Rogue solitons in optical fibers: a dynamical process in a complex energy landscape?: supplementary material

ANDREA ARMAROLI^{1,2}, CLAUDIO CONTI³, AND FABIO BIANCALANA^{1,4,*}

¹Max Planck Research Group 'Nonlinear Photonic Nanostructures', Max Planck Institute for the Science of Light, Günther-Scharowsky-Str. 1/Bau 24 91058 Erlangen, Germany

²FOTON (CNRS-UMR 6082), Université de Rennes 1, ENSSAT, 6 rue de Kerampont, CS 80518, 22305 Lannion CEDEX, France

³Institute for Complex Systems (ISC-CNR) and University of Rome "La Sapienza", Department of Physics, Piazzale Aldo Moro 5 00185, Rome, Italy

⁴School of Engineering and Physical Sciences, Heriot-Watt University, EH14 4AS Edinburgh, United Kingdom

*Corresponding author: f.biancalana@hw.ac.uk

Published 14 May 2015

This document provides supplementary information to "Rogue solitons in optical fibers: a dynamical process in a complex energy landscape?" <http://dx.doi.org/10.1364/optica.2.000497>, including some discussion on the fine points of the analytical and numerical techniques used therein. © 2015 Optical Society of America

<http://dx.doi.org/10.1364/optica.2.000497.s001>

1. WEAK INTERACTION MODEL

In order to derive the weak interaction model, we start from Eq. (1) and use the following *Ansatz* composed by N weakly interacting NLS solitons, and model their pair-wise interaction as [1–8]

$$\begin{aligned}\dot{v}_k &= 16v_k^2 (S_{k,k-1} - S_{k,k+1}) \\ \dot{\mu}_k &= -16v_k^2 (C_{k,k-1} - C_{k,k+1}) \\ \dot{\xi}_k &= 2\mu_k - 4 (S_{k,k-1} - S_{k,k+1}) \\ \dot{\delta}_k &= 2(v_k^2 + \mu_k^2) - 8\mu_k (S_{k,k-1} + S_{k,k+1}) \\ &\quad + 24v_k (C_{k,k-1} + C_{k,k+1})\end{aligned}\quad (\text{S1})$$

where the dot denotes the derivative with respect to z and

$$\begin{aligned}S_{k,n} &= e^{|\beta_{kn}|} v_n \sin s_{kn} \phi_{kn} \\ C_{k,n} &= e^{|\beta_{kn}|} v_n \cos \phi_{kn} \\ \beta_{kn} &= 2v_k (\xi_k - \xi_n) \\ \phi_{kn} &= \delta_k - \delta_n - 2\mu_n (\xi_k - \xi_n)\end{aligned}\quad (\text{S2})$$

and $s_{kn} = \text{sgn}[\beta_{kn}]$.

In the case of homogeneous soliton amplitudes, we recall that the WIM can be reduced to a complex Toda lattice and was further extended to the perturbed NLS, e.g. in [9], to provide a general canonical formulation; nevertheless we proved in the text that the simplest model already provides an insight on the

mechanism of RS generation and perturbations such as Raman effect or higher-order dispersion are important only during collisions.

2. NUMERICAL CALCULATION OF FIXED POINTS

In order to find the fixed point of this system, we set N and compute numerically the fixed points starting from $N_{\text{iter}} = 5,000$ randomly distributed initial conditions: amplitudes v_k are log-normally distributed, velocity μ_k and positions ξ_k are normally distributed, and phases δ_k follow a uniform distribution in $[0, 2\pi]$.

Matlab[®] optimisation toolbox is employed, specifically the nonlinear solver `fsolve` with the Levenberg-Marquardt algorithm. Convergence is reliably achieved up to $N = 40$.

3. WEIBULL DISTRIBUTION

Heavy-tailed distributions are a large class of PDFs which exhibit subexponential tails [10]. In mathematical form the distribution of a random variable X is said to be heavy tailed if

$$\lim_{x \rightarrow \infty} e^{\lambda x} \Pr[X > x] = \infty,$$

for all $\lambda > 0$.

As described in the main part we fit the 95-th percentile of the peak amplitudes of the WIM fixed points with a Weibull

distribution. We employed the Matlab®Statistics Toolbox, which implements the following probability density function (PDF)

$$f(x|a, b) = ba^{-b}x^{b-1}\exp\left[-\left(\frac{x}{a}\right)^b\right]\theta(x),$$

where $\theta(x)$ is the Heaviside function and a and b are the fitting parameters. Due to the limited sample, the frequency of rare events tends to exhibit more discrepancies with respect to the fit.

4. ON THE SPLITTING OF NLS HAMILTONIAN DENSITY

It is well known that the NLS Eq. (1) conserves the integrated Hamiltonian density,

$$H(z) = \int_{-\infty}^{\infty} \mathcal{H}(z, t) dt$$

with $\mathcal{H}(z, t) \equiv \mathcal{H}_K + \mathcal{H}_{NL}$, $\mathcal{H}_K \equiv |u_t|^2/2$ and $\mathcal{H}_{NL} \equiv -|u|^4/2$.

We split $H(z)$ accordingly into two terms $H(z) = H_K(z) + H_{NL}(z)$, where $H_K(z)$ represents the kinetic and $H_{NL}(z)$ the non-linear (interaction) part.

In the presence of higher-order dispersion, we have to include higher-order terms in H_K , but we verified that this does not change qualitatively the conclusions reported in Fig.5.

5. A NOTE ON RAMAN ACCELERATION

With reference to Fig. 5(a), for a soliton with zero transverse velocity, the expected dependence of the kinetic part of the Hamiltonian is $H_K = H_K^0 + bP_0Z^2$, where $H_K^0 = \frac{1}{2} \int_{-\infty}^{\infty} |U_T|^2 dT$ is the kinetic part of the NLS Hamiltonian, $P_0 = \int_{-\infty}^{\infty} |U|^2 dT$ is the conserved total intensity and $b = \frac{4}{15} T_R A^4$ where $T_R = 3$ fs and A is the amplitude of the soliton represents the effect of Raman acceleration. Instead we observe a jump at each collision and a subsequent steady linear increment of H_K , on account of friction-like effects due to inelastic collisions.

REFERENCES

1. G. P. Agrawal, *Nonlinear Fiber Optics*, 5th ed. (Academic Press, 2012).
2. I. M. Uzunov, V. S. Gerdjikov, M. Gölles, and F. Lederer, "On the description of N-soliton interaction in optical fibers," *Opt. Commun.* **125**, 237–242 (1996).
3. V. S. Gerdjikov, D. Kaup, I. Uzunov, and E. Evstatiev, "Asymptotic behavior of N-soliton trains of the nonlinear Schrödinger equation," *Phys. Rev. Lett.* **77**, 3943–3946 (1996).
4. D. Anderson and M. Lisak, "Bandwidth limits due to incoherent soliton interaction in optical-fiber communication systems," *Phys. Rev. A* **32**, 2270 (1985).
5. D. Anderson and M. Lisak, "Bandwidth limits due to mutual pulse interaction in optical soliton communication systems," *Opt. Lett.* **11**, 174–176 (1986).
6. J. M. Arnold, "Soliton pulse-position modulation," *IEEE Proc. J. Optoelectron.* **140**, 359 (1993).
7. V. I. Karpman and V. V. Solov'ev, "A perturbation theory for soliton systems," *Phys. D: Nonlinear Phenomena* **3**, 142–164 (1981).
8. V. I. Karpman and V. V. Solov'ev, "A perturbational approach to the two-soliton systems," *Phys. D: Nonlinear Phenomena* **3**, 487–502 (1981).

9. V. S. Gerdjikov, B. Baizakov, M. Salerno, and N. Kostov, "Adiabatic N-soliton interactions of Bose-Einstein condensates in external potentials," *Phys. Rev. E* **73**, 046606 (2006).
10. S. Foss, D. Korshunov, S. Zachary, *An Introduction to Heavy-Tailed and Subexponential Distributions* (Springer, 2013).