Characterizing an entangled-photon source with classical detectors and measurements: supplementary material

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1. DATA NORMALIZATION AND FITTING

In this section we describe how the intensities that we measured in the lab were used to reconstruct a least-squares estimate of the spontaneously-generated two-photon density matrix. In standard two-photon polarization quantum state tomography (QST), at least 16 (but typically 36) different separable projective measurements are performed on the two photons. From these measurements, the probability of projecting the two-photon state ρ onto each projector is estimated[1]. In a typical experiment (Fig. 2b), each photon is sent to a different polarizing beamsplitter (PBS) and then the rates of both photons being transmitted (R_{HH}), both photons being reflected (R_{VV}), the first being transmitted and the second reflected (R_{HV}), and the second being transmitted and the first reflected (R_{VH}) are measured and used to estimate the probabilities of the different outcomes P_{HH} , P_{HV} , P_{VH} , and P_{VV} as $P_{HH} = R_{HH} / (R_{HH} + R_{HV} + R_{VH} + R_{VV})$ etc. These probabilities are taken to correspond to projecting ρ onto the projection operators $|H\rangle\langle H| \otimes |H\rangle\langle H|$ etc. This is then repeated for several other measurement settings, and a least-squares estimate of ρ , $\hat{\rho}_{LS}$ is constructed by minimizing a cost function

$$C = \sum_{s,i} (\text{Tr} [\hat{O}_{s,i} \hat{\rho}_{LS}] - P_{s,i})^2 \quad (\text{S1})$$

over $\hat{\rho}_{LS}$. In Eq. (S1), s and i label the signal and idler polarizations; i.e.

$$P_{s,i} = \{P_{HH}, P_{HV}, P_{VH}, P_{VV}, P_{DD}, \dots\}, \quad (\text{S2})$$

and

$$\hat{O}_{s,i} = \{|HH\rangle\langle HH|, |HV\rangle\langle HV|, \dots\} \quad (\text{S3})$$

(using $|HH\rangle\langle HH|$ as shorthand for $|H\rangle\langle H| \otimes |H\rangle\langle H|$).

In stimulated emission tomography (SET), these probabilities $P_{s,i}$ are estimated by comparing the stimulated intensity to the seed intensity. Ideally, this is done by seeding with different polarizations and measuring the intensity of the the stimulated light in different polarization bases. Then these stimulated intensities are normalized by the seed intensity to estimate probabilities. For example, if the seed diagonally polarized, and the stimulated beam is measured in the horizontal basis we can compute $P_{D,H}$ (the probability of detecting a diagonally-polarized photon and a horizontally-polarized photon in a spontaneous experiment) as:

$$P_{D,H}^{\text{ideal}} = \frac{I_H^{\text{stim}}}{I_D^{\text{seed}}}, \quad (\text{S4})$$

where I_H^{stim} is the intensity of the simulated light detected at the transmitted port of PBS_{*i*} in Fig. 2c, and I_D^{seed} is the total intensity of the seed light that is coupled into the signal fiber. (Note

we measure I_D^{seed} by summing the intensity that is measured exiting both output ports of PBS_s in Fig. 2c for any waveplate setting).

In practice, however, the coupling efficiencies of the signal and idler modes can be different so $\frac{I_H^{\text{stim}}}{I_D^{\text{seed}}}$ will not always result in the “true probability” $P_{D,H}$. For example, if the idler mode is coupled with efficiency ϵ_i , and the signal mode with efficiency ϵ_s then

$$p_{D,H}^{\text{ideal}} = \frac{I_H^{\text{stim}}}{I_D^{\text{seed}}} = \frac{\epsilon_i}{\epsilon_s} \times P_{D,H}, \quad (\text{S5})$$

which is proportional to the true probability, but has an additional “coupling factor”. To remedy this we take an additional step in our normalization. We first compute the ideal probabilities as defined by Eq. (S5) for all seed polarizations and stimulated measurement combinations. Then we renormalize these probabilities as

$$\tilde{P}_{DH} = \frac{p_{DH}^{\text{ideal}}}{p_{DH}^{\text{ideal}} + p_{DV}^{\text{ideal}} + p_{AH}^{\text{ideal}} + p_{AV}^{\text{ideal}}}. \quad (\text{S6})$$

Now, since the ideal probabilities are all proportional to $\frac{\epsilon_i}{\epsilon_s}$, this coupling factor will cancel out and we are left with

$$\tilde{P}_{DH} = \frac{P_{DH}}{P_{DH} + P_{DV} + P_{AH} + P_{AV}}, \quad (\text{S7})$$

and since $P_{DH} + P_{DV} + P_{AH} + P_{AV} = 1$, we have

$$\tilde{P}_{DH} = P_{DH}. \quad (\text{S8})$$

Thus even with different signal and idler coupling efficiencies we can extract the true probabilities.

We should mention that normalizing experimental data, to account for varying coupling/detection efficiencies, is an issue for both QST and SET. For our renormalization technique, the important point is that the denominator of (S6) should be equal to $\frac{\epsilon_i}{\epsilon_s}$. To achieve this, we used the rule that each probability was renormalized using data taken in the same basis. For example, for P_{DH} (described above) we used data taken when the first photon was measured in the D/A basis, and the second in the H/V basis, i.e. we renormalized by $p_{DH}^{\text{ideal}} + p_{DV}^{\text{ideal}} + p_{AH}^{\text{ideal}} + p_{AV}^{\text{ideal}}$. On the other hand, for (say) P_{DD} we renormalized by $p_{DD}^{\text{ideal}} + p_{DA}^{\text{ideal}} + p_{AD}^{\text{ideal}} + p_{AA}^{\text{ideal}}$, and so on.

In our experiment, we find one other systematic error. Namely, if we prepare diagonally-polarized seed light, the light that is ultimately coupled into the signal mode is not diagonally polarized. For example, we observe a birefringent phase shift, so that if we prepare the seed in $(|H\rangle + |V\rangle)/\sqrt{2}$ we find that the light coupled in the signal fiber has been rotated to $(|H\rangle + e^{i\phi}|V\rangle)/\sqrt{2}$. In addition there is a polarization-dependent coupling efficiency, by which we mean that $(|H\rangle + |V\rangle)/\sqrt{2}$ will go to $|a\rangle|H\rangle + |b\rangle|V\rangle$ with $|a\rangle \neq |b\rangle$. To characterize and correct for this, we reconstruct the polarization state of the seed light coupled that is coupled into the signal fiber, using single-photon QST. Then we perform our least-squared fit with a rotated basis. In other words, when we seed with (say) diagonally-polarized light and then find that this results in a different polarization $\rho_{D'}$ coupled into the signal fiber, we change the corresponding measurement operator from $\hat{O}_{D,i} = |D\rangle\langle D| \otimes |i\rangle\langle i|$ to $\hat{O}_{D',i} = \rho_{D'} \otimes |i\rangle\langle i|$. We do this for all six seed polarizations (H, V, D, A, R, and L), measuring the stimulated light in the same six bases for each seed polarization. A representative example of these seed polarization reconstructions is shown in Table 1.

Then the 36 different probabilities and rotated measurement operators are fed into our least-squares fitting routine to minimize Eq. (S1). As described in the main text, we find that this procedure predicts the amplitudes, purity and concurrence of the entangled state very well.

2. SIGNAL-IDLER POLARIZATION CORRELATIONS

One way to understand stimulated-emission tomography, is in terms of polarization correlations between the seed signal beam and the stimulated idler beam. For example, if an entangled photon-pair source is configured to produce the state $(|HH\rangle_{s,i} + |VV\rangle_{s,i})/\sqrt{2}$ and it is stimulated, then the polarization correlations listed in Table S1 will be observed. Namely, if the signal beam is diagonally polarized the idler beam will also be diagonally polarized, while if the signal beam is right-circularly polarized the idler beam will be left-circularly polarized, etc. However if the source is misaligned, so that it creates an equal mixture of $|HH\rangle$ and $|VV\rangle$ (rather than a quantum superposition) then if the signal beam is diagonally or right-circularly polarized the idler beam will always be incoherently polarized (i.e. a mixture of $|H\rangle$ and $|V\rangle$). Other variations of the spontaneously-generated two-photon state map on to correlations between the signal and idler polarizations as expected. A non-zero phase between $|HH\rangle$ and $|VV\rangle$ results in a rotation of the stimulated idler polarizations, and unequal amplitudes of $|HH\rangle$ and $|VV\rangle$ map onto unequal amplitudes of $|H\rangle$ and $|V\rangle$ in the idler polarization.

3. POLARIZATION-DEPENDENT LOSS

In this section, we address an important experimental consideration: the theory of [2], and all of the discussion above implicitly assumes a single spatial mode, i.e. all of the seed light incident in the signal mode stimulates light into the idler mode, and the light in both modes is detected. In the data presented above this was approximately true; however, it is possible for this assumption to break down. In our experiment, this breakdown occurs if the “seed lens” (Fig. 2) is removed so that the incoming seed beam is approximately the same size as the pump beam. In this configuration we find that it is possible for the seed beam to stimulate light into the idler mode, and then to be “walked out” of the signal mode so that it is not detected. The opposite could also happen – light could be coupled into the signal mode without stimulating any light into the idler mode. Such errors lead to discrepancies between the predictions of SET and QST.

The symptoms and result of this problem are as follows. When a diagonally polarized seed beam is incident, we would expect that the ratio of the intensity of the horizontally-polarized light to the vertically-polarized light that is coupled into the signal fiber is $R_{s_H/s_V} \approx 1$. However, due to birefringent spatial walk-off, either horizontally or vertically polarized light could be coupled into the signal mode more efficiently. This would depend on the incidence angle of the seed beam (θ in Fig. 2 of the main text). In our experiment when the seed beam waist is approximately the same size as the pump beam waist, we find that R_{s_H/s_V} can vary from 0.1 to 10 when θ varies by approximately 10 mrad. In any case, if the spontaneously-generated two-photon state is close to $(|HH\rangle + |VV\rangle)/\sqrt{2}$, then ideally the ratio of the horizontal intensity to vertical intensity for the idler polarization R_{i_H/i_V} would follow the seed polarization, i.e. $R_{i_H/i_V} = R_{s_H/s_V}$ (even if $R_{s_H/s_V} \neq 1$). But we find without the seed lens this is not the case, and so, when the waist of the seed beam is small, the results of SET can vary a great deal.

Table S1. A list of the polarization states that are stimulated into the idler mode (second column) for each seed polarization state (first column) when the SPDC source is configured to produce the nominal state $(|HH\rangle_{s,i} + |VV\rangle_{s,i})/\sqrt{2}$. Example experimentally reconstructed seed and stimulated polarization states are also shown.

Seeding Signal Polarizations			Stimulated Idler Polarizations		
Ideal		Experimental	Ideal		Experimental
$ H\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$		$\begin{pmatrix} 0.996 & -0.020 + 0.058i \\ -0.020 - 0.058i & 0.004 \end{pmatrix}$	$ H\rangle = \begin{pmatrix} 0.936 & -0.062 - 0.053i \\ -0.062 + 0.053i & 0.064 \end{pmatrix}$		
$ V\rangle = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$		$\begin{pmatrix} 0.002 & 0.025 - 0.031i \\ 0.025 + 0.031i & 0.998 \end{pmatrix}$	$ V\rangle = \begin{pmatrix} 0.016 & -0.026 + 0.024i \\ -0.026 - 0.024i & 0.984 \end{pmatrix}$		
$ D\rangle = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$		$\begin{pmatrix} 0.506 & 0.492 - 0.068i \\ 0.492 + 0.068i & 0.494 \end{pmatrix}$	$ D\rangle = \begin{pmatrix} 0.514 & 0.482 - 0.056i \\ 0.482 + 0.056i & 0.486 \end{pmatrix}$		
$ A\rangle = \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$		$\begin{pmatrix} 0.449 & -0.484 + 0.107i \\ -0.484 - 0.107i & 0.551 \end{pmatrix}$	$ A\rangle = \begin{pmatrix} 0.438 & -0.485 + 0.076i \\ -0.485 - 0.076i & 0.562 \end{pmatrix}$		
$ R\rangle = \begin{pmatrix} 0.5 & -0.5i \\ 0.5 & 0.5 \end{pmatrix}$		$\begin{pmatrix} 0.525 & -0.080 - 0.489i \\ -0.080 + 0.489i & 0.475 \end{pmatrix}$	$ L\rangle = \begin{pmatrix} 0.540 & 0.011 + 0.498i \\ 0.011 - 0.498i & 0.460 \end{pmatrix}$		
$ L\rangle = \begin{pmatrix} 0.5 & 0.5i \\ -0.5 & 0.5 \end{pmatrix}$		$\begin{pmatrix} 0.430 & 0.081 + 0.484i \\ 0.081 - 0.484i & 0.570 \end{pmatrix}$	$ R\rangle = \begin{pmatrix} 0.419 & -0.080 - 0.433i \\ -0.080 + 0.433i & 0.581 \end{pmatrix}$		

For example, in one configuration (when the source was configured to produce $(|HH\rangle + |VV\rangle)/\sqrt{2}$) we found $R_{s_H/s_V} = 10.5$ and $R_{i_H/i_V} = 11.6$. In this case the predictions of SET and QST agree: SET predicts a concurrence of 0.915, while QST predicts a concurrence of 0.916. However, changing θ by a few mrad resulted in $R_{s_H/s_V} = 1.16$ and $R_{i_H/i_V} = 0.064$. In this setting SET predicts a concurrence of 0.442 – greatly underestimating the “true concurrence” of 0.916 revealed by QST (which does not change when the seed beam changes). This illustrates that care must be taken so that the assumptions made in [2] are met. In our experiment, this meant making the seed waist larger than the birefringent spatial walk-off. In other systems this particular source of error may not be as prevalent, but care must still be taken to ensure that these sorts of errors are not occurring in other modes, such as temporal or frequency modes. Note that making a single-photon coincidence measurement to estimate the actual ratio of the amplitudes of $|HH\rangle$ to $|VV\rangle$ and comparing to this result to R_{s_H/s_V} and R_{i_H/i_V} would immediately reveal this error. We checked that this was the case for all of our data presented in the paper. However, we stress that in our experiment the seed waist was large, so we were not sensitive to this error. With a large seed waist, θ must to be changed by an amount that would appreciably decrease the intensity of either the signal or idler beam before $R_{i_H/i_V} \neq R_{s_H/s_V}$.

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