Arbitrary two-dimensional spectrally-encoded pattern generation — a new strategy for high-speed patterned illumination imaging: supplementary material

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S1. SPATIAL DISTORTION MEASUREMENT BASED ON LATTICE DEFECT EVALUATION

The combined effect of nonlinear angular dispersion by VIPA and diffraction grating can be considered as a structural defect in the “crystalline lattice”. The 2D spectral pattern is analogous to a 2D plane in a crystal, in which the pattern pixels are treated as atoms cross-linked by chemical bonds, as shown in Fig. S1a. A “perfect” crystal is a spatial replication of an *identical template* called unit cell. If strains are present due to internal stress, the shape of the observed unit cells will deviate from that of the template — resulting in defects or distortion.

To quantify such lattice defect, we follow the atomic strain tensor approach adopted by Mott et al [1]. We assume that each wavelength corresponds to each “atom” and they are separated by the minimally resolvable wavelength set by the VIPA. Each “atom” is located in the object plane by applying Eqs. (1) and (2) in the main text. The spatial arrangement of “atoms” are tessellated into Delaunay triangles (Fig. S1a). For each triangular unit cell, the deformation strain tensor is evaluated from the deviation from the median lengths of the triangle:

$$\epsilon(a_j) = \frac{a_j/c_j - a_{\text{med}}/c_{\text{med}}}{a_{\text{med}}/c_{\text{med}}} \quad (\text{S1})$$

$$\epsilon(b_j) = \frac{b_j/c_j - b_{\text{med}}/c_{\text{med}}}{b_{\text{med}}/c_{\text{med}}}, \quad (\text{S2})$$

where $a_j \leq b_j \leq c_j$ are lengths of three sides of the triangular unit cell Δ_j (see Fig. 2b). a_{med} , b_{med} , and c_{med} refer to the medians

taken across the entire FOV. Fig. S1b–c illustrates the calculated lattice strain distribution of our selected design. Consider the map in Fig. S1b, the cells appear to be compressed toward positive x and y axes. The neutral strain region appears as a narrow V-shaped band spanning roughly along the x -axis. It implies an equal contribution of diffraction grating and the VIPA to the spatial deformation. We can then learn from the strain tensor statistics to estimate the spread of the strain distribution, as in Fig. S1d. The *spatial distortion* metric D is thus the mean of the component magnitudes of strain tensors, which is effectively the sum of *mean absolute deviation* (MAD) of corresponding lengths of Delaunay triangles [Eq. (4) in the main text].

S2. SPECTRAL SHAPE CALIBRATION FOR ARBITRARY PATTERN GENERATION

The simulated wide-field pattern is computed purely in wave optics as in [2], independent of the ray-tracing model in Eqs. (1) and (2) in the main text. The broadband lasing spectrum is oversampled (number of monochromatic wavelength $N > 200$) to overpopulate the FOV of the 2D spatial disperser. For each wavelength, the monochromatic PSF is evaluated and discretized on a regular spatial grid (grid size $M = 256^2 = 65536$). The design parameters are based on the selected optimal design in Section 3. Therefore, we yield a wavelength-to-space projection matrix $\mathbf{A}$ of size $M \times N$. To find corresponding spectral shaping waveform $\mathbf{f}$ for a desired illumination pattern $\mathbf{g}$, we have to solve for $\mathbf{f}$ in the system of equations $\mathbf{g} = \mathbf{A}\mathbf{f}$. Since the problem is

Fig. S1. Method of estimating the spatial distortion of the spectrally-encoded illumination based on a “strain tensor” approach. (a) The spectral lattice tessellated into triangular unit cells. (b–c) Components of strain tensor $\epsilon(a_j)$ and $\epsilon(b_j)$ estimated by Eqs. (S1) and (S2). (d) Strain distribution. Note that based on this statistic, one can obtain the mean absolute deviation (MAD) of corresponding lengths of Delaunay triangles, from which the spatial distortion defined in Eq. (4) of the main text can be evaluated.

over-determined ($M \gg N$), an optimal solution $\hat{\mathbf{f}}$ is sought to fit the desired pattern $\mathbf{g}$. However, a simple least squares fit results in large variations in the waveform $\mathbf{f}$ because of highly-overlapping PSFs (i.e. $\mathbf{A}$ is ill-conditioned). To guarantee a stable solution, we approximate $\mathbf{f}$ by Tikhonov-regularized linear least squares, i.e.

$$\hat{\mathbf{f}} = \arg \min_{0 \leq f \leq 1} \|\mathbf{g} - \mathbf{A}\mathbf{f}\|_2^2 + \gamma \|\mathbf{f}\|_2^2, \quad (\text{S3})$$

where $\gamma > 0$ is the penalty factor. The regularization term suppresses large values in the modulated optical spectrum, which matches the assumption of finite energy of the laser pulse. Eq. (S3) is solved by the iterative quadratic programming routine [3]. Given a wavelength-to-space projection matrix $\mathbf{A}$ and spatial pattern $\mathbf{g}$, the routine converges to a solution within 13 seconds on a workstation with Intel *core-i5* processor and 4 GB memory.

S3. STRUCTURED ILLUMINATION GENERATION BY SPECTRAL INTERFEROMETER

In this section, we estimate the phase, the orientation and the spatial frequency of the 2D sinusoidal pattern of a given temporal delay τ in the spectral interferometer. Although all three parameters are determined by only one input (temporal delay τ), it is possible to independently control each one by choosing the corresponding order of magnitude of the delay adjustment ($\delta\tau$).

From Fig. 7, the intensity output of the spectral interferometer is

$$I(\nu) = |A(\nu - \nu_c)|^2 [1 + \cos(2\pi\nu\tau)], \quad (\text{S4})$$

where τ is the relative time delay in the interferometer, $|A(\nu)|^2$ is the laser spectrum centered at frequency $\nu_c = c/\lambda_c$. Here we denote $\phi_1(\nu) = 2\pi\nu\tau$ as the phase of the 1D sinusoidal pattern in the spectrum (Fig. S2a).

The 2D spectral disperser projects the 1D spectral interferogram to form a sinusoidal pattern in the focal plane of the condenser lens. In ideal condition, the 2D sinusoidal pattern is expressed as

$$I(x, y) = |A(x, y)|^2 [1 + \cos(\phi_2(x, y))] \quad (\text{S5})$$

$$\phi_2(x, y) = 2\pi kx \sin \psi + 2\pi ky \cos \psi, \quad (\text{S6})$$

where $|A(x, y)|^2$ is the laser spectrum mapped from ν to (x, y) ; k and ψ are the spatial frequency and orientation of the generated periodic illumination respectively. If we can relate the 1D phase

Fig. S2. Principle of 2D sinusoidal pattern generation with spectral interferometer. The 1D waveform is segmented at a regular interval $\Delta\nu_{\text{FSR}}$ into four separate diffraction orders (i–iv) by the VIPA, and is then separated along x -axis by the diffraction grating. (a) The 1D spectral waveform of the interferogram and the function $\phi_1(\nu)$ visualized as wrapped phase. (b) 2D sinusoidal pattern generated by the 2D spectral disperser in ideal condition, from which the function $\phi_2(x, y)$ is visualized as wrapped phase. The orientation of the periodic pattern is parallel to the direction of the phase gradient vector $\nabla\phi_2(x, y)$.

profile $\phi_1(\nu)$ to the 2D phase profile $\phi_2(x, y)$ with the space-to-wavelength mapping function $\lambda(x, y)$, we can determine the properties of the sinusoidal pattern (k, ψ, ϕ_0) for temporal delay τ . Note that the value of $\phi_2(x, y)$ is not equal to $\phi_1[\nu(x, y)]$; while $\phi_2(x, y)$ is continuous and smooth in 2D space, $\phi_1[\nu(x, y)]$ is a staircase function along x -axis (Fig. S2). Instead, the two phase profiles can be equated by their first derivatives, i.e. their phase gradients. Hence, we have

$$\begin{aligned} \frac{\partial\phi_2}{\partial x} &= \frac{\Delta\phi_{2,\text{FSR}}}{\Delta\nu_{\text{FSR}}} \cdot \frac{\partial\nu}{\partial x} \\ &= \frac{\text{mod}\{\Delta\phi_{1,\text{FSR}}, 2\pi\}}{\Delta\nu_{\text{FSR}}} \cdot \frac{\partial\nu}{\partial x} \\ &= \frac{\text{mod}\{2\pi\tau\Delta\nu_{\text{FSR}}, 2\pi\}}{\Delta\nu_{\text{FSR}}} \cdot \left(\frac{c}{\lambda_c F_1} \cdot \frac{d\lambda/\lambda_c}{d\theta_x} \right) \end{aligned} \quad (\text{S7})$$

$$\begin{aligned} \frac{\partial\phi_2}{\partial y} &= \frac{d\phi_1}{d\nu} \cdot \frac{\partial\nu}{\partial y} \\ &= \tau \cdot \left(\frac{c}{\lambda_c F_1} \cdot \frac{d\lambda/\lambda_c}{d\theta_y} \right), \end{aligned} \quad (\text{S8})$$

where F_1 is the focal length of the condenser lens; $\Delta\nu_{\text{FSR}}$ denotes

the free-spectral-range of the VIPA in frequency; $(d\lambda/\lambda_c)/d\theta_{x,y}$ is the angular dispersion factor mentioned in the main text. The operator $\text{mod}\{\cdot\}$ evaluates the relative phase shift across one FSR. Note that the finite difference is adopted instead in Eq. (S7) because the wavelength-to-space mapping along x -axis are separated into discrete diffraction orders (Fig. S2b).

Spatial frequency tuning For a 2D sinusoidal pattern nearly parallel to the x -axis, the spatial frequency is given as

$$k = \sqrt{\left(\frac{\partial\phi_2}{\partial x}\right)^2 + \left(\frac{\partial\phi_2}{\partial y}\right)^2} \simeq \left|\frac{\partial\phi_2}{\partial y}\right| \propto \tau, \quad (\text{S9})$$

where $\tau \simeq \ell/\Delta\nu_{\text{FSR}}$, ℓ is a large integer.

Orientation tuning The orientation of the sinusoidal pattern ψ can be derived by finding the 2D phase gradient from Eq. (S6), i.e.

$$\nabla\phi_2(x, y) = 2\pi k \sin\psi \hat{x} + 2\pi k \cos\psi \hat{y}. \quad (\text{S10})$$

Comparing this to the definition of $\nabla\phi_2$, we have

$$\begin{aligned} \tan\psi &= \frac{2\pi k \sin\psi}{2\pi k \cos\psi} \\ &= \frac{\partial\phi_2/\partial x}{\partial\phi_2/\partial y} \\ &= \frac{\text{mod}\{2\pi\tau\Delta\nu_{\text{FSR}}, 2\pi\}}{\tau\Delta\nu_{\text{FSR}}} \cdot \frac{d\theta_y}{d\theta_x}. \end{aligned} \quad (\text{S11})$$

For a sinusoidal pattern nearly parallel to the x -axis, the change in orientation is approximated to

$$\begin{aligned} \delta\psi &\simeq \delta(\tan\psi) \\ &\simeq \frac{2\pi\delta\tau}{\tau} \cdot \frac{d\theta_y}{d\theta_x} \propto \delta\tau, \end{aligned} \quad (\text{S12})$$

where $|\delta\tau| < 1/\Delta\nu_{\text{FSR}} \ll \tau$.

Phase tuning The phase shift of the sinusoidal pattern at an off-axis position (x_0, y_0) is given as

$$\begin{aligned} \delta\phi_2(x_0, y_0) &= \delta\phi_1[v(x_0, y_0)] \\ &= 2\pi v(x_0, y_0)\delta\tau \\ &= \frac{2\pi c \delta\tau}{\lambda(x_0, y_0)} \propto \delta\tau, \end{aligned} \quad (\text{S13})$$

where $|\delta\tau| \ll 1/\Delta\nu_{\text{FSR}}$.

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