period IDT.

The power of SAW wave that is electromechanically excited can be calculated from the S_{11} measurement of the IDT and the input RF power. Figure S2 shows the measured spectrum of $|S_{11}|^2$ centered at the frequency of the R14 mode of the 0.45 μm period IDT. At any frequency Ω , the total power dissipated at the IDT is

$$\begin{aligned}
 P_{\text{dis}} &= P_{\text{in}} \left(1 - |S_{11}(\Omega)|^2 \right) \\
 &= 2P_{\text{SAW}}(\Omega) + P_{\text{R}}(\Omega) \\
 &= P_{\text{in}} \left[2\eta_{\text{SAW}}(\Omega) + \eta_{\text{R}}(\Omega) \right],
 \end{aligned} \tag{S1}$$

$$\text{so, } 1 - |S_{11}(\Omega)|^2 = 2\eta_{\text{SAW}}(\Omega) + \eta_{\text{R}}(\Omega), \tag{S2}$$

where P_{in} is the total input RF power, P_{SAW} (η_{SAW}) is the power (electromechanical coupling efficiency) of the SAW traveling through the photonic crystal nanocavity. The factor of 2 accounts for the fact that the IDTs launch SAW in both directions. P_R (η_R) is the power loss (power loss ratio) at the IDT due to various mechanisms (e.g. Joule heating and electromagnetic radiation loss) other than conversion to SAW. Generally, η_R is slowly varying with respect to frequency, while η_{SAW} is negligibly small outside the narrow resonance dips. Therefore, on the resonance frequency (Ω_{14}) of the R14 mode, $\eta_R(\Omega_{14})$ can be estimated from the interpolation of the adjacent $|S_{11}|^2$ data points outside the resonance dip as shown in Fig. S2. In this case, $|S_{11}(\Omega_{14})|^2 = 2.34 \times 10^{-3}$, $\eta_{SAW}(\Omega_{14}) = 5.3 \times 10^{-4}$ and $\eta_R(\Omega_{14}) = 0.9965$.

The excitation of SAW is generally determined by the overlap integral between the stress field of the SAW mode and the stress field induced by the voltage actually present on the specific IDT design [3]. The propagation loss of the SAW and the efficiency of the RF power delivery from the RF source to the IDT also play important roles in determining the actual excitation of the SAW. Therefore, in our system where piezoelectric material (AlN) is only a thin layer on top of other non-piezoelectric materials (SiO₂ and Si), the SAW modes whose stress field (or almost equivalently, the mechanical energy) distribution is more confined in the AlN layer generally can be more strongly excited.

It is worth noting that, although R15 seems to be the most strongly excited SAW mode, as shown in Fig. S2, we used R14 for the experiments because R14 provides the overall strongest modulation of the photonic cavity among all the SAW modes near 12 GHz, as shown in Fig. 2(a).

4. OPTOMECHANICAL COUPLING COEFFICIENT DEFINITION

The strength of optomechanical interactions in the optical cavity can be quantified using an optomechanical coupling coefficient, G , which is conventionally defined as the shift of the angular resonance frequency per unit displacement. This definition, however, requires that an effective SAW mode amplitude be defined, such that

$$\Delta\omega(t) = \delta\omega \cos(\Omega t) = G A_{\text{eff}} \cos(\Omega t), \quad (\text{S3})$$

where $\Delta\omega(t)$ is the SAW induced sinusoidally time-varying shift of the optical cavity angular resonance frequency, $\delta\omega$ is the amplitude of $\Delta\omega(t)$, Ω is the SAW mode angular frequency and A_{eff} is the effective SAW mode displacement amplitude. Generally, A_{eff} can be arbitrarily defined as long as for any given SAW mode,

$$A_{\text{eff}} \propto \sqrt{P_{\text{SAW}}}, \quad (\text{S4})$$

where P_{SAW} is the traveling power of the SAW mode. In this work we define A_{eff} as

$$A_{\text{eff}} \equiv \sqrt{|A_x|^2 + |A_z|^2} \Big|_{\text{Max in AlN}}, \quad (\text{S5})$$

where A_x (A_z) is the displacement amplitude along the longitudinal (transverse, surface normal) direction of SAW wave. We note in Rayleigh modes, the particle motion in the substrate is in an elliptical trajectory. The right hand side of Eq. (S5) is evaluated in the AlN layer at the depth where it reaches its maximum.

The power of the SAW mode is related to the SAW mechanical energy E by

$$P_{\text{SAW}} = \frac{\Omega}{2\pi} E = \frac{\Omega}{2\pi} \gamma W A_{\text{eff}}^2, \quad (\text{S6})$$

where E is the total mechanical (including potential and kinetic) energy contained in one period Λ of the SAW wave and it is proportional to the IDT aperture W . γ is a proportional factor relating A_{eff}^2 to the total energy of SAW and defined as

$$\gamma \equiv \frac{E}{W A_{\text{eff}}^2} = \frac{1}{W A_{\text{eff}}^2} \int_{x,y,z=0}^{x,y,z=\Lambda,W,\infty} U(x,y,z,A_{\text{eff}}) dx dy dz, \quad (\text{S7})$$

where U is the total mechanical (including potential and kinetic) energy density distribution of a particular mode excited to amplitude A_{eff} . The value of γ for a given SAW mode is calculated from FEM simulation results and the aperture size used in the experiments.

Therefore, the expression of A_{eff} for a given SAW power is

$$A_{\text{eff}} = \sqrt{\frac{2\pi P_{\text{SAW}}}{\gamma W \Omega}}. \quad (\text{S8})$$

Consequently,

$$G = \delta\omega \sqrt{\frac{\gamma W \Omega}{2\pi P_{\text{SAW}}}}. \quad (\text{S9})$$

Alternatively, a more general and convenient way to quantify the interaction between an optical cavity and a traveling SAW mode is

$$\delta\omega = G' \sqrt{P_{\text{SAW}}} = G' A'_{\text{eff}}, \quad \text{where} \quad (\text{S10})$$

$$A'_{\text{eff}} = \sqrt{P_{\text{SAW}}}. \quad (\text{S11})$$

The coefficient G' defined in Eq. (S10) can be determined and used experimentally without the need to solve the SAW mode profile and the ambiguous definition of SAW amplitude. The optomechanical coupling coefficients G and G' are related by

$$G = G' \sqrt{\gamma W \frac{\Omega}{2\pi}}. \quad (\text{S12})$$

5. THEORETICAL ANALYSIS OF THE OPTOMECHANICAL MODULATION IN THE SIDEBAND RESOLVED REGIME

Similar to conventional optomechanical systems [4], the SAW modulated photonic crystal nanocavity transmission spectrum can be derived using temporal coupled mode theory analysis [5–7].

The system can be modeled as a single mode cavity side-coupled to a continuous waveguide. There are three ports in the system: the input and output waveguides, and the cavity loss channel. Under the modulation of the SAW wave, the intra-cavity field amplitude, a , evolves with time as

$$\frac{da}{dt} = \left\{ j \left[\omega_0 + \delta\omega \cos(\Omega t) \right] - \frac{1}{\tau} \right\} a + \sqrt{\frac{1}{\tau_c}} s_{\text{in}} e^{j\omega t}, \quad (\text{S13})$$

where ω_0 and τ are the center frequency and the lifetime of the resonance mode, respectively, $1/\tau_c$ is the total decay rate due to the coupling to both the input and output waveguides. The amplitude a is normalized such that $|a|^2$ corresponds to the energy inside the resonator. The resonance mode is excited (detected) by the incoming (outgoing) wave s_{in} (s_{out}) from the input (output) waveguide, which is normalized in such a way that $|s_{\text{in}}|^2$ ($|s_{\text{out}}|^2$) corresponds to the input (output) power. The quantities in the expression $\delta\omega \cos(\Omega t)$ has been defined in Eq. (S3) in Section 4. Without any loss of generality, we assume that s_{in} is the phase reference, namely, s_{in} is real and positive,

$$s_{\text{in}} = |s_{\text{in}}|. \quad (\text{S14})$$

The SAW modulation induces sidebands. One can write the intra-cavity field amplitude as a Fourier superposition of these sidebands as

$$a = \sum_m a_m e^{j(\omega + m\Omega)t}. \quad (\text{S15})$$

Substituting Eq. (S15) into Eq. (S13) and rearranging, we get

$$\begin{aligned}
& \sum_m j(\omega + m\Omega) a_m e^{j(\omega + m\Omega)t} \\
&= \left(j\omega_0 - \frac{1}{\tau} \right) \sum_m a_m e^{j(\omega + m\Omega)t} \\
&+ \frac{1}{2} j\delta\omega (e^{j\Omega t} + e^{-j\Omega t}) \sum_m a_m e^{j(\omega + m\Omega)t} + \sqrt{\frac{1}{\tau_e}} s_{\text{in}} e^{j\omega t}.
\end{aligned} \quad (\text{S16})$$

In the sideband resolved limit, only the first order sidebands will have significant amplitude in addition to the carrier amplitude. Comparing terms proportional to $e^{j\omega t}$, and taking into account the fact that $|a_0|^2 \gg |a_{\pm 1}|^2$, we get

$$a_0 = \frac{\sqrt{1/\tau_e}}{j\Delta + 1/\tau} s_{\text{in}}, \quad (\text{S17})$$

which is the carrier component of the intra-cavity field amplitude. $\Delta = \omega - \omega_0$ is the cavity detuning. Doing the same for terms proportional to $e^{j(\omega \pm \Omega)t}$, we get the sideband amplitudes as

$$\begin{aligned}
a_{\pm 1} &= \left(\frac{j\delta\omega/2}{j(\Delta \pm \Omega) + 1/\tau} \right) a_0 \\
&= \left(\frac{j\delta\omega/2}{j(\Delta \pm \Omega) + 1/\tau} \right) \left(\frac{\sqrt{1/\tau_e}}{j\Delta + 1/\tau} \right) s_{\text{in}} \equiv |a_{\pm 1}| e^{j\phi_{\pm 1}},
\end{aligned} \quad (\text{S18})$$

which implies that (note that s_{in} is assumed to be real and positive)

$$|a_{\pm 1}| = \frac{\delta\omega/2}{((\Delta \pm \Omega)^2 + 1/\tau^2)^{1/2}} \frac{\sqrt{1/\tau_e}}{(\Delta^2 + 1/\tau^2)^{1/2}} |s_{\text{in}}|, \quad (\text{S19})$$

$$\cos(\phi_{\pm 1}) = \frac{(2\Delta \pm \Omega)/\tau}{((\Delta \pm \Omega)^2 + 1/\tau^2)^{1/2} (\Delta^2 + 1/\tau^2)^{1/2}} \quad \text{and} \quad (\text{S20})$$

$$\sin(\phi_{\pm 1}) = \pm \frac{1/\tau^2 - \Delta(\Delta \pm \Omega)}{((\Delta \pm \Omega)^2 + 1/\tau^2)^{1/2} (\Delta^2 + 1/\tau^2)^{1/2}} \quad (\text{S21})$$

The amplitude and power of the outgoing wave in the output waveguide are, respectively,

$$s_{\text{out}} e^{j\omega t} = -s_{\text{in}} e^{j\omega t} + \sqrt{1/\tau_e} a, \quad (\text{S22})$$

$$|s_{\text{out}}|^2 = |T_0 + \sqrt{1/\tau_e} (a_{-1} e^{-j\Omega t} + a_{+1} e^{j\Omega t})|^2, \quad (\text{S23})$$

where

$$T_0 = -s_{\text{in}} + \sqrt{1/\tau_e} a_0 \equiv |T_0| e^{j\theta}, \quad (\text{S24})$$

$$\theta = \arctan \left(\frac{\Delta/\tau_e}{-(\Delta^2 + 1/(\tau\tau_i))} \right) + \pi. \quad (\text{S25})$$

Taking into account the fact that $|a_0|^2 \gg |a_{\pm 1}|^2$ and retaining the dominant terms, $|s_{\text{out}}|^2$ reduces to

$$\begin{aligned}
|s_{\text{out}}|^2 &\approx |T_0|^2 + 2 \text{Re} \left[T_0 \sqrt{1/\tau_e} (a_{-1}^* e^{j\Omega t} + a_{+1}^* e^{-j\Omega t}) \right] \\
&= |T_0|^2 + 2|T_0| \sqrt{1/\tau_e} \left(|a_{-1}| \cos(-\phi_{-1} + \Omega t + \theta) \right. \\
&\quad \left. + |a_{+1}| \cos(\phi_{+1} + \Omega t - \theta) \right).
\end{aligned} \quad (\text{S26})$$

Lumping together the quadrature components, the total output power takes the expression

$$|s_{\text{out}}|^2 \approx |T_0|^2 + B \cos(\Omega t + \varphi), \quad (\text{S27})$$

where

$$B = 2|T_0| \sqrt{1/\tau_e} C, \quad (\text{S28})$$

$$C = \left\{ \frac{[|a_{-1}| \cos(-\phi_{-1} + \theta) + |a_{+1}| \cos(\phi_{+1} - \theta)]^2}{[|a_{-1}| \sin(-\phi_{-1} + \theta) + |a_{+1}| \sin(\phi_{+1} - \theta)]^2} \right\}^{1/2}, \quad (\text{S29})$$

$$\begin{aligned}
\varphi &= \arctan \left[\frac{|a_{-1}| \sin(-\phi_{-1} + \theta) + |a_{+1}| \sin(\phi_{+1} - \theta)}{|a_{-1}| \cos(-\phi_{-1} + \theta) + |a_{+1}| \cos(\phi_{+1} - \theta)} \right] \\
&+ \begin{cases} 0 & \text{if denominator} \geq 0 \\ \pi & \text{if denominator} < 0 \end{cases}
\end{aligned} \quad (\text{S30})$$

where $1/\tau_i$ is the intrinsic decay rate of the cavity. In the under-coupled regime where $\tau_e \ll \tau \approx \tau_i$, $\theta \approx 0$, the above expressions for C and φ can be further simplified to

$$C = \left\{ \frac{[|a_{-1}| \cos(-\phi_{-1}) + |a_{+1}| \cos(\phi_{+1})]^2}{[|a_{-1}| \sin(-\phi_{-1}) + |a_{+1}| \sin(\phi_{+1})]^2} \right\}^{1/2} \quad \text{and} \quad (\text{S31})$$

$$\begin{aligned}
\varphi &= \arctan \left[\frac{|a_{-1}| \sin(-\phi_{-1}) + |a_{+1}| \sin(\phi_{+1})}{|a_{-1}| \cos(-\phi_{-1}) + |a_{+1}| \cos(\phi_{+1})} \right] \\
&+ \begin{cases} 0 & \text{if denominator} \geq 0 \\ \pi & \text{if denominator} < 0 \end{cases}
\end{aligned} \quad (\text{S32})$$

The first term of the right hand side of Eq. (S27) is the cavity transmission spectrum without SAW modulation, and the second term (the cosine term) is the oscillating power at the SAW frequency due to the beating between the carrier and the two sidebands. The coefficient B of the second term in Eq. (S27) is used to fit the experimental data in Fig. 2(c).

Despite of the lengthy derivation, the physical significance of the curve in Fig. 2(c) is actually quite intuitive. The vertical axis basically corresponds to the amplitude of the beat note produced by the transmitted laser carrier and modulation sidebands. (Only two sidebands are considered. Higher order sidebands are negligible.) The zero beat note amplitude at zero detuning corresponds to the case where the laser carrier is right on resonance and the two sidebands are perfectly balanced. The two peaks at about ± 2 GHz correspond to the cases where the laser carrier is near resonance and the two sidebands are imbalanced. The two peaks at about ± 12 GHz correspond to the cases where one of the sidebands is on resonance hence enhanced by the cavity. At all other detuning values, all of the laser carrier and the two sidebands are off resonance, hence the beat note amplitude is generally small. Especially for detuning beyond ± 20 GHz, the beat note amplitude approaches zero. Four distinct peaks can be observed only when the cavity optomechanical system is deep in the sideband-resolved regime.

6. DETERMINATION OF OPTOMECHANICAL COUPLING COEFFICIENT IN THE SIDEBAND RESOLVED REGIME

In order to determine G and G' from the experimental data shown in Fig. 2(c), we first calculate the amplitude of the oscillating optical power (B) in the output waveguide of the device, using the calibrated lumped RF optical gain of our experimental setup. The lumped gain takes into account the gain and loss of all the experimental equipment from the output grating coupler of the device to the RF amplifiers between the photo detector and the network analyzer. Then the full expression for B , including Eq. (S28) and Eq. (S29) are used to fit the experimental data with $\delta\omega$ as the only free parameter. Meanwhile, the power and effective amplitude of the SAW used in the experiment is

Table S2. Summary of the device parameters for sideband resolved optomechanical experiments.

Photonic Crystal Nanocavity	Resonance Wavelength	1529.7 nm
	Linewidth ($2/\tau$)	$(2\pi) \cdot 3.88$ GHz
	Extinction Ratio	1.64
	Loaded Q	5.1×10^4
	Intrinsic Q	6.5×10^4
	Total Lifetime τ	82 ps
	Intrinsic Lifetime τ_i	105 ps
	External Lifetime τ_e	375 ps
Optical Power	Input Waveguide (DC, On-Chip)	22 μ W
	Output Waveguide (RF, On-Chip)	Data in Fig. 2(c)
SAW Excitation for R14 (IDT Period = 0.45 μm)	Conversion Efficiency η_{SAW}	5.3×10^{-4}
	Power P_{SAW}	0.88 μ W
	Aperture W	100 μ m
	Peak Frequency	12.12 GHz
	Coefficient γ^*	5.7×10^{12} N/m ²
	Effective Displacement Amplitude	0.40 pm
Optomechanical Interaction	Amplitude of Optical Resonance	$(2\pi) \cdot 21$ MHz
	Frequency Oscillation $\delta\omega$	$(2\pi) \cdot 53$ GHz/nm
	Coupling Coefficient G	$(2\pi) \cdot 23$ MHz $\cdot(\mu\text{W})^{-1/2}$
	Coupling Coefficient G'	

** From FEM simulation.

estimated using Eq. (S1). Subsequently Eq. (S3) and Eq. (S10) are used to determine G and G' . The important device parameters used for data processing are summarized in Table S2.

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