K-space design of terahertz plasmonic filters: supplementary material

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1. Methods

A. THz Transmission Measurements

The transmission spectra were obtained using standard THz time-domain spectroscopy measurements [S1]. We used photoconductive antennas for both THz generation and detection stimulated by an ultrafast Ti:Sapphire laser. The broadband, linearly polarized THz radiation generated by the emitter was collected and collimated by an off-axis parabolic mirror, such that the radiation was normally incident on the filters. A second off-axis parabolic mirror placed after the filter was used to focus the transmitted THz radiation onto the photoconductive detector. To test the polarization dependent filters, the printed devices were mounted on a rotation stage and measurements were made at 10° increments over a 90° range.

B. Filter Design and Fabrication

All of the digital images used to create k-space structures were generated with the standard version of MATLAB with no additional toolboxes or add-ons. An example of the k-space design flow for a 720 GHz circularly symmetric band pass filter is demonstrated in Fig. S1.

The digital image in k-space is designed as a 2000 x 2000 array of pixels. Initially a filter frequency response is defined; in this case a brick-wall band pass filter is designed with a center frequency of 720 GHz and a bandwidth of 75 GHz, shown in Fig S1a. Since the filter is designed to be circularly symmetric, the 1D bandpass temporal frequency response is rotated 360° around the

Fig. S1. K-space design details (a) Desired frequency response for a 720 GHz band pass filter. (b) Corresponding k-space or spatial frequency domain image, obtained by rotating (a) about the origin to create an annulus. (c) The spatial frequency domain image is a 2000 x 2000 pixel array; the ring has a radius of 96 pixels and a width of 10 pixels. Each pixel is interpreted to represent a spatial frequency of 25 m^{-1} . (d) The inverse 2D Fourier transformed real space image of (c). The transformed image is also composed of 2000 x 2000 pixels; The resolution of each pixel is interpreted as $20 \mu\text{m}$, corresponding to $1/50000 \text{ m}^{-1}$. The total spatial size of the image is 4 cm corresponding to $1/25 \text{ m}^{-1}$, however only 1cm x 1cm of the image is shown.

origin in k-space. Every point in k-space has a radial coordinate given by $k=2\pi\nu/c$, with a corresponding

value at that point given by the magnitude of the frequency response at that value of v . Thus every symmetric radial cross section in 2D k -space has the desired 1D temporal frequency response. For a brick-wall bandpass filter, this rotation yields a thin annulus in k -space, as seen in Fig. S3b. The annulus has a central radius of 96 pixels and a width of 10 pixels. A single pixel corresponds to a spatial frequency resolution, F_{res} , of 25 m^{-1} , where the spatial frequency is given in general by $F = k/2\pi$. Thus, the highest spatial frequency represented in the k -space image is $F_{\text{max}} = 1000 \cdot F_{\text{res}} = 25000 \text{ m}^{-1}$, as shown in Fig. S1c. The center spatial frequency of the band pass filter, with a radial distance of 96 pixels, is $F_c = 2400 \text{ m}^{-1}$. The resonant wavelength for the filter is given simply by $\lambda_c = 1/F_c = 0.417 \text{ mm}$. Assuming that the filter is free standing in air, the resonant frequency for the filter is given by $\nu_c = c / \lambda_c$, where c is the speed of light in vacuum, yielding a frequency of 720 GHz.

After taking a 2D Inverse Discrete Fourier Transform (IDFT) of the k -space representation shown in Fig. S1c, the real part of the transform is retained and plotted in Fig. S1d. The spatial extent of the filter, shown in Fig. S1d, is given by $1/F_{\text{res}} = 4 \text{ cm}$ and the spatial resolution of the sample image is given by $1/(2F_{\text{max}}) = 20 \text{ }\mu\text{m}$; however, only a $1 \text{ cm} \times 1 \text{ cm}$ section of the image is shown for clarity. The spatial resolution of $20 \text{ }\mu\text{m}$ was chosen to be well below the resolution of the commercial printer used in fabricating the filters. It should be noted that the spatial image obtained from the IDFT contains negative values that cannot be represented by a printer system that deposits varying amounts of ink within a range of 0-255. Thus, the spatial image must be thresholded and normalized to the 0-255 range, in order to be printed. This thresholding process causes higher order combinations of reciprocal vectors to appear in k -space and in the measured spectrum.

Once the digital filter image has been thresholded and normalized, it can be directly printed using a commercial inkjet printer with a conductive ink cartridge. The real space image must be printed with the proper resolution and size to create the correct structure. Most print utilities have a scale option that must be properly set to create a filter with the right spatial size to match the designed spatial frequency resolution. In this example, the required spatial size of 4 cm can be easily verified with a ruler. We used inkjet printable inks from Methode Development Company (9101 conductive silver ink and 3804 resistive carbon ink). The filters were printed using a low cost, commercially available color inkjet printer, Epson Workforce 30. The printed silver ink sinters at room temperature immediately after printing, yielding a final device with no additional post processing steps. The adhesion of the silver ink to the substrate was measured using an industry standard Scotch tape adhesion test in

accordance with the ASTM D3359 standard. No silver flecks were found on the peeled tape. In order to create spatial variations in surface conductivity in a more controlled and linear fashion, we used a combination of silver and carbon ink [S2]. Filters requiring spatially varying conductivities could then be printed using a color-coded image that mapped to the silver and carbon ink cartridges.

The ink-jet printing process was used in two different modes: (1) multi-layer depositions of only conductive silver ink or (2) simultaneous deposition of both silver and carbon ink. In the former mode, a single solid film using only silver ink yields structures with a conductivity of $5 \times 10^7 \text{ S/m}$, which is nearly the same conductivity as that of bulk silver [S3]. Inkjet printers, of course, allow for 8-bit grey scale printing, corresponding to values between 0 and 255.

Fig. S2. Grey scale printing using only silver ink (a) THz transmission as a function of the grey scale printing level. Here, a value of 0, representing black, corresponds to maximum deposition of silver ink, while a value of 255, representing white, corresponds to deposition of no silver ink. The transmission response is approximately linear for grey scale level between 56 to 141 (b) Transmission enhancement as a function of the grey scale level for a 0.25 THz printed hole array using the same grey scale definition. The grey scale values apply to the use of only silver ink, without the addition of any carbon ink. Enhancement is defined as the transmission value of the desired resonant peak divided by the background transmission level.

This corresponds to simply depositing less silver over the same substrate area, which in turn allows varying surface conductivity and THz transmission. In Fig. S2a, we show the change in THz transmission for a printed solid rectangle as a function of the grey scale level. The response was found to be approximately linear for grey scale values between ~ 56 and ~ 141 . In Fig. S2b, we show the enhanced transmission through a periodic hole array at the resonant peak frequency of 0.25 THz. In each case, the geometrical properties of the array are identical, only the grey scale printing level was changed. In the latter mode, where both silver and carbon inks were used simultaneously, the detailed response properties have been discussed elsewhere [S2]. For designed filter images with grey scale variations outside of the linear grey scale range of 56-141, a histogram

Fig. S3. (a) k-space representation of a narrow band-pass filter. (b) Corresponding 2D spatial image obtained from the inverse Fourier transform (c) Measured THz spectrum for the filter in b. (d) k-space representation of a wide band-pass filter. (e) Corresponding 2D spatial image obtained from the inverse Fourier transform. (f) Measured THz spectrum for the filter shown in e.

equalization algorithm can be used to convert the contrast of the image into the proper grey scale range. Alternatively, a structure can be thresholded at different levels and then multiplied by the proper grey scale value to reach the linear region, as was the case with the structure in Figure 3 in the main paper. The plastic substrate upon which the ink structures are deposited creates an asymmetric material stack, with the plastic film acting as the substrate and air acting as the superstrate. The difference in permittivities gives rise to two sets of resonances [S4].

2. Circularly Symmetric Band Pass Filter

Conventional periodic and quasiperiodic hole arrays [S4,S5] can be viewed as band-pass or comb pass filters. In the manuscript, we demonstrated an 8-fold symmetric structure by defining eight points with the same arc spacing and radial length in k-space with a central radius that corresponds to the location of the k_x and k_y resonant peaks of the band pass filter. The ease of the k-space design methodology allows designers to directly control the frequency response of a desired device.

We also created a series of band pass filters in which the k-space pattern was circularly symmetric, as shown in Fig. S3. The circular symmetry in k-space gives rise to bullseye structures in real space that produce surface plasmons that resonate across the surface of each silver ring coupling to radiative modes on the other side of the filter. Other bullseye plasmonic structures have

previously been demonstrated at THz frequencies [S6]. In previous bull's-eye structures, surface plasmons are launched from each successive metal groove toward an aperture at the center of the structure where a signal is transmitted to the other side. The measured spectra of previous bullseye structures [S6] agree with the predicted k-space design approach and are consistent with the results shown in Fig. S2.

3. Real Space Images of the Multi-Resonant Filters

As discussed briefly in regard to Fig. 3 of the manuscript, the transform from k-space to physical space for the different versions of the comb filter show that all three versions have a similar underlying structure but different contrasts in peaks and valleys of the sinusoidal varying structures in the image. By applying a thresholding algorithm, we obtain a 0.4 THz resonant hole array with the intermediate holes (grey holes in Fig. S4 below) darkened according to the weight defined in k-space. In these comb filter variations, the higher frequency weighted filter had higher valued intermediate holes, which corresponds to higher transmission through those particular holes as described earlier in Fig. S2. In Fig. S4, we show the real space images for the three k-space filter designs shown in Figs. 3a-3c of the manuscript. The amount of silver deposited in the odd indexed holes of the array decreases as the relative magnitude of the higher k-space resonance is increased, yet all periodicities remain the same for the three filters.

Fig. S4. (a) Real space image derived from the k-space image in Fig. 3a of the manuscript, where the white holes have a grey scale value of 0 and the grey holes have a grey scale value of 113. (b) Real space image derived from the k-space image in Fig. 3b of the manuscript, where the white holes have a grey scale value of 0 and the grey holes have a grey scale value of 141. (c) Real space image derived from the k-space image in Fig. 3c of the main text, where the white holes have a grey scale value of 0 and the grey holes have a grey scale value of 169.

4. Non Trivial K-Space Spectrum

The k-space design methodology enables a complexity in filter design that cannot be easily achieved by other methods. Both narrow band and broadband responses can be incorporated into a single filter using well-understood and simple operations such as convolution and multiplication. To illustrate this point, Fig. S5 describes a proposed complex filter where a

simple narrow band filter composed of discrete resonant points can be expanded to have an additional complex

Fig. S5a shows the k-space representation of a 4-fold symmetric 0.4 THz filter. The real space structure corresponding to this k-space representation is a simple multiplication of a sinusoid along the x-axis with a sinusoid in the y-axis, which is similar to Fig. 3b but without the stretching effect. By convolving the 4-fold symmetric filter in k-space with the bi-exponential function shown in Fig. S5b, a far more complex filter function can be obtained. Fig. S5c shows the resultant k-space behavior composed of both a narrowband and broadband response. The real space image for the filter is described in Fig. S5d, in where sharp windowing is applied along the x and y axis, which arises from the broadband nature of the bi-exponential function. The sharp windowing requires that fabrication of the real space image have extremely tight tolerances in the grey scale variation. Even small errors in the grey scale gradient will result in errors in the k-space bi-exponential behavior of the complex filter. The inkjet printing technique described here does not have an appropriate level of precision to properly fabricate this structure. However, advances in future fabrication techniques may allow for better fidelity.

Fig. S5. Proposed complex broadband filter created from a simple narrow band filter (a) Simple k-space structure, representing a narrow 400GHz resonant hole array. (b) Bi-exponential function along the k_x and k_y -axis (c) Complex broadband structure resulting from the convolution of the bi-exponential in b with the simple spectrum shown in a. The cross structure shows a complicated spectrum that cannot be created by a superposition of discrete resonant peaks. (d) The real space structure obtained from a 2D inverse Fourier transform of b. This filter cannot be created without very careful control of the grey scale conductivity of the structure.

5. K-Space Notch Filter:

To further show the versatility in k-space design approach, we present a very simple design of a notch filter operating at a frequency of 340 GHz. The k-space representation, shown in Fig. S6a, shows pass bands on either side of the notch. These low and high passbands were not extended over the entire k-space plane because doing so creates a real space image that is very difficult to fabricate. As more of the k-space plane is included in the two passbands, more energy is consolidated into a single delta function in real space. Thus a k-space representation of a unity plane with a single annulus notch will create a real space image with a single high intensity pixel at the center of the image. The structures that create the notch will have such low contrast that they will not be visible in the image, and the filter cannot be fabricated. To work around this issue we used limited low and high pass bands surrounding the notch. The waveform obtained using THz time-domain spectroscopy (THz-TDS) is shown in Fig. S6c. It should be noted that not only the [1,0] mode of the notch is

present in the transmission spectrum of Fig. S5d, but also the [1,1] mode located at 485 GHz. Improved notch filtering can be obtained by using multilayer stacks of these filters [S7].

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Fig. S6. K-space Notch filter (a) K-space representation of a 340GHz notch filter. (b) Real space image derived from the k-space image in a, The small number of visible rings in the real space structure is due to the relatively wide side bands of the designed k-space notch. (c) Measured transmission spectra for the k-space notch filter, obtained from a Fourier transform of c. Along with the [1,0] mode of the notch filter present at 345 GHz, the [1,1] mode of the filter can be seen at approximately 485 GHz.