Quantum vampire: collapse-free action at a distance by the photon annihilation operator: supplementary material

ILYA A. FEDOROV^{1, 2}, ALEXANDER E. ULANOV^{1, 3}, YURY V. KUROCHKIN¹, AND A. I. LVOVSKY^{1, 2, 4, *}

¹Russian Quantum Center, 100 Novaya Street, Skolkovo, Moscow 143025, Russia

²P. N. Lebedev Physics Institute, Leninskiy prospect 53, Moscow 119991, Russia

³Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

⁴Institute for Quantum Science and Technology, University of Calgary, Calgary AB T2N 1N4, Canada

*Corresponding author: LVOV@ucalgary.ca

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The theoretical model used to describe our experiment is shown in Fig. S1. The heralded Fock state produced via parametric down-conversion is combined with the vacuum state $|0\rangle$ on beam splitter $\hat{B}_1$. One of the beam splitter outputs — the mode described by annihilation operator $\hat{a}_1$ — is subjected to photon annihilation using low-reflectivity beam splitter $\hat{B}_2$ and single-photon detector SPCM₃ in the reflected channel. Subsequently modes $\hat{a}_1$ and $\hat{a}_2$ are recombined on beam splitter $\hat{B}_3$. The output mode with constructive interference is subjected to homodyne detection after undergoing loss on an additional beam splitter $\hat{B}_4$.

The density matrix of the output state is calculated in the Fock basis using the matrix of the beam splitter operator, which can be found, for example, in Ref. [2]. Below, we present a more detailed analysis of the photon annihilation operation.

The action of beam splitter $\hat{B}_2$ can be modeled as evolution under Hamiltonian [1]

$$\hat{H} = i\lambda\hat{a}_1(\hat{a}'_1)^\dagger - i\lambda\hat{a}'_1\hat{a}_1, \quad (\text{S1})$$

where λ , assumed real, is related to the amplitude reflectivity r of the beam splitter as $r = \sin(\zeta)$, with $\zeta = \lambda\tau/\hbar$ and τ being the fictitious interaction time. If $|\zeta| \ll 1$, the evolution under Hamiltonian (S1) can be approximated as

$$\hat{B}_2 \equiv e^{-i\hat{H}\tau/\hbar} \approx 1 - i\hat{H}\tau/\hbar. \quad (\text{S2})$$

Suppose the beam splitter input consists of state $|\psi\rangle$ in mode $\hat{a}_1$ and the vacuum state $|0\rangle$ in mode $\hat{a}'_1$. The evolution operator,

acting on this state, yields

$$e^{-i\hat{H}\tau/\hbar}(|\psi\rangle|0\rangle) \approx |\psi\rangle|0\rangle + \zeta(\hat{a}_1|\psi\rangle)|1\rangle. \quad (\text{S3})$$

Conditioning on single-photon detection in mode $\hat{a}'_1$, we obtain state $\hat{a}_1|\psi\rangle$ in mode $\hat{a}_1$.

This approximation is valid as long as higher-order terms in Taylor decomposition Eq. (S2) can be neglected. These terms enter the output state with probabilities on a scale of $r^2 n_{|\psi\rangle} \ll 1$, with $n_{|\psi\rangle}$ being the mean photon number in the target state. Fig. S2 shows the effect of neglecting higher-order terms on the expected output state.

After the beam splitter, mode $\hat{a}'_1$ is subjected to measurement by non-discriminating single-photon detector SPCM₃. This detector is described by positive operator-valued measure [3]

$$\hat{\Gamma}_{\text{off}}(\eta) = \sum_{n=0}^{\infty} (1 - p_{\text{bg}})(1 - \eta)^n |n\rangle\langle n|; \quad (\text{S4a})$$

$$\hat{\Gamma}_{\text{on}}(\eta) = \sum_{n=0}^{\infty} [1 - (1 - p_{\text{bg}})(1 - \eta)^n] |n\rangle\langle n|, \quad (\text{S4b})$$

where η is the quantum efficiency of that detector and p_{bg} is the probability of a “background” click to occur in the absence of input photons. If the detector clicks, the state in mode $\hat{a}_1$ is projected onto

$$S_{\hat{a}_1}(\hat{\rho}) = \text{Tr}_{\hat{a}'_1}[\hat{\Gamma}_{\text{on}}\hat{B}_2(\hat{\rho} \otimes |0\rangle\langle 0|)\hat{B}_2^\dagger], \quad (\text{S5})$$

where $\hat{\rho} = |\psi\rangle\langle\psi|$ is the density matrix of the initial state in mode $\hat{a}_1$.

Fig. S1. Model of the experiment.

Fig. S2. Diagonal elements of the expected output state's density matrix in the Fock basis for one- (a) and two-photon (b) experiment. Purple rectangles: the approximation-free calculation according to Eq. (S5), same as in Fig. 3 of the main text. Black rectangles: a calculation using the first-order approximation (S3) of the beam splitter operator. Gray rectangles: calculation with no background clicks of the SPCM 3.

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