Transient behavior of invisibility cloaks for diffusive light propagation: supplementary material

ROBERT SCHITTNY^{1,*†}, ANDREAS NIEMEYER^{1,†}, MUAMER KADIC¹, TIEMO BÜCKMANN¹, ANDREAS NABER¹, AND MARTIN WEGENER^{1,2}

¹Institute of Applied Physics, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

²Institute of Nanotechnology, Karlsruhe Institute of Technology (KIT), 76021 Karlsruhe, Germany

*Corresponding author: robert.schittny@kit.edu

†These authors contributed equally to this work

Published 21 January 2015

This document provides supplementary information to “Transient behavior of invisibility cloaks for diffusive light propagation,” <http://dx.doi.org/10.1364/optica.2.000084>. We elaborate on the numerical model and its parameters used for the calculations presented in the main paper. © 2015 Optical Society of America

<http://dx.doi.org/10.1364/optica.2.000084.s001>

All fabrication details and the suppliers of the components of the cylindrical core-shell cloak used in the present paper can be found in the Supplementary Material of Ref. [1].

In the same Supplementary Material of Ref. [1], we have derived the scattering mean free path $l_{s,0}$ of the water-paint surrounding from ballistic transmission experiments. For the used white-paint concentration of 0.35% in de-ionized water, we extrapolated $l_{s,0} = 233 \mu\text{m}$. In the meantime, we have refined this value by additional measurements (not depicted) for paint concentrations closer to the used value of 0.35%, leading to a slightly larger value of $l_{s,0} = 252 \mu\text{m}$. Furthermore, we have performed corresponding experiments for the PDMS shell doped with malamine microparticles as well. There, we find a scattering mean free path length of $l_{s,2} = 5.56 \text{ mm}$. Provided the angular scattering patterns in the surrounding and in the shell are the same (also see discussion below), these numbers translate to a ratio of the diffusivities of cloaking shell and surrounding of $D_2/D_0 = l_{s,2}/l_{s,0} = 20.9$. Previously [1], we have inferred this ratio more indirectly, leading to a substantially smaller value of 7.3.

The mathematics of the diffusion equation underlying the numerical modelling presented in Fig. 3 of the present main paper has been described in detail in the Supplementary Material of Ref. [1] and shall not be repeated here. The geometrical parameters $2R_1 = 32.1 \text{ mm}$, $2R_2 = 39.8 \text{ mm}$, and $L = 60 \text{ mm}$ are chosen like in the experiment. The ratio D_2/D_0 is fixed to the value of 20.9 given above. The diffusivity of the core is approximated as zero, i.e., $D_1 = 0$. The full width at half maximum (FWHM) of the illumination spot on the front surface of the tank

has been measured as 3.2 cm. In the numerical modelling, we choose spatially Gaussian illumination with the same FWHM. The temporal profile of the incident pulse is also assumed to be Gaussian with a FWHM of 0.5 ns. All other parameters are considered as free and are used to fit to the entire experimental data set (composed of reference, obstacle, and cloak) shown in Fig. 2 of the present main paper. The parameters chosen here are: $D_0 = 7.58 \times 10^8 \text{ cm}^2/\text{s}$, $\tau_0 = 9.5 \text{ ns}$, $\tau_{12} = 0.2 \mu\text{s}$, and $K = 0.743 \times 10^9 \text{ cm/s}$. Our calculations based on these parameters not only well describe the time-dependent experimental data shown in Fig. 2, but also the time-integrated data shown at the top of each panel in Fig. 2, which are equivalent to the static case. Broadly speaking, the static data can easily be fitted with several different parameter combinations, whereas the time-resolved data put much tighter constraints onto the choice of the parameters. Our choice may still not be unique though.

The parameters chosen here are quite similar to those estimated in the Supplementary Material of Ref. [1], except for the absolute photon diffusivity of the surrounding D_0 , which is 4.33 times larger than previously [1] (the ratio D_2/D_0 has already been discussed above). This aspect can be traced back to the assumptions made in Ref. [1]. There, we have crudely estimated the photon diffusivity D from the scattering mean free path l_s (as measured by the ballistic transmission experiments, see above), which we have assumed to be equal to the transport mean free path l_t , from which the photon diffusivity results via

$$D = \frac{1}{3} v_e l_t, \quad (1)$$

with the energy velocity v_e . This reasoning tacitly assumes isotropic scattering, whereas the Mie scattering off of micrometer sized particles like in the experimental surrounding as well as in the shell is much more pronounced in the forward direction. In general, the scattering mean free path and the transport mean free path are related via [2–4]

$$l_t = \frac{l_s}{1 - \langle \cos(\Theta) \rangle}, \quad (2)$$

where $\langle \cos(\Theta) \rangle$ is the average over the cosine of the scattering angle. Isotropic scattering means $\langle \cos(\Theta) \rangle = 0$, hence $l_t/l_s = 1$. Exclusively forward scattering means $\langle \cos(\Theta) \rangle = 1$, hence $l_t/l_s \rightarrow \infty$. To get to the above factor of 4.33, equivalent to $l_t/l_s = 4.33$, one needs $\langle \cos(\Theta) \rangle = 0.77$. This value appears reasonable for scattering off of micrometer-sized particles, even though the exact value might be different for the two host-particle mixtures we have used for surrounding and cloaking shell, respectively.

Additionally, we briefly comment on backscattering of light. Note that the entire main paper as well as the entire supplementary material is about forward scattering of light (*i.e.*, diffusive transmission). As the diffusive transmission is below 1% for our conditions, more than 99% of the light is backscattered (assuming negligible absorption). This holds true for the reference, the obstacle, and for the cloak. This means that for backscattering, there is little to cloak in the first place, because 1% variations on a 99% background are hard to detect and not very interesting. Our computer simulations with and without absorption confirm this qualitative reasoning. Thus, we have not studied in detail the static and dynamic behavior of our core-shell cloak in backscattering geometry.

For the diffusive cloak designed using the principle of transformation optics (presented in Fig. 4(b) in the main text), we use the following Pendry-type coordinate transformation in polar coordinates:

$$r \rightarrow r'(r, \theta) = R_1 + \frac{R_2 - R_1}{R_2} r \quad (3)$$

$$\theta \rightarrow \theta'(r, \theta) = \theta \quad (4)$$

We choose the ratio of radii to be identical to the one for core-shell cloaks discussed in the main text, *i.e.*, $R_2/R_1 = 1.25$.

Applying this coordinate transformation to the heat-conduction equation (Ref. 19 of the main text)

$$\vec{\nabla} \cdot (\kappa \vec{\nabla} T) - c\rho \frac{\partial T}{\partial t} = 0 \quad (5)$$

with conductivity κ and specific heat $c\rho$, one finds

$$\vec{\nabla}' \cdot \left(\kappa \overset{\leftrightarrow}{J} \overset{\leftrightarrow}{J}^T \frac{1}{\det(\overset{\leftrightarrow}{J})} \vec{\nabla}' T \right) - \frac{c\rho}{\det(\overset{\leftrightarrow}{J})} \frac{\partial T}{\partial t} = 0 \quad (6)$$

with the two-dimensional Jacoby matrix $\overset{\leftrightarrow}{J}$ of the transformation (Ref. 19 of the main text). Obviously, the effects of the transformation can be cast into heat conductivity and specific heat, leading to the transformed equation

$$\vec{\nabla}' \cdot \left(\overset{\leftrightarrow}{\kappa'} \vec{\nabla}' T \right) - c\rho' \frac{\partial T}{\partial t} = 0, \quad (7)$$

with the transformed material parameters

$$\overset{\leftrightarrow}{\kappa'} = \kappa \overset{\leftrightarrow}{J} \overset{\leftrightarrow}{J}^T \frac{1}{\det(\overset{\leftrightarrow}{J})} \quad (8)$$

and

$$c\rho' = \frac{c\rho}{\det(\overset{\leftrightarrow}{J})}. \quad (9)$$

In the case of diffusion, however, there is only a single material parameter (the diffusivity D) in the underlying differential equation (also see Ref. 18 of the main text):

$$\vec{\nabla} \cdot (D \vec{\nabla} n) - \frac{\partial n}{\partial t} = 0. \quad (10)$$

Again applying the coordinate transformation above, one arrives at

$$\vec{\nabla}' \cdot \left(D \overset{\leftrightarrow}{J} \overset{\leftrightarrow}{J}^T \frac{1}{\det(\overset{\leftrightarrow}{J})} \vec{\nabla}' n \right) - \frac{1}{\det(\overset{\leftrightarrow}{J})} \frac{\partial n}{\partial t} = 0. \quad (11)$$

In this case, the transformation cannot be cast into the material parameters, since there is no equivalent to the specific heat in diffusion of light. Hence, the time-dependent diffusion equation is not form-invariant under general coordinate transformations. This fact prohibits the design of transformation-based cloaks for diffusive light under transient conditions (see Fig. 4(b) of the main paper). In the static case, this problem does not arise as the time derivative is zero.

REFERENCES

1. R. Schittny, M. Kadic, T. Bückmann, and M. Wegener, “Invisibility cloaking in a diffusive light scattering medium,” *Science* **345**, 427 (2014).
2. K. Busch, C. M. Soukoulis, and E. N. Economou, “Transport and scattering mean free paths of classical waves,” *Phys. Rev. B* **50**, 93 (1994).
3. J. Galvan-Miyoshi and R. Castillo, “Absolute values of transport mean free path of light in non-absorbing media using transmission and reflectance measurements,” *Rev. Mex. Fis.* **54**, 257 (2008).
4. V. Ntziachristos, “Going deeper than microscopy: the optical imaging frontier in biology,” *Nat. Methods* **7**, 603 (2010).