Tracking nanoscale electric and magnetic singularities through three-dimensional space: supplementary material

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This document provides supplementary information to “Tracking nanoscale electric and magnetic singularities through three-dimensional space,” <http://dx.doi.org/10.1364/optica.2.000540>. Here, we describe the photonic crystal waveguide used in this study, the way in which the electro-magnetic modes of this structure are simulated, and the way in which we extract the in-plane electric and magnetic fields from our near-field measurements. We then discuss the nature of the polarization singularities that we observed, as well as the in- and out-of-plane field amplitudes at the locations of the C-points. © 2015 Optical Society of America

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1. PHOTONIC CRYSTAL WAVEGUIDE

In this work we use a simple W1 photonic crystal waveguide, fabricated into a 220 nm thick silicon membrane. This photonic crystal consists of a hexagonal pattern of air holes perforated into the membrane, with a period $a = 420$ nm and a hole diameter of 240 nm. The waveguide itself is simply a missing row of holes in the crystal. No dispersion engineering was performed on this structure, i.e. all holes are the same diameter and none have been shifted.

We calculate the electromagnetic eigenmodes and dispersion relation of our PhCW (shown in Fig S1) using the freely available MIT Photonic Bands [1] (MPB) software. This software determines the Bloch eigenmodes of a structure using a plane-wave basis and periodic boundary conditions. In the case of our PhCW, we use a supercell of dimensions $a \times 11\sqrt{3}a \times 10h$ and $h = 220$ nm is the thickness of the silicon slab. The photonic crystal is created by a hexagonal pattern of holes (radius $r = 0.29a = 120$ nm) perforated in the silicon slab (index of refraction of 3.48 at a wavelength of 1570 nm), with one row of holes missing to form a W1 waveguide. The size of the supercell was chosen to avoid interactions with adjacent supercells to heights of up to 500 nm above the slab surface. We use a

Fig. S1. Calculated dispersion relation of the PhCW used. The dashed line indicates the wavelength at which the experiments were performed.

grid size of $a/16$ ensuring a convergence of eigenvalues to better than 0.1%.

From the calculations we see that at 1570 nm, where we carry out the experiments, the group index of the mode is ~ 20 .

2. SEPARATING E AND H

Our aim is to separately map the four in-plane components of $\mathbf{E}$ and $\mathbf{H}$, in this case from our near-field measurements above a photonic crystal waveguide (PhCW). This is not a trivial problem since we only detect 2 signals, which we label L_x and L_y (which are associated with fields that are, ideally, orthogonally polarized), but those two contain information from 4 fields: E_x , E_y , H_x and H_y [2]. In our experiment we set the polarization optics such that what the signals detected can be written as

$$L_x(\mathbf{r}) = \eta_E E_x(\mathbf{r}) + \eta_H H_y(\mathbf{r}), \quad (1a)$$

$$L_y(\mathbf{r}) = \eta_E E_y(\mathbf{r}) + \eta_H H_x(\mathbf{r}), \quad (1b)$$

where $\eta_{E,H}$ are the relative sensitivities of our near-field probe to the electric (magnetic) near-fields, which have been previously determined using the method outlined in Ref. 2. Here we have explicitly noted that both the measured signals and the underlying fields are spatially dependent, while the sensitivities are (to first-order) not.

Since we have 4 unknowns and only two equations we clearly require another set of equations. Before we separate the different near-field components from our measurements, however, there are a couple of preliminary steps to our data processing that occur to eliminate unwanted signals (such as, for example, reflections in the waveguide). The results are outlined in Fig. S2, where we show every step of the procedure, beginning with the raw data from L_y 110 nm above the PhCW and ending with the separate E_y and H_x maps. The raw data [Fig. S2(a)] contains both a forward propagating mode as well as a backward propagating mode due to reflection from the end facet of the waveguide. We begin by filtering our data in k -space to isolate the forward propagating mode [3], which is shown in Fig. S2(b). Because of slight polarization mixing in the collection section of our set-up, we measure a small part of the L_x signal in the L_y channel. However, we can exploit the symmetry of the different field components to separate the two contributions [4]. Essentially, we can mirror the signal about the center of the PhCW and hence separate the field distributions with odd symmetry (E_x and H_y), from those that we want, which have even symmetry (for example, the presence of E_y and H_x introduced by polarization mixing in the fiber of the probe, or by asymmetries of the probe itself). The resultant signal is shown in Fig. S2(c).

Having cleaned up the signal by removing reflections and field components from the other channel we are now ready to separate the $\mathbf{E}$ from $\mathbf{H}$. As we noted above, we have 4 unknowns in Eqs. 1 and consequently we need two more equations. In essence what we require is a relation between $\mathbf{E}$ from $\mathbf{H}$ at each position in space. We can recover such a relation from our calculations of the PhCW modes (shown, for example, in Fig. 1 of the main text), which we write as

$$H_y(\mathbf{r}) = \gamma_x(\mathbf{r}) E_x(\mathbf{r}), \quad (2a)$$

$$H_x(\mathbf{r}) = \gamma_y(\mathbf{r}) E_y(\mathbf{r}). \quad (2b)$$

Substituting these equations into Eqs. 1, allows us to solve for

the electric field

$$E_x(\mathbf{r}) = \frac{L_x(\mathbf{r})}{\eta_E + \eta_H \gamma_x(\mathbf{r})}, \quad (3a)$$

$$E_y(\mathbf{r}) = \frac{L_y(\mathbf{r})}{\eta_E + \eta_H \gamma_y(\mathbf{r})}, \quad (3b)$$

where we remind the reader that $\eta_{E,H}$ are the experimentally determined parameters that characterize the efficiencies of our probe, and $\gamma_{x,y}(\mathbf{r})$ are found from numerical simulation of our structure. Once the electric fields are known, we can use Eqs. 2 to solve for the magnetic fields. An example of the results of this procedure, which for L_y yields maps is E_y and H_x , is shown in Fig. S2(d, e).

3. CLASSIFICATION OF C-POINTS

Each of the C-points that we have identified in the near fields of the photonic crystal has, in addition to a handedness, a type and index [5]. Both the type and index can be determined by looking at how the orientation of the ellipse varies and in the area about the c-point. In Fig. S3 we show the long axis of the polarization ellipse for both the electric and magnetic fields, at a height of 20 nm above the PhCW. In each frame we outline the lines traced out by the ellipse orientation, as one moves away from the c-point. From this figure we see that, near c_1 (blue curves), the orientation of the ellipse has a three-fold symmetry, corresponding to a star-type singularity. By tracing out the change in α about this point (in Fig. 4 of the main text, or Fig. S3 above), we see that the index of this singularity is $-1/2$. Likewise, from the area near c_2 (red curve), we see that c_2 is a lemon-type singularity with an index of $+1/2$. Note that for these polarization singularities the type and index are the same for the electric and magnetic fields, although at any point (relative to the C-point) the ellipse orientation is rotated by $\pi/2$ radians.

4. FIELD AMPLITUDE AND OUT-OF-PLANE COMPONENTS

We have, so far, discussed the structure of the 4 in-plane electromagnetic components, E_x , E_y , H_x and H_y , and the trajectories of their associated phase and polarization singularities. There are, however, two more aspects that should not be neglected, namely the amplitude of these fields and the 2 out-of-plane components, E_z and H_z . Although we are not sensitive to E_z or H_z with our measurements [2], they have, in general, non-zero amplitudes. In fact, as we show in Fig. S4, the amplitudes of these out-of-plane components can be comparable or even larger than those of the in-plane components. These out-of-plane components need not effect any potential application, for example, if a nanoscopic object is only non-negligibly polarizable in the $x - y$ plane. However, the questions remain: what is the amplitude of the in-plane field at the C-points relative to (1) the maximum in-plane amplitude, and (2) the out-of-plane amplitude at these positions? Answering question (1) will, for example, tell us how efficiently the C-point could be used for any potential application, while the answer to (2) allows us to determine whether the c-point is property of the entire electric (or magnetic) field, $\mathbf{E}$ (or $\mathbf{H}$), or just two of its components, E_x and E_y (or H_x and H_y).

Figure S5 summarizes both the positions and field amplitudes of the C-points, as extracted from the calculations. In parts (a) and (b) we see the positions of the C-points for the

Fig. S2. Example of the steps taken to separate $\mathbf{E}$ and $\mathbf{H}$ from the raw data. (a) The raw data from L_y 110 nm above the PhCW. (b) After FFT filtering to remove backwards propagating wave. (c) After symmetrization, when only the even symmetry components (E_y and H_x) remain. Using the algorithm outlined in the text results in separate maps of (d) E_y and (e) H_x , which are identical to the experimental data shown in Fig. 1b, c in the main manuscript.

Fig. S3. Calculated orientations of the long axis of the polarization ellipse, 20 nm above the PhCW. The star shape of c_1 is outlined in blue, and the lemon nature of c_2 in red. Note that the unit cell has been shifted by $-0.5 x/a$, relative to Fig. 3 of the main text, to place c_1 in the center of the image.

Fig. S4. Calculated amplitudes of all 6 field components 120 nm above the PhCW. The scaling of each component, relative to E_y or H_y for the electric and magnetic fields, respectively, is shown in the top right corner of each frame.

Fig. S5. Calculated position of the C-points associated with (a) the electric field and (b) the magnetic field, as a function of height. (c, d) The amplitude of the in-plane (c) electric or (d) magnetic field at the positions corresponding to the C-points shown in (a) or (b), respectively. The insets show the in-plane electric field amplitude at the c-point relative to the maximum in-plane amplitude, for each height.

electric and magnetic fields, respectively, which are the same as those summarized by Fig. 4 of the main text. Figure S5 shows the amplitude of the in-plane electric field at the location of the C-points shown in (a), as a function of height. As expected the field amplitude rapidly decreases for increasing heights, as the light fields evanescently decay away from the surface of the PhCW. A closer inspection of this figure reveals that the fall-off of the field amplitude is not a single exponential, but instead has a more complex form [6]. This deviation from exponential decay is particularly evident from the amplitudes found for C-points 2 (blue curve), which appear to have a local minimum at a height of ~ 100 nm and then to *increase* for larger heights. The complex behavior of the field amplitude at the C-points occurs both because, as discussed in the main text, the field distributions themselves are height dependent, and because the position of the c-point within the distribution also changes for different heights. This is easier to see in the insets to Fig. S5(c), which show the amplitude of the in-plane E at the location of the C-points, relative to the maximum in-plane E , for the different heights. As can be seen for C-points 1 (top inset), these singularities actually move towards the points of maximum in-plane E at larger heights. Conversely, C-points 2 scan across an oscillatory feature of the field and so their normalized amplitude, while always much smaller than that of at C-points 1, also oscillates. Similar observations can be made for the C-points in the magnetic field, shown in Fig. S5(d).

Next, we consider the out-of-plane field components, E_z and H_z . In Fig. S6 we show the evolution of the orientation of the fields at the locations of the different C-points. In this figure the orientation of each arrow (see inset) indicates the ratio of the in-plane fields (either $E_{\parallel}$ or $H_{\parallel}$, where recall that the field at the position of the C-points is necessarily circular and hence, for example, $E_{\parallel} = E_x \pm iE_y$) to the out-of-plane fields (either E_z or H_z). From the figure we see that, for the electric field, the field at c-point 1 is mostly in-plane (maximally ~ 0.1 out-of-plane amplitude at a height of 200 nm), while for c-point 2 $E_{\parallel}$ and E_z always have nearly the same amplitude. That is, an electrically polarizable object placed at c-point 1 will begin to spin in place, while at c-point 2 the object must only be polarizable in the $x - y$ plane for it to show this behavior; if the object can also be electrically polarized by E_z then, in addition to spinning in the $x - y$ plane, it will also move in z .

The orientation of the magnetic field at its associated C-

Fig. S6. Orientation of the electric field at the position of the C-points associated with (a) the electric field and (b) the magnetic field, as a function of height. The orientation of the arrow represents the ratio of the in-plane field, for example $\mathbf{E}_{\parallel} = (E_x, E_y)$, to the out-of-plane field, E_z , as shown in the inset to (a). All arrows have been normalized to the same length, which is not indicative of the absolute field amplitude at that position, for clarity. In both frames, C-points 1 are shown in red and C-points 2 in blue, corresponding to Fig. S5(a) and (b), above.

points [Fig. S6(b)] contains a much stronger H_z component than its electric counterpart. This is perhaps not entirely unexpected since the relative amplitude of H_z is almost an order of magnitude stronger than E_z (Fig. S4). In fact, the ratio of $\mathbf{H}_{\parallel}$ to H_z at C-points 1 is ~ 0.5 , while at C-points 2 the magnetic field is predominantly out-of-plane.

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