Discrete Anderson speckle: supplementary material

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This document provides supplementary information to "Discrete Anderson speckle," <http://dx.doi.org/10.1364/optica.2.000201>. We provide additional computational results. Specifically, we carry out a computational study to support the key results reported in the main text, but applied to 1D and 2D waveguide arrays with *diagonal* disorder. After briefly reviewing the difference between diagonal and off-diagonal disorder in our calculations, we proceed in the same order followed in the main text. In addition, we clarify the source for the dephasing of the optical field with propagation in disordered arrays for both disorder classes. The computations indicate that there are no qualitative differences between the behavior exhibited by lattices with diagonal or off-diagonal disorder. 2015 Optical Society of America

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MODEL

Consider an array of parallel waveguides on a 1D lattice with evanescent nearest-neighbor-only coupling. The field propagation is described by the coupled equations

$$i \frac{dE_x(z)}{dz} + \beta_x E_x + C_{x,x-1} E_{x-1} + C_{x,x+1} E_{x+1} = 0, \quad (\text{S1})$$

where $E_x(z)$ is the complex optical field in the x^{th} waveguide ($x = -N, \dots, N$) at axial position z , β_x is the propagation constant of waveguide x , and $C_{x,x+1}$ is the coupling coefficient between adjacent waveguides x and $x+1$. These equations may be written as

$$i \frac{d\mathbf{E}}{dz} + \hat{\mathbf{H}}\mathbf{E} = 0,$$

where $\mathbf{E}(z)$ is a vector with components $\{E_x(z)\}_{x=-N}^N$ and $\hat{\mathbf{H}}$ is the Hermitian coupling matrix given by

$$\hat{\mathbf{H}} = \begin{pmatrix} \ddots & \ddots & 0 & \dots & 0 \\ \ddots & \beta_{x-1} & C_{x-1,x} & \ddots & \vdots \\ 0 & C_{x,x-1} & \beta_x & C_{x,x+1} & 0 \\ \vdots & \ddots & C_{x+1,x} & \beta_{x+1} & \ddots \\ 0 & \dots & 0 & \ddots & \ddots \end{pmatrix} \quad (\text{S2})$$

The evolution of the input field $E_i(x_i) = E_{x_i}(z=0)$ to the output $E_o(x_o) = E_{x_o}(z)$ may be written as

$$E_o(x_o) = \sum_{x_i} h(x_o, x_i) E_i(x_i), \quad (\text{S3})$$

where $h(x_o, x_i)$ corresponds to the matrix elements of $e^{i\hat{\mathbf{H}}z}$ and represents the impulse response function of the system. The point spread function (PSF) is $|h(x_o, x_i)|^2$ and corresponds to the output intensity. This formulation is readily extended to 2D lattices.

In this supplement, we focus on *diagonal* disorder, which is characterized by constant coupling coefficients ($C_{x,x+1} = \bar{C}$) and random propagation constants β_x having a uniform probability distribution of mean $\bar{\beta}$ and half width $\Delta\beta$. We use the same normalization scheme used in the main text. The coupling coefficients are written in units of their average $\bar{C}$, and the distance z in units of the coupling length $\ell = 1/\bar{C}$. Throughout, $\Delta\beta$ ranges from 0 to 2, and the lattice size is $N_t = 151$ for 1D and $N_t \times N_t = 101 \times 101$ for 2D (hexagonal honeycomb) lattices ($N_t = 2N + 1$). The sizes are chosen such that reflections from the array boundaries do not affect the conclusions. Statistical averages are taken for an ensemble with 10^6 and 10^4 realizations for 1D and 2D simulations, respectively. Unless otherwise stated these parameters are identical to those used for the calculations in the off-diagonal disorder case presented in the main text unless otherwise stated.

RESULTS

Anderson Localization

We first report the spatial distribution of the mean output intensity for single-waveguide excitation in 1D and 2D waveguide lattices for selected disorder levels in Fig. S1 and Fig. S2(a), respectively. In both cases, ballistic spread in periodic arrays leads to extended output states and gradually introducing disorder results in Anderson localization. The spatial width σ_s of the mean output intensity as a function of disorder level is depicted for 1D and 2D arrays in the inset of Fig. S1(b) and Fig. S2(b), respectively.

Fig. S1. (a) PSF $I(x_0) = |h(x_0, 0)|^2$ at $z = 10$ for a 1D periodic array with single-waveguide excitation at $x_i = 0$. Inset is a schematic of the configuration. (b) Mean PSF $\langle I(x_0) \rangle = \langle |h(x_0, 0)|^2 \rangle$ for disordered 1D arrays. Insets show the localization length σ_s as a function of $\Delta\beta$ and z . For the values of σ_s in the insets, 21 points for $\Delta\beta$ and 200 for z are chosen.

Discrete Anderson Speckle - Transverse Coherence

The focus in our paper is on the case of coherent uniform illumination extending across the whole array – in *diagonally* disordered Anderson lattices here. Examples of individual realizations for 1D and 2D lattices are shown in Fig. S3 and Fig. S4, respectively. Similar behavior in these discrete-speckle realizations is observed in the case of off-diagonal disorder.

Next, we calculated the magnitudes of $g^{(1)}(x)$ and $g^{(1)}(r)$ for 1D and 2D lattices, respectively, revealing a non-zero pedestal $|g^{(1)}(\infty)|$ riding on which is a finite-width distribution. The results are presented in Figs. S5. From these results we obtained the long-range-order pedestal $|g^{(1)}(\infty)|$ for 1D and 2D lattices, which are plotted in Fig. S5(c). Finally, we present the axial evolution of the long-range-order coherence pedestal $|g^{(1)}(\infty)|$ for different diagonal disorder levels in a 1D lattice in Fig. S5(d).

Axial Coherence

Figure S6(a) depicts the axial evolution of the intensity along individual realizations of 1D lattices with different disorder levels. The normalized axial coherence function $|g^{(1)}(\Delta z)|$ for different disorder levels and its width σ_a are given in Fig. S6(b,c).

Dephasing

Here, we present the probability distributions for the phase of the output field due to extended coherent illumination for both disorder classes in Fig. S7. The probability distribution depends on the propagation distance and disorder level. In all cases,

Fig. S2. (a) Mean PSF $\langle I(x_0, y_0) \rangle = \langle |h(x_0, 0; y_0, 0)|^2 \rangle$ for 2D hexagonal (honeycomb) arrays with increasing disorder (from left to right) at $z = 10$. For clarity, each panel is normalized separately and convolved with a gaussian function of width 1.6 for visualization. (b) Localization radius σ_s as a function of disorder level $\Delta\beta$. For the values of σ_s shown, 11 points for $\Delta\beta$ are chosen.

Fig. S3. (a) Realizations of the intensity distribution of discrete Anderson speckle at various disorder levels ($z = 10$) for extended uniform coherent input light. The speckle grain size decreases with disorder. The dotted lines are ensemble averages that confirm the statistical homogeneity. (b) Same as (a) except that $z = 20$.

Fig. S4. Individual realizations of discrete Anderson speckle at various disorder levels corresponding to the panels in Fig. S2(a). The excitation is extended uniform coherent light. Note that the speckle grain size decreases with disorder. Insets are magnified by a factor of 2. Speckle contrasts are 0, 0.67, 1.17, 1.53 with increasing disorder level.

Fig. S5. (a) Magnitude of the degree of the normalized transverse coherence function $|g^{(1)}(x)|$ for 1D and (b) $|g^{(1)}(r)|$ for 2D arrays at various disorder levels at $z = 10$. (c) The long-range-order coherence pedestal $|g^{(1)}(\infty)|$ for 1D and 2D arrays. The circles in (a) correspond to those in (c) for 1D lattices. Similarly, the hexagons in (b) correspond to those in (c) for 2D lattices. In (c), 21 and 11 points for $\Delta\beta$ are taken for 1D and 2D arrays, respectively. (d) $|g^{(1)}(\infty)|$ as a function of propagation distance z at various disorder levels for 1D waveguide arrays.

Fig. S6. (a) Axial evolution of the intensity along individual realizations of 1D lattice with different disorder levels. (b) The normalized axial coherence function $|g^{(1)}(\Delta z)|$ for different disorder levels and (c) its width σ_a . (a-b) For z , 200 points are taken. In (c), 11 points are taken for $\Delta\beta$.

Fig. S7. Probability distribution function of the phases φ of the field in (a) off-diagonal and (b) diagonal 1D disordered arrays at $z = 10$. For periodic arrays, the phase has a constant (z -dependent) value (black vertical line at $\varphi = -0.372\pi$ at $z = 10$). In this case, the distribution is simply a delta function of height 1.

the probability distributions evolve into a uniform distribution as the propagation distance increases. This transition happens quicker for higher disorder levels. Figure S8 depicts the transverse coherence function calculated for the *magnitude* and *phase* of the field separately for both disorder classes. The results confirm that the statistical fluctuations of the field is in fact due primarily to dephasing, not due to fluctuations of the magnitude of the field. Similar behavior can also be observed in the axial coherence function as shown in Fig. S9.

CONCLUSION

As we have seen, the qualitative behavior of the field statistical characteristics for 1D and 2D lattices with diagonal disorder is similar to those with off-diagonal disorder. The main quantitative difference that we observed is that we usually double the amount of disorder when going from off-diagonal to diagonal disorder. That is, the quantitative results presented here approach those in the main text most when we set $\Delta\beta = 2\Delta C$. We conclude that discrete Anderson speckle produced in waveguide arrays with *diagonal* and *off-diagonal* have the same qualitative statistical characteristics.

Fig. S8. Transverse coherence function $\Gamma_1(x) = \langle |E(0)| |E^*(x)| \rangle$ for the field magnitude in (a) off-diagonal and (b) diagonal 1D disordered arrays at $z = 10$. Transverse coherence function $\Gamma_2(x) = \langle \varphi(0) \varphi(x) \rangle$ for the field phase in (c) off-diagonal and (d) diagonal 1D disordered arrays at $z = 10$.

Fig. S9. Axial coherence function $\Gamma_1(\Delta z) = \langle |E(0)| |E^*(z)| \rangle$ for the field magnitude averaged over all transverse positions x in (a) off-diagonal and (b) diagonal 1D disordered arrays. (b) Axial coherence function $\Gamma_2(\Delta z) = \langle \varphi(0) \varphi(\Delta z) \rangle$ for the field phase in (c) off-diagonal and (d) diagonal 1D disordered arrays also averaged over all transverse positions x .