Classically entangled optical beams for high-speed kinematic sensing: supplementary material

STEFAN BERG-JOHANSEN^{1,2,+}, FALK TÖPPEL^{1,2,3,+}, BIRGIT STILLER^{1,2}, PETER BANZER^{1,2,4}, MARCO ORNIGOTTI⁵, ELISABETH GIACOBINO^{1,6}, GERD LEUCHS^{1,2,4}, ANDREA AIELLO^{1,2}, AND CHRISTOPH MARQUARDT^{1,2,*}

¹Max Planck Institute for the Science of Light, Günther-Scharowsky-Straße 1/Bldg. 24, 91058 Erlangen, Germany

²Institute for Optics, Information and Photonics, Universität Erlangen-Nürnberg, Staudtstraße 7/B2, 91058 Erlangen, Germany

³Erlangen Graduate School in Advanced Optical Technologies (SAOT), Paul-Gordan-Straße 6, 91052 Erlangen, Germany

⁴Department of Physics, University of Ottawa, 25 Templeton, Ottawa, Ontario, K1N 6N5 Canada

⁵Institute of Applied Physics, Friedrich-Schiller University Jena, Max-Wien Platz 1, 07743 Jena, Germany

⁶Laboratoire Kastler Brossel, Université Pierre et Marie Curie, École Normale Supérieure, CNRS, 4 place Jussieu, 75252 Paris, France

*Corresponding author: christoph.marquardt@mpl.mpg.de

+These authors contributed equally to this work.

Published 28 September 2015

This document provides supplementary information to “Classically entangled optical beams for high-speed kinematic sensing,” <http://dx.doi.org/10.1364/optica.2.000864>. Details are provided on the experimental methods and theoretical models, as well as the numerical algorithm used for position inference. © 2015 Optical Society of America

<http://dx.doi.org/10.1364/optica.2.000864.s001>

1. EXPERIMENTAL METHODS

We employed a continuous-wave diode laser (Sumitomo SLT5411) with wavelength $\lambda = 1550$ nm, amplified by an erbium-doped fiber amplifier (OPREL OFA20-1231S) to a power of 50 mW. The initially Gaussian beam profile was converted to a radially polarized mode using a commercially available liquid crystal device (Arcoptix S.A.) with custom anti-reflection coatings. The mode then passed two further waveplates in order to suppress residual circular polarization components resulting from the mode converter. The rotor was a Cu-coated wire of width 0.79(1) mm, placed in a rotation mount which was actuated by a stepper motor. The sphere had diameter 1.00(1) mm and was made from steel. It was attached to an uncoated glass substrate of thickness 1 mm by a meniscus of epoxy resin which was small enough to be neglected optically. The substrate was mounted in a vertical position on a linear precision stage (PI M-410.DG, minimum incremental motion 0.1 μ m) and moved in steps of 50 μ m. The plane of object interaction was located at 50 cm after the mode converter. The distances between object-camera and object-detectors were 10 cm and 80 cm, respectively. The beam had a width of 2.84(10) mm in the plane of object interaction, and a Rayleigh range of approximately 2 m. All beam widths given here and in the main text are 90-10 knife-edge widths. The detectors used to measure the horizontal, vertical,

diagonal and anti-diagonal polarization components were two Soliton ET-3500, an Alphasalas UPD-35 and an Alphasalas UPD-70, respectively. Plano-convex focusing lenses were used in front of each detector in order to confine the mm-sized beam to the detector sensitive areas with diameters of tens of μ m. Calibration measurements with static test targets and a verification camera were carried out to verify that the beams are fully coupled to their detector's sensitive area (i.e. that no part of the beam is cut off). The detectors were temporally matched to within 1.3 ns, with the remaining offset being compensated by the oscilloscope up to the measurement bandwidth. For the rotor measurement only, a pair of BPDV2120R fiber-coupled balanced photoreceivers with 43 GHz 3 dB-bandwidth from the company u2t were used. Tailored phase plates were used to convert the the linearly polarized Hermite-Gauss projection modes into approximate fundamental modes in order to fiber-couple them to these detectors. For the real-time measurement, the beam was focused using a pair of aspheric lenses with NA 0.65. While it is known that radially polarized beams display a strong longitudinal electric field component on axis when focused tightly [1], such components are not yet predominant in the moderate focusing regime found here. Comparative measurements with an azimuthally polarized beam, for which no longitudinal field components appear during focusing, showed very simi-

lar overall behavior and duration. All signals were recorded with a digital storage oscilloscope (Agilent DSO81204B in half-channel mode, 8 GHz analog bandwidth, sinc- x -interpolation disabled). An InGaAs camera with resolution 320×256 px (Xenics XS) was used for beam inspection and for generating spatially resolved beam polarization tomographies prior to the actual sensing measurement. The Bayesian tracking algorithm described below used these tomographies in order to improve its estimate for the object position. For a successful measurement, the polarization structure of the beam must be preserved under propagation through the apparatus. Custom-coated mirrors were used that preserve both amplitude and relative phase of s and p polarization components and this was verified through polarization tomographies for each individual mirror. In order to account for differences in the coupling efficiencies and sensitivities between the individual detectors, the detector signals were normalised based on polarization tomography data of the unobstructed mode. Additionally, in our treatment the measured object must be opaque and independent of polarization. Transparent objects altering the polarization of transmitted light (e.g. through birefringence) are not considered.

2. THEORETICAL MODEL

In order to obtain the theoretical predictions and simulated values shown in Figure 3 of the main text we developed the following simple model. Consider a radially polarized beam of light of angular frequency ω , propagating along the z -axis of a Cartesian reference frame and polarized in the xy -plane. In the paraxial approximation its electric field takes the form $\mathcal{E}(\mathbf{r}, t) = \text{Re}[E(\boldsymbol{\rho}, z) \exp(-i\omega t)]$ with time-independent mode function $E(\boldsymbol{\rho}, z)$ (Jones vector). In the ideal case of a perfect radially polarized beam the mode function is given by

$$E(\boldsymbol{\rho}, z) = \frac{1}{\sqrt{2}} [\hat{x} \psi_{10}(\boldsymbol{\rho}, z) + \hat{y} \psi_{01}(\boldsymbol{\rho}, z)]. \quad (1)$$

Therein $\psi_{mn}(\boldsymbol{\rho}, z)$, with $m, n \in \{0, 1, 2, \dots\}$, is the Hermite-Gaussian solution of the paraxial wave equation of order $N = m + n$, while $\boldsymbol{\rho} = \hat{x}x + \hat{y}y$ denotes the transverse position vector. Please note that Eq. (1) is a particular case of the general expression in Eq. (1) of the main text.

When an opaque obstacle is placed into the beam, covering a region $\mathcal{A}$ in the xy -plane, the electric field directly behind the obstacle is given by

$$E'(\boldsymbol{\rho}, z) = [1 - \mathcal{I}_{\mathcal{A}}(\boldsymbol{\rho})]E(\boldsymbol{\rho}, z), \quad (2)$$

where

$$\mathcal{I}_{\mathcal{A}}(\boldsymbol{\rho}) = \begin{cases} 1 & \text{if } \boldsymbol{\rho} \in \mathcal{A}, \\ 0 & \text{if } \boldsymbol{\rho} \notin \mathcal{A}, \end{cases} \quad (3)$$

denotes the transmission function of the aperture complementary to the obstruction (Babinet's principle).

We are interested in the perturbed beam's intensity after passing a linear polarizer making an angle $\varphi \in \{0^\circ, 90^\circ, 45^\circ, 135^\circ\}$ with the x -axis, as the Stokes parameters s_0, s_1, s_2 are easily calculated from these intensities. Representing Eqs. (1) and (2) as two-component Jones vectors and describing the action of the linear polarizer by the projection matrix

$$P_\varphi = \begin{bmatrix} \cos^2 \varphi & \sin \varphi \cos \varphi \\ \sin \varphi \cos \varphi & \sin^2 \varphi \end{bmatrix}$$

yields

$$\begin{aligned} I_{\mathcal{A}}(\varphi) &= \int |P_\varphi E'(\boldsymbol{\rho}, z)|^2 d^2 \boldsymbol{\rho} \\ &= \int [1 - \mathcal{I}_{\mathcal{A}}(\boldsymbol{\rho})] |P_\varphi E(\boldsymbol{\rho}, z)|^2 d^2 \boldsymbol{\rho} \end{aligned} \quad (4)$$

for the beam's intensity behind the polarizer. With the help of Eq. (4), the Stokes parameters, measured in the plane of the obstructing object, take the form

$$s_0(\mathcal{A}) = I_{\mathcal{A}}(0^\circ) + I_{\mathcal{A}}(90^\circ), \quad (5a)$$

$$s_1(\mathcal{A}) = I_{\mathcal{A}}(0^\circ) - I_{\mathcal{A}}(90^\circ), \quad (5b)$$

$$s_2(\mathcal{A}) = I_{\mathcal{A}}(45^\circ) - I_{\mathcal{A}}(135^\circ), \quad (5c)$$

and are functions of $\mathcal{A}$, the region covered by the object within the beam. In simple cases, these equations may allow for straightforward algebraic inversion. To obtain the theoretical predictions shown in Figure 3a of the main text (rotor), we have used the time-independent mode function of the ideal radially polarized beam from Eq. (1). The rotation angle can then be obtained analytically as $\theta_0 = \arcsin(s_1/s_2)$. The simulated reference in Figure 3b of the main text (tracking) relies on experimentally measured distributions of $|P_\varphi E(\boldsymbol{\rho}, z)|^2$ that have been substituted into Eq. (4). This allows for the numerical evaluation of a likelihood function L which is used by the position tracking algorithm (see following section).

3. POSITION TRACKING ALGORITHM

Here we explain the algorithm used to infer the transverse central coordinates (x_0, y_0) of a spherical object from measurements of the polarization Stokes parameters $\mathbf{s} = (s_0, s_1, s_2)$. Among the advantages of a numerical implementation are that experimental beam imperfections and electronic detector noise can be accounted for in a straightforward manner. Eqs. (5) showed how to compute the individual Stokes parameters as a function of the transverse position of an object covering a known region $\mathcal{A}$. Before the actual sensing measurement, we perform this computation for each possible object position in discrete steps using the experimentally determined spatially resolved beam polarization tomographies $|P_\varphi E(\boldsymbol{\rho}, z)|^2$, effectively yielding a look-up table $(x_0, y_0) \rightarrow \mathbf{s}$ as visualised by the surfaces in Figure S1a. By taking into account the normally distributed, experimentally determined statistical noise of each detector, the look-up table can be turned into a likelihood distribution $L(\mathbf{s} | (x_0, y_0))$. During the real-time sensing measurement, given a measured Stokes vector $\mathbf{s}$, the position can be inferred by finding the coordinates (x_0, y_0) maximizing this likelihood distribution (see Figure S1b).

The rotational symmetry of the radially polarized mode implies a twofold ambiguity in determining the object's position, since rotating any object by 180° about the beam's center leaves the Stokes parameters invariant. This ambiguity can be lifted by making a number of additional assumptions, such as specifying the half-plane where the particle is located at time $t = 0$ (initial position), and that its velocity is small compared to the sampling frequency (continuity of position), or that its acceleration stays within physically motivated bounds (continuity of velocity). Such assumptions can easily be accommodated within the framework of Bayesian probability theory (BPT) [2] where they take the role of *prior information*. BPT also assigns a formal role to other assumptions such as the size of the object to be tracked, and provides the means to estimate such parameters when they are not known *a priori*. However, in our laboratory

Fig. S1. Position tracking (experimental data). **a.** For a known beam and object geometry, a look-up table $(x_0, y_0) \rightarrow s_{\{0,1,2\}}$ may be computed from Eqs. (5), giving the expected Stokes parameters as a function of the object's transverse position within the beam. One such look-up table, obtained from experimental data, is visualized here as a set of three surfaces. The measurement of a particular Stokes vector may be regarded as a set of planes intersecting these surfaces. The intersection of the corresponding contours (shown on the bottom) indicates the position of the object. **b.** The upper plane shows the likelihood distribution $L(s|(x_0, y_0))$ for the measured Stokes vector s . The maximum of this distribution corresponds to the inferred position. Note that although the theory for an ideal mode predicts two solutions, this symmetry may be broken in practice due to slight irregularities in the mode pattern, as demonstrated by this particular example.

setting we observe that the beam's symmetry is already broken by small beam imperfections (including a slight, but not complete, four-fold symmetry in intensity). Interestingly, we found that taking into account this overall asymmetry helps to determine the trajectory unambiguously with high certainty without such additional assumptions as described before. If necessary, the symmetry could also be broken in more controlled ways, e.g. by introducing a second beam, or by using a beam with a less symmetric polarization structure [3].

We emphasize that the algorithm used is non-recursive and does not require any repeated iterations on the same data. Rather, it relies mainly on data which can be pre-computed in the form of look-up tables to any desired accuracy in a computationally efficient way. Hence, the algorithm is highly parallelizable and thus compatible with the requirements of high-speed real-time signal processing. The algorithm's performance is rather insensitive to the assumed shape of the object: assuming a square of the same area instead of a circle leads to identical inferred positions (up to the width of the 68% credibility region). Changing the assumed object size generally leads to a radial shift of the inferred trajectory, but leaves the main features of the trajectory intact.

For the measurements shown here, we accounted for diffraction by assuming an effective sphere diameter at the detector plane which is 35% larger than the sphere's actual physical dimensions. It is likely that the accuracy can be further increased

by incorporating a more detailed model of diffraction. However, the actual impact of diffraction and the appropriate model to incorporate it will depend on the particular application under consideration.

REFERENCES

1. R. Dorn, S. Quabis, and G. Leuchs, "Sharper Focus for a Radially Polarized Light Beam," *Phys. Rev. Lett.* **91**, 233901 (2003).
2. U. von Toussaint, "Bayesian inference in physics," *Rev. Mod. Phys.* **83**, 943 (2011).
3. A. M. Beckley, T. G. Brown, and M. A. Alonso, "Full Poincaré beams," *Opt. Express* **18**, 10777 (2010).