Conservation of orbital angular momentum in air core optical fibers: supplementary materials

P. GREGG,¹ P. KRISTENSEN,² S. RAMACHANDRAN,^{1,*}

¹Boston University, 8 St. Mary's St, Boston, Massachusetts, USA

²OFS-Fitel, Priorparken 680, Brøndby 2605, Denmark

*Corresponding author: sidr@bu.edu

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This document provides supplementary information to “Conservation of orbital angular momentum in air core optical fibers,” <http://dx.doi.org/10.1364/optica.2.000267>. The supplementary material is organized as follows. In §1 we list in detail the materials used for these experiments. In §2-3 we briefly review fiber vector and OAM modes, and fiber mode coupling. In §4 we provide additional detail on our fiber design methodology. In §5 we describe spiral interference images as a diagnostic tool. In §6 describe in detail our experimental OAM excitation scheme. In §7 we provide additional time domain data and describe measurement uncertainties. © 2015 Optical Society of America
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1. Materials and Methods

To fabricate the air core fiber, standard modified chemical vapor deposition (MCVD) is used to deposit the high-index ring region. However, contrary to standard fiber draws, the preform tube is not collapsed on the MCVD lathe. During the draw, the preform is pressurized on the order of a few hundred mm H₂O and nitrogen is passed through the fiber during draw, which is performed at a few m/s. Fabrication is achieved with standard production equipment for non-zero dispersion shifted fiber (NZDF), and hence much higher draw speeds may be achievable.

The fiber index profile in **Fig. 1b** was obtained using an interferometry-based fiber profiler (IFA-100 Interfiber Analysis) at 633nm and is plotted with respect to the refractive index of silica. Since the profiler detects changes in the index of refraction interferometrically, an abrupt transition between air and glass is too large to be correctly calculated. Thus, the fiber is cleaved and allowed to soak in, via capillary action, index-matching oil (Cargille Laboratories) with $n \sim n_{\text{Silica}}$ to minimize the index discontinuity at that interface. Simulations were performed with a numerical finite-difference method waveguide solver [1].

The fiber is cleaved with a Fujikura CT-100 fiber cleaver at tension ~200 grams. Care is taken in cleaving the fiber as cleaving at an angle will result in a discontinuity where the cleave's shearing action meets across the air core opposite to the cleaver blade. In phase, this discontinuity resembles an orbital angular momentum (OAM) perturbation and should be avoided.

The CW external cavity laser (ECL) used is a continuously tunable HP 8168F. The picosecond laser used is a PriTel FFL passively mode-locked fiber laser with a 3dB bandwidth of 0.6nm.

Conventional SMF-28e is used for all single mode fiber links. The air core fiber is held on a Thorlabs Nanomax stage. Input coupling is performed with an $f \sim 4.5$ mm mid-IR coated aspheric lens.

The CPBS is comprised of a zero-order quarter wave plate and a polarization beam displacing prism. The camera is an InGaAs Allied Vision Technologies “Goldeye” camera. The fast detector is a New Focus 1444-50 picosecond photo-detector. The oscilloscope used was an Agilent infiniiium DCA wide bandwidth oscilloscope, 86100A, with detector unit HP 83485B, 40 GHz port. The spatial light modulator (SLM) used was a Hamamatsu LCOS-SLM x10468-08, with 600x800 pixels of pitch 20 μ m.

Fiber loss measurements are conducted via the conventional cutback method, measuring power as a function of fiber length at 1550nm from the ECL, after ensuring that the desired mode is excited with high purity – more details on launching pure OAM modes are provided in §6. Measured losses were 1.9 and 2.2 dB/km for the $|\ell| = 5$ and 6 mode groups, respectively. This is higher than the first reported ring fiber that guided only the $|\ell| = 0$ and 1 modes (1.3 and 1.6 dB/km, respectively) [2] whose index gradients were limited to those achievable with Ge-doped silica fibers. We attribute the higher losses for the air-core fibers to lengthwise fabrication imperfections and fundamental loss limits arising from high (glass-air) index contrasts [3]. It is conceivable that this number may reduce considerably in future realizations since the index contrast of the ring-core to glass-cladding is only 3.5×10^{-2} , which is comparable to that of commercial dispersion compensating fibers (DCF), whose losses can be as low as 0.5 dB/km [4].

2. Vector and OAM Modes

Conventionally, for azimuthal mode number $L > 1$, fiber vector modes are designated as:

$$HE_{L+1,m}^{e,o} = F_{L,m}(r) \begin{Bmatrix} \hat{x} \cos(L\varphi) - \hat{y} \sin(L\varphi) \\ \hat{x} \sin(L\varphi) + \hat{y} \cos(L\varphi) \end{Bmatrix} e^{ik_z HE^z} \quad (S1)$$

$$EH_{L-1,m}^{e,o} = F_{L,m}(r) \begin{Bmatrix} \hat{x} \cos(L\varphi) + \hat{y} \sin(L\varphi) \\ \hat{x} \sin(L\varphi) - \hat{y} \cos(L\varphi) \end{Bmatrix} e^{ik_z EH^z} \quad (S2)$$

$F_{L,m}(r)$ is the mode's electric field envelope, where m is the radial mode number and $m-1$ the number of zeroes in $F(x,y)$ and (r,φ) are transverse coordinates, and k_z is the mode's longitudinal wave vector, related to the effective index by $k_z = 2\pi n_{eff}/\lambda$. The $HE^{e,o}$ (even and odd) modes are degenerate with each other in circular fibers, as the vector fields are related by a rotation. Similarly, the $EH^{e,o}$ modes are degenerate; however, the pairs are not degenerate with each other, $k_{z,HE} \neq k_{z,EH}$.

The HE/EH polarization structure is spatially varying, as shown in **Fig. S1(a-b)**, and difficult to excite. An equivalent representation considers combinations of *degenerate* vector modes:

$$V_{L,m}^{\pm} = HE_{L+1,m}^e \pm iHE_{L+1,m}^o = \hat{\sigma}^{\pm} F_{L,m}(r) e^{\pm iL\varphi} e^{ik_z HE^z} \quad (S3)$$

$$W_{L,m}^{\pm} = EH_{L-1,m}^e \mp iEH_{L-1,m}^o = \hat{\sigma}^{\mp} F_{L,m}(r) e^{\pm iL\varphi} e^{ik_z EH^z} \quad (S4)$$

Here, $\hat{\sigma}^{\pm} = \hat{x} \pm i\hat{y}$ indicates left and right handed circular polarizations. We define an orbital angular momentum (OAM) state of topological charge L as:

$$\psi_L = \hat{e} G(r) e^{iL\varphi} \quad (S5)$$

where $\hat{e}$ is an arbitrary but spatially uniform polarization, and $G(r)$ the field's radial envelope. It is evident from (Eqn. S 3-5) that these fiber modes are OAM states, with $L = \pm L$ for $V_{L,m}^{\pm}$ and $L = \mp L$ for $W_{L,m}^{\pm}$, and have spatially uniform polarizations (**Fig. S1c**). The two kinds of OAM states, $V_{L,m}^{\pm}$ and $W_{L,m}^{\pm}$, are distinguished by comparing the handedness of OAM and circular polarization, with $V_{L,m}^{\pm}$ possessing spin and orbital angular momentum of the same handedness, and $W_{L,m}^{\pm}$ the opposite handedness. We refer to these two classifications as “spin-orbit aligned” and “spin-orbit anti-aligned,” respectively. We define the 4 modes, $V_{L,m}^{\pm}$ and $W_{L,m}^{\pm}$, of the same (L,m) as an “OAM family.” The HE/EH modes and the OAM modes, are *equivalent* descriptions of fiber eigenmodes of the vector wave equation under the weak-guidance approximation[5].

For fibers with small refractive index differences between core and cladding, $n_{eff,HE} \approx n_{eff,EH}$. In this case, an OAM family is susceptible to strong mode coupling from birefringent perturbations (detailed further in §3) and the fiber output typically resembles scalar linearly polarized (LP) “modes” which have a characteristic “beady” intensity pattern (see **Fig. S4c**). However,

Fig. S1 Plot of the electric field of (a) the HE_{31}^o and (b) EH_{21}^o mode in an air core fiber (similar polarization patterns exist for any weakly-guiding circular fiber). (c) Plot of the electric field of an OAM mode made of complex combinations of the HE_{31} even and odd modes. Note that the complicated spatially-dependent polarization has given way to a spatially uniform circular polarization, and this now resembles a free-space Laguerre-Gaussian beam.

the resulting field is not a fiber mode as it does not possess one distinct effective index. The splitting between degenerate pairs of OAM modes may be enhanced by designing a waveguide such that a large field coincides with a large index contrast [6]. Thus, ring fibers are advantageous, as they mimic the field structure of OAM modes. In addition the ring thickness can be chosen to restrict guidance to primarily the $m=1$ radial order, thus sculpting the density of states for guided modes.

3. Brief overview of fiber mode coupling

For any realistic waveguide, coupling between bound modes is expected, due either to deterministic perturbations or random fluctuations in the fiber. Following Bjarklev [7], we write the refractive index profile of a fiber as $n(r,z) = n_o(r) + n_b(r,\varphi,z)$, where n_o is the fiber's ideal profile and n_b , which is separable into transverse and longitudinal parts, the latter denoted $f(z)$. The coefficient of power coupling between modes j and k is then determined by:

$$\langle P_{j,k} \rangle = \frac{\omega^2}{c^2} \Phi(k_{z,j} - k_{z,k}) \left(\iint r dr d\varphi N_b E_j^* E_k \right)^2 \quad (S6)$$

where ω and c are the frequency and speed of light, Φ is the spatial power spectrum of the autocorrelation of $f(z)$, E_j is the normalized electric field of the j^{th} mode, and $N_b(r,\varphi) = n_b(r,\varphi,z)/f(z)$ is the transverse perturbation. If the autocorrelation of $f(z)$ is assumed to be of Gaussian form, then:

$$\Phi(k_{z,j} - k_{z,k}) = \sqrt{\pi} \sigma^2 L_c e^{-\frac{1}{2} L_c (k_{z,j} - k_{z,k})^2} \quad (S7)$$

where σ is the RMS deviation of $f(z)$, and L_c is its correlation length.

The angular part of the perturbation can be separated by

$$N_b(r, \varphi) = \sum_{p=-\infty}^{\infty} \tilde{N}_p(r) e^{ip\varphi} \quad (S8)$$

If the j^{th} and k^{th} modes are OAM states, as specified by (S 5), we can separate the integrals in (r,φ) , and simplify the angular part of the integral in (S 6):

$$\iint r dr d\varphi N_b E_j^* E_k \propto \sum_{p=-\infty}^{\infty} \int_0^{2\pi} d\varphi e^{i(L_j - L_k + p)\varphi} \quad (S9)$$

(S 9) vanishes unless

$$p - (L_j - L_k) = 0 \quad (S10)$$

This selection rule implies overall conservation of orbital angular momentum. Knowledge of the angular Fourier series coefficients [8], $\tilde{N}_p$, enables prediction of the results of a perturbation. For instance, it is evident that for an untilted fiber bragg grating no transfer of OAM between the input mode and perturbation should be possible, since $\tilde{N}_p = 0 \forall p \neq 0$, although if the grating is tilted, transfer is permitted.

In (S 7), since Φ achieves maximum values for $\Delta n_{eff} = n_{eff,j} - n_{eff,k} = 0$, maximum exchange of power between modes will be achieved when they are phase matched. In theory, two degenerate modes will never couple if the integral in (S6) is zero. In practice any real perturbation will have some Fourier spectrum in both azimuthal and radial coordinates, as in equation (2); thus, coupling between degenerate states is generally expected. A simple manifestation of this is coupling between the two polarization modes in conventional single mode fibers because of which polarization cannot be maintained. The question becomes one of coupling length scale.

In view of phase matching considerations only, strong coupling between the spin-orbit aligned states, $V_{L,m}^+$ and $W_{L,m}^-$, would be expected. However, to achieve this coupling a perturbation, n_b , must couple *both* opposing SAMs and OAMs of arbitrary $|L|$, in order to conserve total angular momentum flux. As indicated in

Fig 3a, such a perturbation is unlikely for high $|\mathcal{L}|$. In view of coupling strength relative to the integral in (S6), coupling is likely between the modes $V_{L,m}^+$ and $W_{L,m}^-$, since any birefringent perturbation will be sufficient to couple $\hat{\sigma}^+$ and $\hat{\sigma}^-$. Indeed, this coupling is evident in even a few meters of conventional fibers. However, for OAM states in our air core fiber, this coupling is inhibited because states of the same $\mathcal{L}$ are non-degenerate.

4. Modes of air core fibers

Desired OAM states possess quantum numbers $(L,m) = (\mathcal{L},1)$. Modes with higher radial quantum numbers should be suppressed because they can lead to so-called “accidental degeneracies,” in which a mode with quantum numbers L,M may be degenerate, over some wavelength range, with a mode with quantum numbers L',M' , where L and L' may be several integers apart (**Fig. S2a**; note that modes of different azimuthal symmetry have no avoided-crossings that would prevent these accidental degeneracies). Coupling between such states necessitates a perturbation with OAM content $\mathcal{L}' - \mathcal{L}$ which, although still potentially large, is generally more feasible than a perturbation with content $2\mathcal{L}$, which will destroy a fiber’s ability to transmit higher-order OAM states over long distances.

The most obvious design methodology for suppressing $m>1$ coupling is to make a thin, high-contrast ring, for which the $m>1$ states simply are not guided. Recent work has theoretically proposed such waveguides [9], and experimental work has claimed OAM propagation over cm-lengths [10]. A more comprehensive review of recent work on OAM in ring fibers may be found in [11]. However, when the index-ring is made very thin, differences in electromagnetic continuity conditions between **s** and **p** polarized electromagnetic fields will cause spin-orbit coupling, a fundamental effect preventing km-scale OAM state

Fig. S2 (a) Accidental degeneracy in a particular fiber design between the $(L,m) = (6,1)$ and $(2,2)$ modes. Even though the $|\mathcal{L}| = 6$ OAM modes are sufficiently split in effective index, they are degenerate with another OAM family, preventing pure propagation of either family. (b) Radial envelope for the transverse field, $E(r)$, of one of the $|\mathcal{L}| = 6$ OAM modes. The radial envelopes for every mode within an OAM family are nearly identical. (c) Radial envelope of the mode field of an $m=2$ mode. Due to the possibility for accidental degeneracy these modes should be avoided, although they are valid OAM states. (d) n_{eff} splitting as a function of $|\mathcal{L}|$ and r_{core} for a constant ring width. Depending on the chosen radii, an ensemble of OAM states with different members is supported. This forms a band OAM states as a function of r_{core} , and is generally designable as desired.

propagation in such thin-ring air-core fibers [12]. In order to avoid this debilitating effect, we allow for a thicker ring, for which the presence of $m>1$ modes may be unavoidable. In such waveguides, the additional design requirement is the avoidance of accidental degeneracy in the wavelength region of interest. Examples of $m=1$ and $m=2$ modes in the air core fiber are shown in **Fig. S2(b and c)**, where we wish to avoid operation in regions in which the $m=2$ modes are closely spaced in n_{eff} to desired OAM modes.

The design yields a band of OAM states in $|\mathcal{L}|$, for which the n_{eff} splitting values are large enough ($\geq 1e-4$) to enable propagation of pure OAM states (see discussion in [6] for a phenomenological reasoning behind this difference as a near-universal metric in optical fibers). Below the lowest $|\mathcal{L}|$ for which OAM states are supported, the n_{eff} splitting is too small and LP-like behavior is obtained. Above the largest $|\mathcal{L}|$, modes are cut-off. The specific set of $|\mathcal{L}|$ states which support OAM propagation are a function of the dimensions of the air core fiber: the outer radius of the air core, r_{core} , outer radius of the ring, r_{ring} , and the index contrast between the ring and the silica cladding, Δn . This waveguide design optimization is illustrated in **Fig. S2d** for fiber with large index contrast, $\Delta n = 0.035$. As r_{core} is increased for constant ring widths ($r_{\text{ring}} - r_{\text{core}}$), the mode volume increases and higher $|\mathcal{L}|$ OAM states are supported. A similar effect is observed when mode volume is changed by allowing r_{ring} to vary. However, as mode volume is increased, the number of $m=2$ modes also increases in a nonlinear manner, thus, r_{core} and r_{ring} must be chosen in tandem to support the desired OAM ensemble while avoiding accidental degeneracy. For the fabricated air core fiber tested in this work, $r_{\text{core}} = 3\mu\text{m}$ and $r_{\text{ring}} = 8.25\mu\text{m}$, designed for an OAM ensemble of $|\mathcal{L}| = \{5,6,7\}$ in the 1550nm wavelength range. Fluctuations in waveguide parameters over the course of a fiber draw may limit the long-length conservation of OAM for fiber length scales $>> 1\text{km}$.

Modal dispersions and effective areas are given in **Fig. S3** for the $|\mathcal{L}|=5$ and 6 mode families. Effective areas are roughly double that of standard SMF in the same wavelength region.

Fig. S3 (a) Modal dispersions and (b) effective areas for the $|\mathcal{L}|=5$ and 6 mode families across C-band. All modes are similar due to nearly identical radial mode profiles. Mode designations (V, W) as in Eqs. S3, S4.

5. Brief overview of OAM-Gaussian (spiral) interfere

Fig. S4 a depicts a simulated interference between a pure, defocused $\mathcal{L} = -6$ OAM state and a Gaussian beam. The right-handed spiral indicates, in the convention we have chosen, $\mathcal{L} < 0$, while the six spiral arms, or parastiches, indicates the absolute value of $\mathcal{L}$. Comparing the simulated behavior shown in **Fig. S4a** with that shown in **Fig. S4b**, we observe that when the OAM state degrades in purity by the addition of a meager amount (~ -14 dB) of $\mathcal{L} = 6$, the arms become “beady.” In the case where $\mathcal{L} = 6$ and $\mathcal{L} = -6$ have equal contributions (**Fig. S4c**), the arms give way entirely to a circularly symmetric constellation of $2|\mathcal{L}|$ beads, somewhat similar to the field of an LP state; this is indeed the origin of observation of only LP-like states in conventional fibers, where mixing between all modes of the same OAM family readily occurs. If an OAM state is degraded by the presence of a neighboring $\mathcal{L}$ state (**Fig. S4d**), the intensity distribution of the interference pattern becomes asymmetric, although the number of spiral arms is unchanged. One expects the same effect to occur in an interference pattern between a pure OAM mode and an off-centered Gaussian. In contrast to the degraded behaviors discussed above and shown in **Fig. S4**, we find that the experimentally recorded images in **Fig. 2** demonstrate clean OAM state propagation in air core fibers.

Fig. S4 (a) Simulated interference pattern between an expanded Gaussian and an $\mathcal{L} = -6$ OAM mode. The number of spiral arms indicates $|\mathcal{L}|$ for a pure OAM state, and the handedness indicates whether $\mathcal{L}$ is positive or negative. (b) Interference pattern between a Gaussian and an OAM beam with dominant component $\mathcal{L} = -6$ and ~ 14 dB MPI into $\mathcal{L} = 6$. Such impurity is manifest in beadieness along the spiral arms. (c) Interference between a Gaussian beam and a beam with equal parts $\mathcal{L} = 6$ and $\mathcal{L} = -6$. Spiral arms give way to a LP mode-like constellation. (d) Gaussian interference with a beam containing $\mathcal{L} = -6$ with 6dB MPI in $\mathcal{L} = -5$. Note that the same effect is expected if the Gaussian is off-center.

6. Experimental coupling scheme

Excitation of fiber OAM modes requires matching one or a set of free-space OAM states to the corresponding fiber state in size and relative dimensions for a variety of $\mathcal{L}$, and ensuring that the incident beam is both tilt-free and on-axis relative to the fiber to avoid projection into other fiber OAM states. To achieve our goal we use two holograms. A collimated Gaussian beam illuminates the first pattern, SLM1 (equipment details in §1), on which is displayed:

$$SLM1(x, y, r, \varphi) = \left(\mathcal{L}\varphi + k_{SLM}x + \frac{r^2\pi}{\lambda F} + M(r, \varphi) \right) * LG_{\mathcal{L}0}(r; w) \quad (S11)$$

Here $(x, y) = (m, n) \cdot \Delta x$ are the SLM's spatial coordinates, which are equal to some integer (m, n) multiples of the SLM pixel pitch, Δx , where $m=n=0$ indicates the SLM's center. (r, φ) are the (approximate) polar coordinates of the SLM, k_{SLM} is a blazed grating wave vector of programmable magnitude, λ is the wavelength of operation, F is the focal length of some lens to which the phase curvature (third term in brackets in (S11)) corresponds, $LG_{\mathcal{L}0}(r; w)$ is the radial envelope of the $LG_{\mathcal{L}0}$ Laguerre Gauss mode of beam waist, w , scaled such that the peak amplitude is 1, and $M(r, \varphi)$ is a phase function related to the $LG_{\mathcal{L}0}$ mask. The azimuthal phase, in conjunction with the phase mask, M , and amplitude mask, $LG_{\mathcal{L}0}$, optimize coupling into the $LG_{\mathcal{L}0}$ mode in the far field [13,14]. Since this conversion happens with sub-optimal efficiency, a phase tilt, $k_{SLM}x$, is added to spatially isolate the desired diffraction order. Additionally, a phase curvature corresponding to F is added, such that a distance F from the SLM, the far-field is obtained as though the beam were passed through a lens. In our experiments, $F \sim 1$ m.

The diffracted beam, a distance F from SLM1, is multiplied by a second phase hologram:

$$SLM2(x, y, r, \varphi) = \left(\frac{r^2\pi}{\lambda F_2} \right) \text{circ} \left(\frac{r}{R} \right) + \text{attn} * \left(1 - \text{circ} \left(\frac{r}{R} \right) \right) \quad (S12)$$

Where F_2 is, again, the focal length of the lens with phase curvature corresponding to the first term in (S12), and R is a scale factor determined *in situ*. This pattern is designed to, within the region of interest $r < R$, multiply by a spherical phase. Outside this region of interest, an attenuation pattern, *attn*, is applied, such that light in the *attn* region is scattered in all directions [15]. This completely separates the first diffracted order of SLM1 from the rest. In our experiments, $R \sim 2$ mm, dependent on F and w in (S11).

The near field of SLM2 is then imaged by an $f \sim 4.5$ mm aspheric lens into the fiber under test, through a zero-order quarter wave plate (aligned to negligibly degrade or displace the incident OAM state). F_2 is chosen to offset the additional phase curvature accrued by such an imaging operation. Higher diffracted orders from SLM2 are vignetted, either by the clear aperture of the coupling lens, or by the air core fiber itself.

The primary advantage of this method is as follows: in free space, the ring radius of an LG beam after focus is determined by the beam's topological charge, $\mathcal{L}$, initial beam waist, and lens focal length. Thus, beams of different $\mathcal{L}$ map to different sizes in the focal plane of a given imaging system. In the air core fiber, however, modes with different $\mathcal{L}$ are constrained by the waveguide to have roughly the same radius. Thus, by performing only a focusing operation, beam size and lens focal length cannot be simultaneously optimized for coupling different $|\mathcal{L}|$ into the air-core fiber. The two-stage excitation system, on the other hand, enables exciting a wider range of desired OAM states of the fiber by digitally manipulating the SLM parameters.

We have found that the shape of the radial envelope is less critical to mode excitation compared with beam axis-overlap. If the fiber is offset from the incident beam by distances on the order of $0.1\mu\text{m}$, mode purity can drop by an order of magnitude (10dB). Similarly, tuning the incident wavelength without changing the phase tilt, k_{SLM} , on SLM1 by as little as 5nm can result in input coupling mode purity degradation of ~ 10 dB.

While our coupling scheme is necessarily component intensive and alignment sensitive, it suffices for the purposes of the primary research goals herein – of studying fiber propagation and stability of OAM. Alternative coupling/multiplexing strategies include OAM mode sorters [16], integrated silicon photonic interferometric couplers [17], and cascaded dove prism interferometers [18], and we note that the input/output coupling problem is beginning to

receive greater attention from the community, as practical applications of OAM, including using fibers that can support them, develop.

7. Supplementary time domain measurements and error estimation

For the time domain measurements described in discussion related to **Fig. 4**, data was taken for all 8 modes of the $|\mathcal{L}| = 5$ and $|\mathcal{L}| = 6$ OAM families. Data for the $\mathcal{L} = -5$ and $\mathcal{L} = -6$ states is shown in **Fig. S5a**, with individual traces in **Fig. S5(b-d)**. Each mode overlaps in time with its degenerate partner from **Fig. 4**, and exhibits similar MPI compared to states shown in the main text as expected; the sign of $\mathcal{L}$ should not alter mode purity.

The reason that simple peak power ratios obtained from the temporal traces suffice in calculating mode purity or MPI is that the high $|\mathcal{L}|$ OAM states in air core fibers have similar group-velocity dispersion values (see **Fig S3a**) The dispersion values, as simulated from the measured refractive index profile of the air-core fiber, are 60.0 and 58.0 ps/nm/km for the spin-orbit aligned and anti-aligned modes of the $|\mathcal{L}| = 5$ family, and 64.3 and 62.4 ps/nm/km for the aligned and anti-aligned modes of the $|\mathcal{L}| = 6$ OAM family. So, our ~ 7 ps pulsed source (FWHM ~ 0.6 nm), broadens to similar output pulse widths of 36.7 and 35.5ps for the $|\mathcal{L}| = 5$ spin-orbit aligned and anti-aligned modes, and 39.2 and 38.0ps for the $|\mathcal{L}| = 6$ aligned and anti-aligned modes, respectively, resulting in similar peak power reductions after 1km of propagation.

Fig. S5 (a) Time domain traces for the remaining 4 OAM modes in the $|\mathcal{L}| = 5$ and $|\mathcal{L}| = 6$ OAM families not shown in **Fig 5a**. Traces are offset for vertical clarity, in order of increasing group delay. Each mode overlaps in time with its degenerate partner, within the resolution limit of the measurement, and has comparable MPI. (b-e) Close-ups of time domain trace for $\mathcal{L} = -5 \hat{\sigma}^+$, $\mathcal{L} = -5 \hat{\sigma}^-$, $\mathcal{L} = -6 \hat{\sigma}^+$, and $\mathcal{L} = -6 \hat{\sigma}^-$. In each case, the excited mode is at least 15dB pure in relation to the background.

One limit to this time-of-flight measurement is laser pulse drift in time with respect to the electrical trigger signal on the order of 10s of picoseconds (**Fig S6a**). Whether this drift is intrinsic to the laser or is a consequence of thermal/environmental fluctuations of the optical system as a whole is unclear. The spurious peak shown approximately 1.5ns after each main measurement peak is a result of electrical ringing in the detector + oscilloscope receiver. **Fig. S6b** shows similar impulse response behavior for pulsed transmission through 2m of single mode fiber at 1550nm, in which this behavior cannot be a result of multimode effects in the fiber under test. Thus, we conclude that the spurious peaks must be considered part of the measurement background.

Additionally, the detector for these measurements is polarization-sensitive on the level of about 8~10%. Assuming the worst case error (the parasitic mode is detected 10% less efficiently than the signal mode, or vice versa), this leads to an error in measured mode purity of ± 0.5 dB.

Fig. S6 (a) Several traces of the same state over a period of a few minutes. Drift from peak to peak provides an accuracy limitation for this measurement and may be caused by fluctuations inside the laser relative to its electrical trigger, or from temperature/environmental fluctuations of the optical system as a whole. (b) Time of flight measurement for transmission over 2m of single mode fiber, plotted on log scale. The ringing around 548.9ns is part of the impulse response of the receiver, and is evident (around 1.5ns after the main peak) in all time of flight measurements.

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