Incoherent self-accelerating beams: supplementary material

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This document provides supplementary information to “Incoherent self-accelerating beams,” <http://dx.doi.org/10.1364/optica.2.000886>. First, we provide analysis of accelerating beams created from high-order Hermite-Gaussian modes. This provides, through the modal decomposition of incoherent light, further insight into the results provided in the paper. Second, we expand on the numerical results of the paper regarding the value of coherence length in which acceleration ceases to exist. Lastly, we give an illustrative example of an additional method of constructing incoherent accelerating beams via the modal decomposition. © 2015 Optical Society of America

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1. Features of accelerating beams generated using higher order Hermite-Gaussian modes

In the paper, we have the following coherence function in Fourier space, describing quasi-monochromatic, paraxial, partially-spatially-incoherent accelerating beams (see Eq (9)):

$$W(k_1, k_2) = I_0 \exp\left(-\frac{(k_1^2 + k_2^2)}{w^2}\right) \times \exp\left(-\frac{i}{3}c_0^3(k_1^3 - k_2^3) - \frac{(k_1 - k_2)^2}{2\sigma^2}\right) \quad (\text{S1})$$

Decomposing Eq (S1) into orthogonal modes, one can show that the modes are Hermite-Gaussians, with the cubic phase and with exponentially decreasing powers [1]:

$$W(k_1, k_2) = I_0 \sum_n \lambda_n \phi_n^*(k_1) \phi_n(k_2) \quad (\text{S2})$$

with

$$\phi_n(k) = \left(\frac{2}{\pi \tilde{w}}\right)^{1/4} \frac{1}{(2^n n!)^{1/2}} H_n\left(\sqrt{2} \frac{k}{\tilde{w}}\right) \times \exp\left(-\frac{k^2}{\tilde{w}^2} + i \frac{1}{3} c_0^3 k^3\right) \quad (\text{S3})$$

$$\tilde{w}^2 = \frac{w^2}{\sqrt{1 + w^2 / \sigma^2}} \quad (\text{S4})$$

$$\lambda_n = \sqrt{\frac{2\pi w^2 \sigma^2}{w^2 + 2\sigma^2 + 2\sigma^2 \sqrt{1 + w^2 / \sigma^2}}} \times \left(\frac{w^2}{w^2 + 2\sigma^2 + 2\sigma^2 \sqrt{1 + w^2 / \sigma^2}}\right)^n \quad (\text{S5})$$

where H_n are Gauss-Hermite polynomials. Thus, in order to understand the propagation dynamics of incoherent accelerating beams, it is important to understand the features of accelerating beams generated by high-order Hermite-Gauss modes, compared to accelerating beams generated using simple Gaussians (H_0

mode). The accelerating beams generated from the first four Hermite-Gaussian modes are illustrated in Fig S1.

First we note that, in general, the extent of the beam in the Fourier plane (spatial spectrum) determines the acceleration distance. For an infinite spatial spectrum, the beam is truly shape-preserving and the acceleration distance is infinite, while for a narrow spectrum the acceleration rate and acceleration distance are small. In this context, high-order Hermite-Gauss modes contain sign flips, hence some of the power of the beam is diffracted away at large angles, and is not contained within the localized accelerating portion of the beam. However, the support of high order Hermite-Gaussians in Fourier plane is larger than the zeroth-order Gaussian, therefore the beam accelerates over larger distances. This effect is mitigated by the fact that the minimum waist parameter of the Gaussian mode, for significant incoherence (small σ) varies like $\tilde{w}^2 \approx w \cdot \sigma$, which means smaller support. Thus, by decreasing σ and increasing the number of modes with appreciable energy, the acceleration distance remains approximately constant, despite the growing incoherence of the beam. This behavior persists up until the limit where the coherence length l_c is smaller than half the width of the main lobe of the Airy function. In such case, the acceleration distance drops, and the beam undergoes normal, non-accelerating diffraction.

2. Presenting results according to Fig 2(d) of the main article, for various values of exponential truncation

In Fig (S2) we present the acceleration distance z_{acc} vs coherence length l_c for additional values of the exponential truncation factor a . In general, a larger a means a smaller aperture from which the accelerating beam is launched, hence smaller acceleration distance. In Fig S2 (a), the accelerating distance for the coherent beam is $8.8z_0$, and in Fig S2 (b) the accelerating distance for the coherent beam is $6.75z_0$. We see in Fig S2 the same behavior as in Fig 2(d). That is, at least within the Schell model, incoherence affects the acceleration distance only when the correlation distance is approximately half the width of the main lobe. The threshold correlation distance increases slightly when a is increased.

3. An illustrative example of the difference between "wrong" selection of modes and "right" selection of modes

In this section, we present an illustrative example of the meaning of choosing the "right" modes of the accelerating incoherent beam. For simplicity, we do so in the paraxial regime. In figure S3, we present to different options for the choice of modes. In Figure S3 (a) we present modes that follow Eq (8) of the main article, i.e. modes with the same acceleration that are shifted relative to one another. For clarity, we choose $a = 0$, and so the modes are written as:

$$\psi(x, z) = \sum_n c_n Ai \left(\frac{x - \Delta_n}{x_0} - \frac{1}{4} \left(\frac{z}{kx_0^2} \right)^2 \right) \times \exp \left(\frac{i}{12} \left(\frac{z}{kx_0^2} \right)^3 - \frac{i}{2} \left(\frac{z}{kx_0^2} \right) \left(\frac{x - \Delta_n}{x_0} \right) \right) \quad (S6)$$

In Figure S3 (b) we present modes that have the same origin, but different acceleration. Again, we choose $a = 0$, and so the modes are written as:

$$\psi(x, z) = \sum_n c_n Ai \left(\frac{x}{x_n} - \frac{1}{4} \left(\frac{z}{kx_n^2} \right)^2 \right) \times \exp \left(\frac{i}{12} \left(\frac{z}{kx_n^2} \right)^3 - \frac{i}{2} \left(\frac{z}{kx_n^2} \right) \left(\frac{x}{x_n} \right) \right) \quad (S7)$$

It is clearly seen in Figure S3 (a) that when choosing modes that follow the same parabolic trajectory, that are shifted to one another, the incoherent beam accelerates along the same trajectory. However, when choosing modes that follow different parabolic trajectories, the incoherent beam's acceleration lasts only a short distance, and it diffracts rapidly after that. This can be clearly seen in Figure S3 (b).

References

1. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995).

Figure S1: Intensity profile for accelerating beams generated using various Hermite-Gaussian modes in the Fourier plane. (a) HG_0 (b) HG_1 (c) HG_2 (d) HG_3 .

Figure S2: Acceleration distance z_{acc} vs coherence length l_c , for paraxial incoherent beams, varying exponential truncation factor a , for beams obeying the Schell model (blue) and for beams constructed by the modal decomposition of Eq. (8) (green).

Figure S3: (a) An incoherent Airy beam comprised of the "right" set of modes, according to Eq (S6). (b) An incoherent Airy beam comprised of the "wrong" set of modes, according to Eq (S7)