Shifting the quantum-classical boundary: theory and experiment for statistically classical optical fields: supplementary material

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This document provides supplementary information to "Shifting the quantum-classical boundary: theory and experiment for statistically classical optical fields," <http://dx.doi.org/10.1364/optica.2.000611>. The growing recognition that entanglement is not exclusively a quantum property, and does not even originate with Schrödinger's famous remark about it [Proc. Camb. Phil. Soc. 31, 555 (1935)], prompts examination of its role in marking the quantum-classical boundary. We have done this by subjecting correlations of classical optical fields to new Bell-analysis experiments, and report here values of the Bell parameter greater than $\beta = 2.54$. This is many standard deviations outside the limit $\beta = 2$ established by the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [Phys. Rev. Lett. 23, 880 (1969)], in agreement with our theoretical classical prediction, and not far from the Tsirelson limit $\beta = 2.828\dots$. These results cast a new light on the standard quantum-classical boundary description, and suggest a reinterpretation of it. © 2015 Optical Society of America

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1. SHIMONY'S BELL-TEST CONDITIONS

In his extended analyses [1] of Bell inequalities and their testing, Shimony recognized that in order to deserve serious attention, an alternative non-quantum theory entering what is considered a quantum domain (in our case, the domain of Bell test violation) needs to embrace in some way aspects of Nature that appear completely random, i.e., purely probabilistic. These aspects are dealt with by quantum theory in well known ways, and this is the reason Bell once raised the issue of classical indeterminism [2], but without going as far as classical entanglement. Shimony summarized these considerations by naming three key features, all of which should be considered, in his words, "... as established parts of physical theory: (I) In any state of a physical system S there are some eventualities which have indefinite truth values. (II) If an operation is performed which forces an eventuality with indefinite truth value to achieve definiteness ... the outcome is a matter of chance. (III) There are 'entangled systems' (in Schrödinger's phrase) which

have the property that they constitute a composite system in a pure state, while neither of them separately is in a pure state." Here by eventualities Shimony means measurement outcomes. We have identified the electric field as it is dealt with in the standard classical theory of partial coherence and partial polarization as a physical system satisfying all three conditions.

2. PARTIALLY POLARIZED FIELDS

Classical statistical light of any degree of polarization can be treated exactly as the unpolarized light in the text, with a small change. One needs to insert the parameters that allow the two orthogonal components provided by the Schmidt decomposition to have different intensities. These parameters, κ_1 and κ_2 , are the (real) Schmidt eigenvalues. They satisfy $\kappa_1^2 + \kappa_2^2 = 1$, and both equal $1/\sqrt{2}$ in the completely unpolarized case treated in the text. Any intensity-normalized partially polarized field can then be written as in text Eqn. (3), but with κ_1

and κ_2 attached [3]:

$$|\mathbf{e}\rangle = \kappa_1|u_1\rangle|f_1\rangle + \kappa_2|u_2\rangle|f_2\rangle. \quad (1)$$

It is clear that the field is entangled between polarization and amplitude unless one of the κ 's is zero, in which case $|\mathbf{e}\rangle$ is plainly separable. All of the formulas for unpolarized light will change, but only to the extent that the presence of the κ 's requires. The conventional degree of polarization P is given in terms of the κ 's by $P = |\kappa_1^2 - \kappa_2^2|$ [4], and this can be used to find the Schmidt coefficients. For the experimental classical statistical optical light field we obtained $\kappa_1, \kappa_2 = 0.750, 0.661$. Another interesting formula is the full result for the partially polarized Bell parameter:

$$\begin{aligned} \mathcal{B} &= \cos 2a(\cos 2b - \cos 2b') \\ &+ \cos 2a'(\cos 2b + \cos 2b') \\ &+ 2\kappa_1\kappa_2\{\sin 2a(\sin 2b - \sin 2b') \\ &+ \sin 2a'(\sin 2b + \sin 2b')\}. \end{aligned} \quad (2)$$

One sees two uncomplicated limits: when either κ is zero (no entanglement) no result higher than $\mathcal{B} = 2$ can be achieved, and when $\kappa_1 = \kappa_2 = 1/\sqrt{2}$ (maximal entanglement) the Tsirelson bound can be reached: $\mathcal{B} = 2\sqrt{2}$. Also, if one follows Gisin's approach [5] by choosing the rotation angles as $a = 0, a' = \pi/4$, and $\cos 2b = -\cos 2b' = 1/\sqrt{1 + 4|\kappa_1\kappa_2|^2}$, the Bell parameter becomes

$$\mathcal{B} = 2/\sqrt{1 + 4|\kappa_1\kappa_2|^2}. \quad (3)$$

Apparently, there will be a Bell violation ($\mathcal{B} > 2$) as long as $\kappa_1\kappa_2 \neq 0$, i.e., when there is non-zero entanglement.

3. BASIS STRIPPING, ROTATION, AND PROJECTION IN AMPLITUDE FUNCTION SPACE

To observe correlation, it is essential to be able to access and measure both polarization and amplitude function degrees of freedom. Unfortunately, unlike the polarization degree of freedom, there is no systematic technology working directly in the infinite-dimensional amplitude function space that can project a non-deterministic field onto an arbitrary basis $|f_1^b\rangle$ in that space. This requires an innovation using an indirect measurement such as incorporated in the experimental setup sketched in Fig. 1 of the text. In our setup we employ an auxiliary beam that contains only the $|f_1^b\rangle$ basis to interfere with the primary test beam (which in general contains both $|f_1^b\rangle$ and $|f_2^b\rangle$). By interference measurements one is able to determine the $|f_1^b\rangle$ information from the test beam, and finally obtain the needed c_{11} value given in equation (9) of the main text.

This section shows specifically how to "strip" off and rotate a basis in the statistical amplitude function space of the auxiliary field $|\mathbf{\bar{E}}\rangle$, which shares the same statistical properties as the primary test beam. For generality, we take the auxiliary beam as initially in the form of Eq. (1) with arbitrary κ_1, κ_2 . Such a beam can always be rewritten in the rotated amplitude function space basis $|f_1^b\rangle, |f_2^b\rangle$, i.e.,

$$\begin{aligned} |\mathbf{\bar{E}}\rangle &= \sqrt{I}[(\kappa_1 \cos b|u_1\rangle - \kappa_2 \sin b|u_2\rangle)|f_1^b\rangle \\ &+ (\kappa_1 \sin b|u_1\rangle + \kappa_2 \cos b|u_2\rangle)|f_2^b\rangle]. \end{aligned} \quad (4)$$

One notes from the second term of the equation that a properly chosen polarizer that blocks completely the polarization-space component $\kappa_1 \sin b|u_1\rangle + \kappa_2 \cos b|u_2\rangle$ will effectively strip off the amplitude function space basis vector $|f_2^b\rangle$.

Such a stripping polarizer $|u_1^s\rangle$ is oriented with respect to the polarization space basis $|u_1\rangle, |u_2\rangle$ by a specific angle we have called s , i.e.,

$$|u_1^s\rangle = \cos s|u_1\rangle - \sin s|u_2\rangle. \quad (5)$$

Then the stripping condition is directly given as

$$\langle u_1^s | (\kappa_1 \sin b|u_1\rangle + \kappa_2 \cos b|u_2\rangle) = 0, \quad (6)$$

which specifies the rotation angle s according to

$$\tan s = (\kappa_1/\kappa_2) \tan b, \quad (7)$$

so s is determined by the values of κ_1 and κ_2 for any choice of rotation angle b in the amplitude function space.

As a result of this stripping polarizer, the beam (4) becomes

$$\begin{aligned} |\mathbf{\bar{E}}_1^s\rangle &= |u_1^s\rangle\langle u_1^s|\mathbf{\bar{E}}\rangle \\ &= \sqrt{I}(\kappa_1 \cos b \cos s + \kappa_2 \sin b \sin s)|u_1^s\rangle|f_1^b\rangle, \end{aligned} \quad (8)$$

where the amplitude function space component $|f_2^b\rangle$ of the transmitted beam is completely stripped off.

One notes immediately that arbitrary rotations of the function space basis $|f_1^b\rangle$ (i.e., arbitrary change of angle b) for the auxiliary beam can be effectively realized by the change of angle s through Eq. (7). Consequently, as described in the main text, by further interference of this auxiliary beam with the primary test beam, one is able to project the test beam onto any function space basis $|f_1^b\rangle$ with an arbitrary angle b . This is exactly how we make measurements in the amplitude function space of the statistical light beam.

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