Focusing through dynamic tissue with millisecond digital optical phase conjugation: supplementary material

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Published 07 August 2015

This document provides supplementary information to “Focusing through dynamic tissue with millisecond digital optical phase conjugation,” <http://dx.doi.org/10.1364/optica.2.000728>. It includes a detailed diagram of the optical setup and details pertaining to the framework of binary phase digital optical phase conjugation (DOPC) with theoretical calculations for PBR and an algorithm for single-shot binary phase retrieval. © 2015 Optical Society of America

<http://dx.doi.org/10.1364/optica.2.000728.s001>

1. OPTICAL DIAGRAM

Fig. S1. Setup diagram.

BD: beam dump, BE: beam expander, BS1: 90/10 cube beam-splitter, BS2: 90/10 cube beam-splitter, BS3: 50/50 cube beam-splitter, CL: camera lens (Nikon AF-S VR Micro-NIKKOR 105 mm f/2.8G IF-ED), DMD: digital micromirror device (W4100,

Wintech), FPC: fiber port connector, HWP: half wave plate, L1: 50 mm planoconvex lens, L2: 100 mm planoconvex lens, L3: 100 mm planoconvex lens, L4: 150 mm planoconvex lens, L5: 15 mm planoconvex lens, M: mirror, ND: neutral density filter, PB1: 50/50 plate beam-splitter, PB2: 90/10 plate beam-splitter, PB3: 50/50 plate beam-splitter, FS: fast shutter, PoBS: polarized beam splitter, P: polarizer, S: sample, SLF: spatial light filter.

2. DMD CURVATURE COMPENSATION

The performance of an OPC system is highly dependent on the fidelity between the playback and recording wavefronts. Therefore, curvature of the DMD surface must be taken into account and digitally corrected by finding a compensation phase map for the DMD and adding it to the playback phase map. The compensation phase map is acquired in three steps. First, the playback beam is turned off and the intensity of the reference beam is captured. Second, all the DMD pixels are set to the “on” position, the playback beam is turned on, and the interference pattern between the reference beam and playback beam is captured. Third, the single-shot binary phase retrieval technique is used to compute the binary wavefront difference between the two beams and find the wavefront distortion due to the curvature of the DMD. In this case, the playback beam is modulated by the curved surface of the DMD and acts as the sample

beam in the binary phase retrieval computation. The result of this calculation is a binary phase map which is then applied to each subsequently calculated wavefront in the DOPC procedure before playback.

3. PBR DERIVATION

The theoretical performance of optical phase conjugation is well described in framework of Vellekoop [1]. Scattering by tissue essentially maps the input electric field to the output electric field and can be described by a transmission matrix. Assuming the incident electric field on a scattering sample A is E_a and the electric field coming out from the other surface B is E_b , the transformation between E_a and E_b can be described as,

$$E_b = T_{AB} E_a \quad (1)$$

where T_{AB} is the total transmission matrix describing the propagation of E_a from plane A to B. In perfect phase conjugation, T_{AB} is unitary, which reflects the time symmetry $(T_{AB})^\dagger = (T_{AB})^{-1} = (T_{BA})^*$. If we play back conjugate field E_a^* on plane B, the outcome E'_a on plane A will be,

$$E'_a = T_{BA} E_b^* = T_{BA} (T_{AB} E_a)^* = T_{BA} T_{BA}^{-1} E_a^* = E_a^* \quad (2)$$

where “ $+$ ” stands for conjugate transpose. While in practice T_{AB} is only part of the total transmission matrix, we can still derive an approximate phase conjugation solution so that E_a^* is played back on surface A with a background. For a single mode input E_a with unit intensity (without loss of generality), the phase conjugate field in the original position a with background m at plane A will be,

$$E'_a = \sum_{b=1}^N t_{ba} E_b^* = \sum_{b=1}^N t_{ba} (t_{ab} E_a)^* = E_a^* \sum_{b=1}^N |t_{ab}|^2 \quad (3)$$

$$E'_{m,m \neq a} = \sum_{b=1}^N t_{bm} E_b^* = \sum_{b=1}^N t_{bm} (t_{ab} E_a)^* = E_a^* \sum_{b=1}^N t_{bm} t_{ab}^* \quad (4)$$

In the case of phase-only modulation with amplitude A , the phase conjugate field is,

$$\begin{aligned} E'_a &= A \sum_{b=1}^N t_{ba} e^{i \arg(E_b^*)} = A \sum_{b=1}^N t_{ab} e^{i \arg(t_{ab}^* E_a^*)} \\ &= A E_a^* \sum_{b=1}^N |t_{ab}| \end{aligned} \quad (5)$$

$$E'_{m,m \neq a} = A \sum_{b=1}^N t_{bm} e^{i \arg(E_b^*)} = A E_a^* \sum_{b=1}^N t_{bm} e^{i \arg(t_{ab}^*)} \quad (6)$$

If a DMD is used as the spatial light modulator in DOPC, the expression for the phase conjugate field at the original position needs to be modified to account for the binary modulation of the DMD. This can be written as,

$$E'_a = \sum_{b=1}^N t_{ba} A f(|\Delta\theta|) \quad (7)$$

where $|\Delta\theta| = |\alpha - \theta_p|$, $\alpha = \arg(E_b^*)$, θ_p is the phase of playback light from the DMD, and $f(|\Delta\theta|)$ is the phase modulation function of the DMD given by,

$$f(|\Delta\theta|) = \begin{cases} 0, & |\Delta\theta| > \phi \\ \cos |\Delta\theta| e^{i\alpha} + \sin |\Delta\theta| e^{i\beta}, & |\Delta\theta| \leq \phi \end{cases} \quad (8)$$

where β is the argument of the decomposed phasor orthogonal to $e^{i\alpha}$. Pixels with an absolute phase difference less than or equal to the upper bound ϕ are turned on for playback. With phase stepping or off-axis holography, we can easily select any ϕ . When $0 \leq |\Delta\theta| < \phi$, we can decompose E'_a into two terms. The first term is a phase only conjugation term modulated by $\cos |\Delta\theta|$. As the phase distribution of speckles is uniform within the selected range, the second term, which has an orthogonal phase, will make no contribution to recovery of the peak. From the above, we can intuitively come to the result that DMD based DOPC is phase only DOPC with an amplitude modulated by the cosine term as shown in Eq. 9 below.

$$\begin{aligned} E'_a &= \sum_{b=1}^N t_{ba} A f(|\Delta\theta|) \\ &= \sum_{b=1}^N t_{ba} A e^{i \arg(E_b^*)} \cos |\Delta\theta| (|\Delta\theta| \leq \phi) \\ &= \sum_{b=1}^N t_{ba} A e^{i \arg(t_{ab}^* E_a^*)} \cos |\Delta\theta| (|\Delta\theta| \leq \phi) \\ &= A E_a^* \sum_{b=1}^N |t_{ab}| \cos |\Delta\theta| (|\Delta\theta| \leq \phi) \end{aligned} \quad (9)$$

Since the transmission matrix elements can be assumed to follow complex circular Gaussian distribution [1], the amplitudes of the individual matrix elements $|t_{ab}|$ follow a Rayleigh distribution $|t_{ab}| : \text{Rayleigh}(\sigma)$ and their intensities $|t_{ab}|^2$ follow an exponential distribution $|t_{ab}|^2 \sim e^{-1/(2\sigma^2)}$ with $2\sigma^2$ as the ensemble average intensity of each element. The absolute phase difference $|\Delta\theta|$ follows a uniform distribution from 0 to π . Then, based on the derivation of phase-only phase conjugation described by Wang [2], the peak intensity and its ensemble average are derived as,

$$\begin{aligned} I'_a &= \left| A \sum_{b=1}^N |t_{ab}| \cos |\Delta\theta| (|\Delta\theta| \leq \phi) \right|^2 \\ &= A^2 \sum_{b=1}^N |t_{ab}|^2 (\cos |\Delta\theta| (|\Delta\theta| \leq \phi))^2 \\ &\quad + A^2 \left\{ \sum_{b=1}^N \sum_{b' \neq b}^N |t_{ab}| |t_{ab'}| (\cos |\Delta\theta| (|\Delta\theta| \leq \phi))^2 \right\}^2 \end{aligned} \quad (10)$$

$$\begin{aligned} \langle I'_a \rangle &= A^2 N \langle |t_{ab}|^2 \rangle \langle (\cos |\Delta\theta| (|\Delta\theta| \leq \phi))^2 \rangle \\ &\quad + A^2 N(N-1) \langle |t_{ab}|^2 \rangle \langle \cos (|\Delta\theta|) (|\Delta\theta| \leq \phi) \rangle^2 \\ &= 2NA^2 \sigma^2 \left(\frac{\sin 2\phi/2 + \phi}{2\pi} \right) \\ &\quad + \frac{\pi}{2} N(N-1) A^2 \sigma^2 \left(\frac{\sin \phi}{\pi} \right)^2 \end{aligned} \quad (11)$$

Fig. S2. Normalized theoretical PBR with respect to ϕ (the upper bound of the absolute phase difference).

Similarly, the background can be derived as,

$$E'_{m,m \neq a} = \sum_{b=1}^N t_{bm} A e^{i\theta_p} (|\Delta\theta| \leq \phi) \quad (12)$$

$$I'_{m,m \neq a} = A^2 \left| \sum_{b=1}^N t_{bm} (|\Delta\theta| \leq \phi) \right|^2 \quad (13)$$

$$\begin{aligned} \langle I'_{m,m \neq a} \rangle &= A^2 \left\langle \left| \sum_{b=1}^N t_{bm} (|\Delta\theta| \leq \phi) \right|^2 \right\rangle \\ &= 2NA^2\sigma^2 \frac{\phi}{\pi} \end{aligned} \quad (14)$$

Putting these two expressions together, we can find the PBR as,

$$\begin{aligned} \text{PBR}_{\text{DMD}} &= \frac{\langle I'_a \rangle}{\langle I'_{m,m \neq a} \rangle} \\ &= \frac{2NA^2\sigma^2 \left(\frac{\sin 2\phi/2 + \phi}{2\pi} \right) + \frac{\pi}{2} N(N-1) A^2\sigma^2 \left(\frac{\sin \phi}{\pi} \right)^2}{2NA^2\sigma^2 \frac{\phi}{\pi}} \\ &= \frac{1}{2} + \frac{\sin 2\phi + (N-1) \sin^2 \phi}{4\phi} \\ &\approx \frac{N \sin^2 \phi}{4\phi} \end{aligned} \quad (15)$$

For $\phi = \pi/2$, the PBR is,

$$\text{PBR}_{\text{DMD}, \phi = \pi/2} = \frac{1}{2} + \frac{(N-1)}{2\pi} \approx \frac{N}{2\pi} \quad (16)$$

which is consistent with derivation in iterative binary iterative wavefront optimization [3]. As shown in Fig. S2, the PBR is a unimodal function of ϕ , reaching its maximum at $\phi = 0.371\pi$, with a 13.8% higher value than at $\phi = \pi/2$. When there are M nonzero optical modes in focus, the PBR is scaled by M so that the new PBR is,

$$\text{PBR}_{\text{DMD}} \approx \frac{N \sin^2 \phi}{4M\phi} \quad (17)$$

4. SINGLE-SHOT BINARY PHASE RETRIEVAL

Using a reference beam with an electric field $E_{ref}(x, y)$ (whose intensity is $I_{ref}(x, y)$ and phase is zero without loss generality), the complex information (phase and amplitude) of the sample field $E_{sam}(x, y)$ (whose intensity is $I_{sam}(x, y)$ and phase is α) is coupled into the intensity of their interference pattern I_i as,

$$I_i(x, y) = I_{ref}(x, y) + I_{sam}(x, y) + 2\sqrt{I_{ref}(x, y)I_{sam}(x, y)} \cos \alpha \quad (18)$$

For phase retrieval in DMD based DOPC, only the condition $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ or $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ is needed. This allows a single-shot binary phase retrieval technique to be used to determine which DMD pixels should be turned on.

$$\text{DMD}(x, y) = \begin{cases} 1, & I_i(x, y) < I_{ref}(x, y) \rightarrow B(\alpha) = \pi \\ 0, & \text{else} \rightarrow B(\alpha) = 0 \end{cases} \quad (19)$$

It can be shown that $I_i(x, y) < I_{ref}(x, y) \rightarrow \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$. For this phase range of interest, the absolute phase difference is defined as $|\Delta\theta| = |\alpha - \pi|$. To derive the PBR, new notations are defined $r = I_{sam}/4I_{ref}$ and $g = I_{ref}/\langle I_{sam} \rangle$. For a pixel with specific phase α , its status is controlled by the intensity ratio of the reference and sample signal as derived from Eqs. 18 and 19,

$$\text{DMD}(x, y) = \begin{cases} 1, & \cos |\Delta\theta| > \sqrt{r} \\ 0, & \text{else} \end{cases} \quad (20)$$

As the absolute phase distribution of $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ is uniform within $(0, \frac{\pi}{2})$, given an intensity I_{sam} , the upper phase difference bound of “on” pixels is,

$$\phi = \begin{cases} \arccos(\sqrt{r}) & r \leq 1 \\ 0, & \text{else} \end{cases} \quad (21)$$

Since the sample intensity follows a Rayleigh distribution $I_{sam} : \text{Rayleigh}(\sigma)$, the ratio r also follows a Rayleigh distribution with a scale parameter given by:

$$\sigma_r = \sigma_{I_{sam}}/4I_{ref} = \frac{\sigma_{I_{sam}}}{4g\langle I_{sam} \rangle} = \frac{\sigma}{2\sqrt{2\pi}g\sigma} = \frac{1}{2\sqrt{2\pi}g} \quad (22)$$

The peak intensity can then be derived from Eq. 11,

$$\begin{aligned} \langle I_{peak} \rangle &\approx \frac{\pi}{2} N(N-1) A^2\sigma^2 \left\langle \left(\frac{\sin \phi}{\pi} \right)^2 \right\rangle \\ &= \frac{\pi}{2} N(N-1) A^2\sigma^2 \left(\int_0^1 \frac{1-r}{\pi^2} f(r) dr + \int_1^\infty \frac{0}{\pi^2} f(r) dr \right) \\ &= \frac{1}{2\pi} N(N-1) A^2\sigma^2 \int_0^1 (1-r) \frac{r}{\sigma_r^2} e^{-r^2/(2\sigma_r^2)} dr \\ &= \frac{1}{2\pi} N(N-1) A^2\sigma^2 \left[1 - \sqrt{\frac{\pi}{2}} \sigma_r^2 (1 - e^{-1/\sigma_r^2}) \right] \\ &= \frac{1}{2\pi} N(N-1) A^2\sigma^2 \left[1 - \sqrt{\frac{1 - e^{-8\pi g^2}}{16g^2}} \right] \end{aligned} \quad (23)$$

In the same way, the background can be derived from Eq. 14,

$$\begin{aligned}
 \langle I_{background} \rangle &= \frac{2NA^2\sigma^2}{\pi} \langle \phi \rangle \\
 &= \frac{2NA^2\sigma^2}{\pi} \left(\int_0^1 \arccos(\sqrt{r})f(r)dr + \int_1^\infty 0f(r)dr \right) \\
 &= \frac{2NA^2\sigma^2}{\pi} \int_0^1 \arccos(\sqrt{r}) \frac{r}{\sigma_r^2} e^{-r^2/(2\sigma_r^2)} dr \\
 &= 16Ng^2NA^2\sigma^2 \int_0^1 \arccos(\sqrt{r})re^{-4\pi g^2r^2} dr \quad (24)
 \end{aligned}$$

Fig. S3. Single shot binary phase retrieval. (a) The normalized theoretical PBR (relative to $PBR_{DMD,\phi=\pi/2}$) with respect to the intensity ratio g of the reference and sample beam. (b) The playback ratio function for different intensity ratios g .

So, calculating the PBR from these two quantities gives,

$$\begin{aligned}
 PBR_{DMD,g=I_{ref}/\langle I_{sam} \rangle} &= \frac{\langle I_{peak} \rangle}{\langle I_{background} \rangle} \\
 &= \frac{\frac{1}{2\pi}N(N-1)A^2\sigma^2 \left[1 - \sqrt{\frac{1-e^{-8\pi g^2}}{16g^2}} \right]}{16Ng^2NA^2\sigma^2 \int_0^1 \arccos(\sqrt{r})re^{-4\pi g^2r^2} dr} \\
 &= \frac{(N-1) \left[1 - \sqrt{\frac{1-e^{-8\pi g^2}}{16g^2}} \right]}{32\pi g^2 \int_0^1 \arccos(\sqrt{r})re^{-4\pi g^2r^2} dr} \\
 &\approx \frac{N \left[1 - \sqrt{\frac{1-e^{-8\pi g^2}}{16g^2}} \right]}{32\pi g^2 \int_0^1 \arccos(\sqrt{r})re^{-4\pi g^2r^2} dr} \quad (25)
 \end{aligned}$$

which is a unimodal function of the intensity ratio g . For $\langle I_{sam} \rangle \ll I_{ref}$, $g \rightarrow \infty$ and the PBR is $PBR_{DMD,\phi=\pi/2}$. As shown in Fig. S3(a), the function reaches its maximum at $g = 1.61$, having a value 12.6% higher than $PBR_{DMD,\phi=\pi/2}$. This is consistent with Eq. 15, where the maximum value position is at $\phi = 0.371\pi$ equal to $\arccos(1/(2\sqrt{g}))$. The inherent statistical phase selection capacity of single-shot phase modulation allows us to achieve a PBR enhancement nearly equivalent to that offered by selecting the optimal bound ϕ from the exact phase map. According to Eq. 20, given a pixel with absolute phase difference $|\Delta\theta| < \pi/2$ and intensity I_{sam} , it will be turned on for playback when,

$$\phi = \arccos(\sqrt{r}) = \arccos\left(\sqrt{I_{sam}/4I_{ref}}\right) > |\Delta\theta| \quad (26)$$

For all pixels with an absolute phase difference $|\Delta\theta|$, as $r = I_{sam}/4I_{ref}$ follows a Rayleigh distribution, the fraction of pixels on will be,

$$\begin{aligned}
 p_{on}(|\Delta\theta|) &= \int_{|\Delta\theta|}^{\pi/2} f(\phi)d\phi \\
 &= \int_0^{\cos^2|\Delta\theta|} \frac{r}{\sigma_r^2} e^{-r^2/(2\sigma_r^2)} dr \\
 &= 1 - e^{-4\pi g^2 \cos^4|\Delta\theta|} \quad (27)
 \end{aligned}$$

The playback ratio function is plotted for different intensity ratios in Fig. S3(b).

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