Wide-field optical model of the human eye with asymmetrically tilted and decentered lens that reproduces measured ocular aberrations: supplementary material

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This document provides supplementary information to “Wide-field optical model of the human eye with asymmetrically tilted and decentered lens that reproduces measured ocular aberrations,” <http://dx.doi.org/10.1364/optica.2.000124>. Further detail is provided on the optimization process used to generate the eye model. A representative subset of the full merit function is included to illustrate the types of constraints and relative weights of the operands used during optimization. Discussion of the limitations of GRIN surface representation in optical design software is provided. Additional figures compare the measured wavefront data along the vertical meridian to those values reproduced in the newly proposed and existing eye models. Detail and discussion are provided on the wavefront phase profile of the model eye in the pupil plane. © 2015 Optical Society of America

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Eye Model Optimization Merit Function

The reverse building optimization technique used to create the proposed eye model is depicted schematically in Fig. S1. The model was optimized in Zemax optical design software using a custom merit function (Table S1) that incorporated information about anatomical boundary constraints, GRIN lens properties, and measured wavefront aberration data.

Anatomical boundary constraints, derived from multiple sources and compiled more succinctly in [1, 2], limited the range of variation of anatomical features, such as the radii of curvature, thicknesses, and conic constants. Boundary constraints were divided qualitatively into “hard” and “soft” limits according to their weights within the merit function. Hard limits, which constrained the associated properties with very high weight, were used for all surface properties except for those pertaining to the lens. The hard limits forced the parameters into a desired target range without

any preference for particular values within that range. Soft limits, which penalized variation outside the target range with much lower weight, were used for the anterior and posterior lens surfaces because anatomical values reported for the lens [1], including radius of curvature, thickness, and conic constants, often included simplifying assumptions, only described the lens over a small field-of-view, or were not measured *in vivo*. The maximum, minimum, and range of the index of the lens were constrained using hard limits based on the values derived in [3]. Tilt and decentration of the lens were not constrained in the merit function, but rather, they were allowed to vary freely.

Measured ocular aberration data as a function of retinal eccentricity was incorporated into the merit function as soft constraints with target values (as opposed to the target ranges used for the anatomical constraints), where the target values were the mean aberration magnitudes derived from our measured data sets (101 eyes along the horizontal meridian [4] and 10 eyes along the vertical meridian). The first 15 Zernike terms (through 4th

order), excluding piston, tip, and tilt, were inserted into the merit function with soft limits from -40° to $+40^\circ$ in 10° steps along the horizontal meridian and -20° to $+20^\circ$ in 10° steps along the vertical meridian. The values under 10° from the on-axis (0°) point were given 10x greater merit function weight than the values in the periphery. Also, since there was twice the number of horizontal to vertical constraints, the vertical operands were given twice the weight.

The complete optimization merit function contained 201 total operands. Table S1 includes the complete set of anatomical boundary constraints and GRIN lens properties, but only a representative subset of the wavefront constraints. For brevity, the full set of wavefront constraints is listed for one angle (40° nasal); additionally, the relative weights (but not the specific values) are listed for the Zernike terms (Z3-Z14) of the other measured angles.

Optical dispersion parameters for the cornea, aqueous humor, lens, and vitreous humor were based on the findings of Atchison and Smith [5] (Table 2). The chromatic response of the GRIN lens (Table 3) was implemented using the information provided in [6], which examined specifically the chromatic aberrations across a wide field of view for human eyes. The dispersion parameters were hard set in the glass type category of Zemax, and they were not allowed to vary with the optimization routine.

Hammer optimization, which is a global optimization feature in Zemax, was used to reproduce the measured Zernike aberrations while keeping the anatomical parameters within their boundary limits. Note that all of the values listed in Table 1 were allowed to vary simultaneously during global optimization. Hammer optimization was allowed to run for over 72 hours on a 3.20 GHz Intel Core i7 processor with 12 cores, which accounts for approximately 2.5 million variations of the system. Therefore, we posit that our optimization technique reached a unique solution for a specific set of weighted operands.

While a variety of similar eye models could be generated by varying the relative weights of the different operands, we settled on the proposed model weights because the result matched well the measured data throughout the field of view, including those meridians not constrained directly in the merit function (Figure 4). If a specific feature, such as corneal biometry or peripheral defocus, needed to be constrained with greater precision, this could be achieved through adjustment of the associated merit function weights.

Fig. S1. Flow-diagram illustrating the eye model optimization process. The design method implemented a combination of experimentally measured data and ray-tracing optimization.

GRIN Surfaces in Zemax

In Zemax optical design software, GRIN lenses are defined as a piecewise series of infinitely thin refractive index plates where light does not refract between adjacent plates. The spatial distribution of refractive index is calculated only at a plate, and after refracting from a plate, each ray becomes associated with a refractive index until the next plate, regardless of whether the ray changes height or direction. Therefore, in order to reverse a GRIN-based lens model, the refractive index in the reverse direction would need to be a function of both height and angle or the spacing of the index plates needs to be small. However, Zemax does not include surfaces that allow for refractive index to be defined as a function of both height and angle.

The ΔT value from Table 1 corresponds to the axial sampling of the GRIN. While a ray trace of a highly sampled GRIN ($\Delta T = 0.010$ mm) would more accurately simulate a continuous gradient, our reverse building process arrived at a large ΔT value in order to match best the measured aberration data. Since there were limited gradient profiles available in Zemax and gradient 5 was the only profile that supported chromatic dispersion, we believe that our results could be improved even further with a more realistic axial GRIN profile. This treatment of GRIN profiles may help to explain why we obtained a slightly larger ΔT and Δn (0.056) than expected (0.050) [3]. Previous works have not specified their chosen value of ΔT , and it is possible that others may have used a course sampling value too (the default value is 1 mm). Thus, we believe caution should also be used in reversing prior eye models as well.

Discussion of Wavefront Phase Profile

Figure S4 demonstrates that the wavefront of the new eye model closely resembles that of the measured data. This implies that the optical path length of any individual ray across the entire pupil stop correctly mimics the optical path length differences in the average human eye. This feature has important implications for many common ophthalmic imaging systems, such as optical coherence tomography [7, 8], which incorporates interferometric optical depth ranging. Knowing the optical path length of an individual ray in the pupil at a given angle of incidence potentially could allow aberration corrected images to be obtained without the use of adaptive optics [9, 10]. Computationally reconstructing aberrated images has been shown to be successful in examples such as deconvolution imaging [11-14], digital refocusing [15, 16] and synthetic aperture microscopy [17, 18].

Additional Figures

Fig. S2. RMS wavefront error of the various eye models as compared to the measured wavefront data set along the vertical meridian. The on-axis (0°) defocus magnitude for each eye model was subtracted from the defocus value of the other eccentricities in order to illustrate more clearly the variation in RMS

wavefront error with retinal eccentricity. The standard deviation of the measured data set was $\sim 0.093 \mu\text{m}$.

Fig. S3. Comparison of the same metrics as Fig. 3, but along the vertical meridian. The measured data in this figure corresponds to the vertical data taken from the supplementary group consisting of 10 eyes. The most significant aberration terms in the vertical direction were oblique astigmatism (a), defocus (b), vertical astigmatism (c), vertical coma (d), and spherical aberration (e). Like many of the other figures, the on-axis (0°) defocus magnitude for each eye model was subtracted from the defocus value of the other eccentricities in order to emphasize the variation in RMS wavefront error with retinal eccentricity. The error bars correspond to the standard deviation in the measured data set for the group of 10 individuals.

Fig. S4. Wavefront phase profile in the pupil plane for two other wide-field schematic eyes (Navarro [19], G&D 30S [20]), our proposed eye, and the average of the measured data set [4]. The phase profiles displayed represent the phase profile at various retinal eccentricities. The profiles are presented in units of wavelengths for 780 nm light.

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Table S1. Subset of Optimization Merit Function

Surface/Angle	Property	Type	Target	Weight	Description
Anatomical Boundary Constraints					
Anterior Cornea	Radius of Curvature	Greater than	7.56 mm	1,000,000	Hard limit, from [1, 2]
Anterior Cornea	Radius of Curvature	Less than	8.16 mm	1,000,000	Hard limit, from [1, 2]
Posterior Cornea	Radius of Curvature	Greater than	6.05 mm	1,000,000	Hard limit, from [1, 2]
Posterior Cornea	Radius of Curvature	Less than	6.86 mm	1,000,000	Hard limit, from [1, 2]
Anterior Lens	Radius of Curvature	Greater than	9.03 mm	1	Soft limit, from [1, 2]
Anterior Lens	Radius of Curvature	Less than	12.41 mm	1	Soft limit, from [1, 2]
Posterior Lens	Radius of Curvature	Greater than	5.75 mm	1	Soft limit, from [1, 2]
Posterior Lens	Radius of Curvature	Less than	7.71 mm	1	Soft limit, from [1, 2]
Retina	Radius of Curvature	Greater than	11.00 mm	1,000,000	Hard limit, from [1, 2]
Retina	Radius of Curvature	Less than	13.10 mm	1,000,000	Hard limit, from [1, 2]
Anterior Cornea	Conic Constant	Greater than	-0.30	1	Soft limit, from [1, 2]
Anterior Cornea	Conic Constant	Less than	0.00	1	Soft limit, from [1, 2]
Posterior Cornea	Conic Constant	Greater than	-0.30	1	Soft limit, from [1, 2]
Posterior Cornea	Conic Constant	Less than	0.16	1	Soft limit, from [1, 2]
Anterior Lens	Conic Constant	Greater than	-3.20	1	Soft limit, from [1, 2]
Anterior Lens	Conic Constant	Less than	0.25	1	Soft limit, from [1, 2]
Posterior Lens	Conic Constant	Greater than	-1.00	1	Soft limit, from [1, 2]
Posterior Lens	Conic Constant	Less than	1.00	1	Soft limit, from [1, 2]
Cornea	Thickness	Greater than	0.50 mm	1,000,000	Hard limit, from [1, 2]
Cornea	Thickness	Less than	0.56 mm	1,000,000	Hard limit, from [1, 2]
Anterior Chamber	Thickness	Greater than	2.91 mm	1,000,000	Hard limit, from [1, 2]
Anterior Chamber	Thickness	Less than	3.65 mm	1,000,000	Hard limit, from [1, 2]
Lens	Thickness	Greater than	3.46 mm	1	Soft limit, from [1, 2]
Lens	Thickness	Less than	4.06 mm	1	Soft limit, from [1, 2]
Posterior Chamber	Thickness	Greater than	16.00 mm	1,000,000	Hard limit, from [1, 2]
Posterior Chamber	Thickness	Less than	17.00 mm	1,000,000	Hard limit, from [1, 2]
GRIN Lens Properties					
GRIN of Lens	Index Min	Greater than	1.350 RIU	1,000,000	Hard limit, from [3]
GRIN of Lens	Index Max	Less than	1.420 RIU	1,000,000	Hard limit, from [3]
GRIN of Lens	Index Change	Less than	0.065 RIU	1,000,000	Hard limit, from [3]
Measured Wavefront Aberration Data					
40° Nasal	Z3 (Ob. Astig.)	Equal	0.167 λ	1	Soft limit, from [4]
40° Nasal	Z4 (Defocus)	Equal	0.703 λ	1	Soft limit, from [4]
40° Nasal	Z5 (Vert. Astig.)	Equal	1.138 λ	1	Soft limit, from [4]
40° Nasal	Z6 (Vert. Trefoil)	Equal	0.063 λ	1	Soft limit, from [4]
40° Nasal	Z7 (Vert. Coma)	Equal	0.052 λ	1	Soft limit, from [4]
40° Nasal	Z8 (H. Coma)	Equal	0.406 λ	1	Soft limit, from [4]
40° Nasal	Z9 (Ob. Trefoil)	Equal	0.110 λ	1	Soft limit, from [4]
40° Nasal	Z10 (Ob. Quadrafoil)	Equal	0.007 λ	1	Soft limit, from [4]
40° Nasal	Z11 (Ob. 2 nd Astig.)	Equal	0.001 λ	1	Soft limit, from [4]
40° Nasal	Z12 (Spherical)	Equal	0.031 λ	1	Soft limit, from [4]
40° Nasal	Z13 (Vert. 2 nd Astig.)	Equal	0.020 λ	1	Soft limit, from [4]
40° Nasal	Z14 (Vert. Quadrafoil)	Equal	0.012 λ	1	Soft limit, from [4]
30° Nasal	Z3 – Z14	Equal	--	1	Soft limit, from [4]
20° Nasal	Z3 – Z14	Equal	--	1	Soft limit, from [4]
10° Nasal	Z3 – Z14	Equal	--	10	Soft limit, from [4]
0° (on-axis)	Z3 – Z14	Equal	--	10	Soft limit, from [4]
10° Temporal	Z3 – Z14	Equal	--	10	Soft limit, from [4]
20° Temporal	Z3 – Z14	Equal	--	1	Soft limit, from [4]
30° Temporal	Z3 – Z14	Equal	--	1	Soft limit, from [4]
40° Temporal	Z3 – Z14	Equal	--	1	Soft limit, from [4]
20° Inferior	Z3 – Z14	Equal	--	2	Soft limit, from new 2D data
10° Inferior	Z3 – Z14	Equal	--	20	Soft limit, from new 2D data
10° Superior	Z3 – Z14	Equal	--	20	Soft limit, from new 2D data
20° Superior	Z3 – Z14	Equal	--	2	Soft limit, from new 2D data