Intrinsic linewidth of quantum cascade laser frequency combs: supplementary material

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1. FREQUENCY-TO-AMPLITUDE CONVERTER CALIBRATION

In this section, the procedure for the calibration of the frequency-to-amplitude (FA) converter is discussed. This procedure has already been applied giving reliable results as ascertained by comparing the results given by different types of FA converters (optical cavities resonances and molecular absorption lines) [1, 2]. The obtained frequency noise power spectral densities (FNPSDs) are reliable up to the limit given by the detection noise floor set by the detection system. The set-up is shown in figure 1 of the main text and is mainly composed of the laser source under test, an optical cavity with a mirror connected to a piezoelectric actuator, a detector, an oscilloscope and a spectrum analyzer. The laser current and temperature must stay constant in order to keep the optical frequency constant. In order to calibrate the FA converter, two ingredients are needed:

- The cavity transmission profile, obtained by scanning the cavity length acting with a voltage ramp on the piezoelectric actuator. The cavity is usually scanned over few free spectral ranges FSR (typically three) in order to avoid nonlinearities. The cavity transmission profile is acquired with a detector and the photo-voltage is sampled with an oscilloscope, triggered with the signal delivered by the voltage ramp controlling the piezoelectric actuator.
- The photo-current voltage noise power spectral density (VNPSD) of the light transmitted by the cavity, collected with the same detector and acquired with a spectrum analyzer. The VNPSD contains the FNPSD if the cavity length is adjusted in order to have the optical frequency of the laser source corresponding to the side of a cavity

resonance, where the intensity of the transmission is half the maximum (see figure 2a and b). The VNPSD unities are (V^2/Hz) because VNPSD are voltage fluctuations square mean values normalized to the radio-frequency bandwidth.

The frequency scale of the cavity transmission profile is obtained observing the resonance peaks, which fall at a specific Δt that corresponds to the cavity FSR . After this conversion we have the profile of the cavity transmission in terms of (V/Hz) . At this point we focus on the resonance peak closer to the center of the transmission profile, we measure its FWHM and we determine the slope of the sides fitting the profile around the half maximum with a beeline. This slope is expressed in (V/Hz) . As a FA converter, the cavity behaves as a second order low-pass filter. The cutting is due to the photon lifetime within the cavity. The cutoff frequency is given by $f_c = FWHM/2$ and the compensating factor for this cutoff is $(1 + (f/f_c)^2)$. At the end we obtain the following formula:

$$FNPSD(Hz^2/Hz) = VNPSD(V^2/Hz) \left(\frac{1 + \left(\frac{f}{f_c}\right)^2}{\text{slope}(V/Hz)} \right)^2 \quad (S1)$$

The detection noise floor is acquired and elaborated exactly in the same way, just with the laser beam cut before reaching the cavity.

Moreover, the intensity noise (amplitude noise) of the laser is also measured. For this measurement the cavity is removed and the laser beam is shined on the detector, just attenuated with a proper attenuator to take advantage of the whole detection dynamical range. Obviously in this case the cavity cutoff

is not present. In order to compare the intensity noise with the frequency noise, a proper conversion is needed. The intensity noise has to be scaled to the level of the cavity transmission. This is done by dividing the intensity noise by its DC value and multiplying it for the peak value of the cavity resonance (RP). Finally it is multiplied for the same slope. The contribution of the intensity noise to the frequency noise is therefore given by

$$\text{FNPSD}_{\text{intensity}}(\text{Hz}^2/\text{Hz}) = \text{VNPSD}_{\text{intensity}}(\text{V}^2/\text{Hz}) \cdot \left(\frac{RP(V)}{\text{DC}_{\text{intensity}}(V)} \right)^2 \left(\frac{1}{\text{slope}(V/\text{Hz})} \right)^2 \quad (\text{S2})$$

2. DEMONSTRATION OF THE MULTIMODE FREQUENCY-TO-AMPLITUDE CONVERSION

In this section we will show that, by satisfying the right conditions, it is possible to use an optical cavity as *multimode* frequency-to-amplitude converter to retrieve the frequency noise of an optical frequency comb (see figure 2b in the main text). The intensity of the i -th comb mode transmitted by the optical cavity is given by

$$I_i = I_{i0} T(\nu_{ij}) \quad (\text{S3})$$

where I_{i0} is the intensity of the i -th comb mode, $\nu_{ij} = \nu_j - \nu_i$, ν_i is the center frequency of the i -th comb mode, ν_j is the center frequency of the j -th cavity resonance and $T(\nu_{ij})$ is the convolution between the i -th comb mode shape and the j -th cavity resonance profile. The total intensity transmitted by the cavity is given by

$$I_{\text{tot}} = \sum_i I_i = \sum_i [I_{i0} T(\nu_{ij})] \quad (\text{S4})$$

If

$$\nu_{ij} = \Delta\nu = \text{const.} \quad \forall i, j \quad (\text{S5})$$

we are allowed to bring $T(\nu)$ out of the sum, yielding

$$I_{\text{tot}} = T(\Delta\nu) \sum_i I_{i0} = T(\Delta\nu) I_{0 \text{ tot}} \quad (\text{S6})$$

This proves that in this conditions the transmission and consequently the frequency-to-amplitude conversion follows the same rules of the single-mode case. Actually, the condition expressed by equation S5, where both the comb modes and the cavity resonances have the same dispersion in frequency, is not so strict. It is in fact sufficient that

$$|\nu_{ij} - \nu_{lm}| \ll \frac{1}{2} \text{FWHM}[T(\Delta\nu)] \quad (\text{S7})$$

for all the transmitted modes. In other words it is sufficient that the accumulated dispersion over the whole comb spectrum is small compared to the width of the cavity resonances.

3. HENRY LINEWIDTH ENHANCEMENT FACTOR ESTIMATION

In order to compute the Schawlow-Townes limit

$$\delta\nu = \frac{h\nu}{P} \frac{\alpha_{\text{tot}} c^2}{4\pi n_g^2} \alpha_m n_{\text{sp}} (1 + \alpha_e^2) \quad (\text{S8})$$

a proper estimation of the Henry linewidth enhancement factor is needed [3]. As reported in [4], assuming a Lorentzian line-shape for the intersubband transition, the Henry factor for a QCL can be expressed as

$$\alpha_e(\nu) = \frac{\nu_{32} - \nu}{\gamma_{32}} \quad (\text{S9})$$

where ν_{32} and γ_{32} are the center frequency and the half width at half maximum of the transition respectively, while ν is the frequency of the radiation emitted by the laser. Concerning ν , we have to consider that the emitted radiation is a comb of frequencies, so α_e has to be computed as average over the whole emitted spectrum

$$\langle \alpha_e^2 \rangle_{\text{laser spectrum}} = \frac{1}{\Delta\nu \gamma_{32}^2} \int_{\text{las. sp.}} (\nu_{32} - \nu')^2 d\nu' \quad (\text{S10})$$

where $\Delta\nu$ is the width of the comb spectrum. Assuming that the center of the emitted spectrum corresponds to ν_{32} we obtain directly

$$\langle \alpha_e^2 \rangle_{\text{laser spectrum}} = \frac{1}{3\gamma_{32}^2} \left(\frac{\Delta\nu}{2} \right)^2 \quad (\text{S11})$$

Finally, knowing from electroluminescence measurements that $\gamma_{32} = 150 \text{ cm}^{-1}$ and that $\Delta\nu = 25 \text{ cm}^{-1}$ in the actual operating conditions of the laser ($P = 25 \text{ mW}$) we have

$$\langle \alpha_e^2 \rangle_{\text{laser spectrum}} = 0.0023 \quad (\text{S12})$$

value that can be neglected in equation S8. In case of a multi-stack device the shape of the gain curve on the top is even flatter than a Lorentzian, consequently the given value for $\langle \alpha_e^2 \rangle$ has to be considered as an upper limit.

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