High-speed nonreciprocal magnetoplasmonic waveguide phase shifter: supplementary material

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1. CALCULATION OF THE MAGNETIC FIELDS FOR A CRITICALLY-DAMPED MAGNETIZATION RESPONSE

As a sample calculation, we consider the L-configuration as shown in Fig. 4(a) in the main text. To determine the magnetic field values required to critically-damp the magnetization response, Eqn. (3) (in the manuscript) is numerically solved to track the temporal evolution of the magnetization vector in response to a transient pulse, $\mathbf{h}(t) = \langle h_x(t), 0, 0 \rangle$, and a static biasing field $\mathbf{H}_{static} = \langle 0, 0, H_z \rangle$. The pulse is characterized by a full-width-at-half-maximum (FWHM), τ_p , and a peak value, h_{pk} , such that the transient field pulse has the following super-Gaussian form, resembling a practical logic signal:

$$h_x(t) = h_{pk} \exp \left[-\frac{|t - t_0|^5}{2\sigma^5} \right] \quad (\text{S1})$$

where the parameter σ is related to τ_p via:

$$\sigma = \sqrt[5]{\frac{\tau_p^5}{2^6 \ln 2}} \quad (\text{S2})$$

While all three magnetization components, M_x , M_y , and M_z are determined simultaneously, only the M_x component contributes to the nonreciprocal phase shift (NRPS). Considering the typical M_x response shown in Fig. 5(b) of the main text, two parameters must be optimized for a critically-damped response. Here, the magnitude of the primary peak of M_x must be maximized, while the amplitude of the residual precessional oscillations must be minimized. Ideally, the peak value of M_x/M_S equals 1 and the oscillation amplitude is equal to 0, corresponding to a critically-damped response.

The Landau-Lifshitz-Gilbert (LLG) vector equation is solved for various combinations of static and transient peak fields. Both the magnitude of the primary peak and the oscillation amplitude are plotted as functions of H_z for different values of h_{pk} ,

as shown in Fig. S1(a)-(b) for the $\tau_p = 500\text{ps}$ exemplary scenario. As can be seen in Fig. S1(a), the primary peak in the response will exhibit a unity maxima that shifts to higher static fields as the transient field peak is increased. Conversely, the amplitude of the oscillations, shown in Fig. S1(b), exhibit minima at multiple values of the static field. Only the first minimum is of interest, as the others correspond to higher harmonics of the oscillation. These minima manifest at lower static field values as the peak field of the transient pulse increases. Thus, with successive iterations, a parameter set can be found such that both the observed features occur for the same combination of static/dynamic field values. Figure S1(c) depicts both curves for a pulse with $\mu_0 h_{pk} = 54\text{mT}$. For a static field of 47.8mT, both features occur simultaneously, and thus the critical damping point has been determined. Similar analysis yields the critical damping parameters for other values of τ_p in the L-configuration.

When considering the T-configuration, analogous expressions are taken for the appropriate field components, and the LLG vector equation is again numerically solved to determine the motion of the magnetization vector, $\mathbf{M}$. The parameters that must be optimized in this configuration differ slightly from the L-configuration. Here, one must consider both the value of the primary peak in the M_x response and the value that M_x returns to immediately following excitation. The final state of M_x is intimately related to the precessional oscillations that will occur in M_z . Since the magnitude of $\mathbf{M}$ is a constant (M_S) and this particular arrangement forces the precession of the magnetization around the x-axis, a large amplitude of oscillations in M_z implies that M_x has not returned to its initial state. Note that over a time scale of several hundred nanoseconds, the initial magnetization state will be restored as the oscillations in M_z diminish; however, this is undesirable for efficient operation at high speeds. Thus, in this configuration, it is desirable to have a peak response of $M_x/M_S = -1$, after which M_x returns back

Fig. S1. (a) Magnitude of the primary peak of the M_x/M_S response as a function of static field for $\tau_p = 500$ ps in the L-configuration. (b) Amplitude of the residual precessional oscillations as a function of static field strength for $\tau_p = 500$ ps in the L-configuration. (c) Primary peak magnitude and oscillation amplitude for a $\tau_p = 500$ ps pulse with $\mu_0 h_{pk} = 54$ mT in the L-configuration. Note that the maximum of the primary peak response occurs simultaneously with the first minimum of the oscillation amplitude at a static field of $\mu_0 H_z = 47.8$ mT, as indicated by the dashed red line.

to its initial saturated state of $M_x/M_S = 1$, implying minimal precession, and hence a critically-damped response.

In this case, it is illustrative to consider parameters as functions of the peak magnetic field, h_{pk} . Figure S2(a) displays the peak in the M_x/M_S response as a function of h_{pk} for various values of the static field, at $\tau_p = 500$ ps. For a set static field, H_x , increasing the peak field strength deflects the magnetization further toward $M_x/M_S = -1$, as desired. Decreasing H_x further enhances this effect, and allows more deflection to be achieved at a lower transient field. Note that the limiting case of $\mu_0 H_x$ is 9mT, as the static field must be able to saturate the magnetization. Figure S2(b) depicts the value that M_x/M_S returns to after deflection, having oscillatory behaviour as a function of h_{pk} . Thus, the response exhibits multiple maxima of $M_x/M_S = 1$. As well, these maxima tend to shift to lower transient field values as the static field is increased. However, the trends for the two parameters are opposing. To maximize the peak M deflection at lower transient field strengths, H_x must be reduced, while conversely, in order to maximize the end state of M_x at lower values of h_{pk} , one requires H_x to be increased.

In the current application, achieving the maximum possible phase shift is desirable, and thus, the operating conditions are taken with $\mu_0 H_x = 9$ mT. This ensures that the peak deflection is as large as possible. At $\mu_0 H_x = 9$ mT, the peak field is then chosen to maximize the M_x end state at the lowest possible value of

h_{pk} , i.e., h_{pk} then corresponds to the first maximum in the end state response, as shown in Figure S2(c). Accordingly, the peak deflection is $M_x/M_S = -0.96$. Note that for the $\tau_p = 100$ ps case, higher fields are naturally required, so when considering operating conditions at $\mu_0 H_x = 9$ mT, the peak deflection is essentially unity.

2. CALCULATION OF THE MAGNETIC FIELD DISTRIBUTION AND AG TRANSMISSION LINE DESIGN

The dimensions of the Ag slabs, as well as the required currents, are determined based on the criterion that they must produce a magnetic field distribution that exhibits little variation over the waveguide's volume (i.e., <2.5% variation), and thus, can be considered uniform. The spatial dependence of the magnetic fields is highly dependent upon the current distribution within the Ag wire. At the frequencies or bit rates examined in the main text (1.21GHz and 6.06GHz), the current density is concentrated near the surface of the conductor due to the skin effect, as shown in Fig. S3 for the $\tau_p = 100$ ps, L-configuration case. Note that the skin depths at 1.21GHz and 6.06GHz are $1.823\mu\text{m}$ and 815nm , respectively. Accounting for this, the magnetic fields induced by these currents at any point in space are numerically calculated. In the L- and T- configurations, the transient x - and z -components are of interest, respectively. An example, the spa-

Fig. S2. (a) Primary peak of the M_x/M_S response as a function of the peak transient field for a $\tau_p = 500$ ps pulse in the T-configuration. (b) M_x state achieved after deflection as a function of the peak transient field for a $\tau_p = 500$ ps pulse in the T-configuration. (c) Both the normalized peak and the end state response of M_x for a $\tau_p = 500$ ps pulse with a static field of $\mu_0 H_x = 9$ mT in the T-configuration. The dashed red line indicates the optimum operating conditions.

Fig. S3. Current density distribution (top) and $|\mu_0 h_x|$ distribution (bottom) for a device operating in the L-configuration with $\tau_p = 100$ ps pulses, $I_{peak} = 24.19$ A, and Ag transmission line dimensions of $40\mu\text{m} \times 10\mu\text{m} \times 49.06\mu\text{m}$.

tial distribution of the x-component of the magnetic fields due to the aforementioned current distribution is shown in Fig. S3.

The fields present over the cross section of the waveguide normal to the direction of current flow are examined, and the maximum variation over this region is calculated. For devices operating in the L-configuration, this cross section is $270\text{nm} \times 300\text{nm}$, while in the T-configuration the cross section is $49.06\mu\text{m} \times 300\text{nm}$. Dimensions of the transmission line and the total current are then iteratively optimized until the required magnetic field variation around the critical point has been achieved, and the resultant parameters for each case are detailed in Table 1 in the main text. With the aforementioned parameters, the variation of the magnetic field is 0.74%-2.42%, and thus magnetic fields over the waveguide volume can be considered constant.

The required high current pulses can be generated in a variety of ways. In addition to commercial current sources (Montena model GP5-01-05), compact circuits have been developed to drive laser diodes with currents in excess of 10A, and nanosecond pulse lengths [1, 2]. In this manner, GaAs transistors operating in avalanche mode have been shown to produce short current pulses up to 120A [3]. Additionally, common transmission line transformers can be incorporated into the chip design to step currents up to a sufficiently high value [4].

We can estimate the power dissipation by first determining the root-mean-square (RMS) current. This is done by approximating the super-Gaussian pulses as rectangular pulses with a duration, τ_p , and repetition rates, τ_r , of either 825ps (1.21Gbit/s with $\tau_p = 500$ ps) or 165ps (6.06Gbit/s with $\tau_p = 100$ ps). The RMS current of such a pulse is given by the relation $I_{RMS} = I_{pk}\sqrt{D}$, where D is the duty cycle of the pulse, defined as the ratio of τ_p and τ_r . The effective resistance of each transmission line is then determined considering an effective area within one skin depth from the conductor's edges. The $I_{RMS}^2 R$ power dissipation is estimated to be 3.51W and 22.71mW for the $\tau_p = 100$ ps and $\tau_p = 500$ ps pulses in the L-configuration. Similarly, in the T-configuration, the estimated power dissipation is 210.74mW

Fig. S4. Temperature profile of the device operating in the L-configuration with $\tau_p = 100$ ps pulses, and $I_{peak} = 24.19$ A. The maximum temperature increase over the device volume is $\Delta T_{ss,max} = 21.85$ K above room temperature.

and 2.37mW for the two pulse widths respectively. Thus, power dissipation should not be a significant issue with the incorporation of a sufficient sink.

3. THERMAL ANALYSIS

With currents passing through small metal structures, the temperature of each device needs to be carefully assessed for practical implementation. As such, temperature distributions are calculated for the magnetoplasmonic devices with the dimensions and current parameters determined earlier.

Since the phase shifter will typically be operated with multiple pulses representing an encoded data sequence, the steady state temperature that will be reached by the device due to repetitive pulsing must be considered. To calculate the heat dissipated under continuous AC operation, a pulse train is approximated as a sinusoidal signal with a frequency equal to the pulse repetition rate (i.e., 1.21GHz for $\tau_p = 500$ ps pulses and 6.06GHz for $\tau_p = 100$ ps pulses). Utilizing finite element calculations, the current distribution in the transmission line is determined at the required frequency, allowing for the incorporation of Joule heating due to the nonuniform current distribution caused by the skin effect. The temperature profile is then numerically calculated to determine the maximum temperature in the structure, $T_{ss,max}$, and the maximum difference from a room temperature of 293.15K, $\Delta T_{ss,max}$. Here, standard convective boundary conditions are employed, while the bottom surface is held at room temperature.

Maximum steady state temperatures of the device are calculated to be $T_{ss,max} = 315.00$ K ($\Delta T_{ss} = 21.85$ K) and $T_{ss,max} = 293.29$ K ($\Delta T_{ss} = 0.14$ K) for $\tau_p = 100$ ps and $\tau_p = 500$ ps pulses respectively in the L-configuration. In the T-configuration, the steady state temperatures are $T_{ss,max} = 298.05$ K ($\Delta T_{ss} = 4.90$ K) and $T_{ss,max} = 293.29$ K ($\Delta T_{ss} = 0.14$ K) for the two pulse widths respectively. Clearly, even in the worst case scenario, the L-configuration with $\tau_p = 100$ ps (as shown in Fig. S4), this temperature increase is far below any compromising value.

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