Quantum teleportation over 100 km of fiber using highly efficient superconducting nanowire single-photon detectors: supplementary material

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1. TIME-BIN ENTANGLED PHOTON PAIR SOURCE

Fig. S1. Coincidence fringes obtained with the time-bin entanglement source. Squares and circles shows the data for signal interferometer phase set at 0 and $\pi/2$, respectively.

We evaluated the quality of the time-bin entanglement generated by our source with a Franson interferometry experiment [1]. The signal and idler photons were received by unbalanced interferometers whose output ports were connected to superconducting nanowire single photon detectors (SNSPD). Note that in the evaluation the setup is slightly different from the one used in the experiments described in the main text: we

employed a mode-locked laser with a repetition frequency of 100.2 MHz and SNSPDs based on NbN nanowires. The detection efficiencies of the SNSPDs for the signal and idler photons were 19 and 21%, respectively, both with a background count rate of ~ 10 cps. Since the quality of the entanglement based on the spontaneous parametric process is dominantly determined by the photon statistics [2], we can estimate the quality of the entanglement by setting the average photon per qubit at the same value as in the teleportation experiments (0.016). Also note that the use of a different mode-locked laser did not change the width of the pump pulse for the second harmonic generation (SHG) process or the coherence time of the photon pairs, since these parameters were determined by the optical filter bandwidths. We undertook a coincidence measurement on a superposition basis while sweeping the idler interferometer phase for two non-orthogonal phase settings at the signal interferometer. The result is shown in Fig. S1. The visibilities of the fitted curves were $94.6 \pm 2.3\%$ (signal interferometer phase: 0) and $94.1 \pm 2.5\%$ ($\pi/2$), which confirmed that our source generates high-quality time-bin entangled photon pairs.

2. QUANTUM TELEPORTATION WITH TIME-BIN QUBITS

We assume that the quantum state of the input photon, namely the photon whose quantum state is to be teleported, is given by $\alpha|1\rangle_{in} + \beta|2\rangle_{in}$, where α and β are complex numbers that satisfy $|\alpha|^2 + |\beta|^2 = 1$, and $|x\rangle_y$ denotes a state where there is a photon at a time slot x in a mode y . The quantum state of the time-bin entangled photon pairs that we use in our experiment is

expressed as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_s|1\rangle_i + |2\rangle_s|2\rangle_i), \quad (\text{S1})$$

where subscripts s and i denote the signal and idler modes, respectively. Using Eq. (S1), the state of the whole system is expressed as

$$\begin{aligned} & \frac{1}{\sqrt{2}}(|1\rangle_s|1\rangle_i + |2\rangle_s|2\rangle_i)(\alpha|1\rangle_{in} + \beta|2\rangle_{in}) \\ &= \frac{1}{2} [|\Phi^+\rangle(\alpha|1\rangle_i + \beta|2\rangle_i) + |\Phi^-\rangle(\alpha|1\rangle_i - \beta|2\rangle_i) \\ & \quad + |\Psi^+\rangle(\beta|1\rangle_i + \alpha|2\rangle_i) + |\Psi^-\rangle(\beta|1\rangle_i - \alpha|2\rangle_i)], \quad (\text{S2}) \end{aligned}$$

where $|\Psi^\pm\rangle$ and $|\Phi^\pm\rangle$ are the Bell states for the signal and input photons given by

$$\begin{aligned} |\Psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_s|2\rangle_{in} \pm |2\rangle_s|1\rangle_{in}), \\ |\Phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_s|1\rangle_{in} \pm |2\rangle_s|2\rangle_{in}). \end{aligned}$$

The right hand side of Eq. (S2) shows that the amplitudes of the input photon α and β are transferred to that of the idler photon with some unitary transformation depending on the results of Bell state measurements. We implement Bell state measurements using a beamsplitter followed by single photon detectors without a photon number resolving capability as in [3]. With this scheme, $|\Phi^\pm\rangle$ cannot be observed because the two photons are always output from the same port of the beamsplitter due to the Hong-Ou-Mandel (HOM) effect [4]. When the state $|\Psi^+\rangle$ is input into a beamsplitter, the two photons are output from the same port but in different time slots, so in theory this event can be observed by detecting consecutive clicks in the same detector. However, in our experiment, the SNSPD recovery time is ~ 100 ns and consecutive photon pulses cannot be detected, and thus projection to the $|\Psi^+\rangle$ state cannot be implemented. On the other hand, the $|\Psi^-\rangle$ state launched into a beam splitter gives a coincidence between the two detectors at different time slots, which means that only this state can be distinguished. As a result, we can teleport the quantum state of the input photon when the signal and input photons are projected onto the $|\Psi^-\rangle$ state with a probability of $1/4$.

According to the corresponding $|\Psi^-\rangle$ term on the RHS of Eq. (S2), the states of input photon $|1\rangle_{in}$, $|2\rangle_{in}$, $|L\rangle_{in}$, $|R\rangle_{in}$, and $|\pm\rangle_{in}$ are transformed to $|2\rangle_i$, $|1\rangle_i$, $|L\rangle_i$, $|R\rangle_i$, and $|\mp\rangle_i$, respectively. Here, $|\pm\rangle$, $|L\rangle$ and $|R\rangle$ are defined as $(|1\rangle \pm |2\rangle)/\sqrt{2}$, $(|1\rangle + i|2\rangle)/\sqrt{2}$ and $(|1\rangle - i|2\rangle)/\sqrt{2}$, respectively.

3. MOSI SNSPD

The SNSPDs are based on a $15 \mu\text{m} \times 15 \mu\text{m}$ MoSi meander whose nominal wire width and pitch are 160 and 210 nm, respectively. These devices are placed in a cryostat and cooled to 0.7 K. A 3.5-cm diameter, 5-turn fiber coil, which works as a blackbody filter, efficiently eliminates the photons whose wavelengths are longer than $2 \mu\text{m}$ [5]. The coil is formed in a fiber pigtail in front of each SNSPD inside the cryostat. The system detection efficiencies and background count rates as a function of bias current are plotted in Fig. S2 (a) and (b), respectively. We obtained $>80\%$ detection efficiencies for all four detectors used in the experiment. In the experiment, the bias currents of the SNSPDs were set to give system detection efficiencies of 80%

Fig. S2. System detection efficiency (a) and dark count rate (b) of the SNSPD system. Squares: SNSPD1, circles: SNSPD2, crosses: SNSPD3, and x symbols: SNSPD4.

(SNSPD1), 86% (SNSPD2 and 3) and 81% (SNSPD4). With these detection efficiencies, the background count rates were $\sim 10^2$ cps for all the detectors, which is an order of magnitude smaller than that reported in [6] ($\sim 10^3$ cps) thanks to the blackbody filter. In addition, the timing jitter of the SNSPDs was ~ 90 ps, and thus the measurement errors caused by inter-symbol interference were negligible in our experiment where the temporal difference between time bins was 1 ns. We also observed that the polarization dependence of the detection efficiency was less than 5% for all the detectors. This polarization insensitivity is probably due to the larger fill factor of the meander, which is achieved by designing the wire width and pitch so that their sizes are similar. The polarization insensitivity of the SNSPDs contributed to the stable multi-fold coincidence measurement for a long time in our quantum teleportation experiment over 100 km of fiber described below. The polarization dependence of the SNSPDs will be reported in detail elsewhere.

The operational conditions of SNSPDs are summarized in Table S1.

Table S1. Operational conditions of SNSPDs

SNSPD	Bias current [μA]	Detection efficiency [%]	Dark count rate [cps]
1	-10.3	80.0	110
2	-9.8	86.0	120
3	-10.9	86.2	80
4	-9.8	80.6	160

4. DETAILED CONDITIONS OF QUANTUM TELEPORTATION EXPERIMENT

The triple coincidence measurements were performed using a time interval analyzer (TIA). The detection signals from the four SNSPDs together with the synchronization signal from the mode-locked laser were input into the TIA and monitored in a time-tagged time-resolved (TTTR) mode. In a 100 km teleportation experiment, we performed 20-s TTTR data acquisition cycles, repeated 300 times for each input state and phase setting of MZI2, which means that the overall measurement time for each input state was 12000 s. The TTTR data were processed using Matlab to count triple coincidences. A 500-ps time window was used when counting the coincidences. To compensate for

the imbalance of the detection efficiencies, the number of triple coincidences obtained at SNSPD3, whose detection efficiency was higher than that of SNSPD4, was divided by the ratio of the detection efficiencies of SNSPD3 and 4. Note that this procedure decreases the effective number of triple coincidences at SNSPD3, and does not falsely improve the statistical error.

In the 100-km teleportation experiment, we observed a significant drift of the temporal positions of the teleported photons, which was caused by the fiber length fluctuation induced by temperature changes in the laboratory. For example, the temporal position shifted by 850 ps in a 6000-s projection measurement (whose result was used to construct the density matrix shown in Fig. S3 (a)), which exceeded the width of the time window. To cope with the temporal drift, we searched for the temporal position of the two-fold coincidences originating from the detections of the entangled pairs by SNSPD1 and 3 for each data file (for 20 s data acquisition), with which we could obtain the shift in the temporal position of the triple coincidences. As a result, we could count the triple coincidences stably by changing the position of the time window synchronously with the change in the temporal position of the teleported photons.

To estimate the collection efficiencies of the entangled photon pair source, we measured the heralding efficiencies of the signal and idler photons [7]. From the obtained heralding efficiencies and the measured system detection efficiencies of the SNSPDs, the effective collection efficiencies of the signal and idler photons (including other optical losses such as the filter insertion loss and fiber connection losses) were both estimated to be 0.13. These relatively low collection efficiencies probably result from the narrow-band filtering used to suppress the frequency correlation between the signal and idler photons [8]. When we take the collection efficiencies and other losses into consideration, the triple coincidence rate in our quantum teleportation experiment is given by the following equation.

$$R_{3fold} = \frac{1}{8} f_c \mu_{pp} \mu_{in} \eta_1 \eta_2 \eta_x c_s c_i 10^{-(2\alpha_c + \alpha_{pol} + \alpha_{pc} + \alpha_{mzi} + \alpha_f)/10} \quad (\text{S3})$$

The meanings of the symbols in the above equation and their values in our setup are shown in Table S2. The factor 1/8 on the right hand side of Eq. (S3) comes from the success probability of the Bell state measurement (1/4) and the probability of observing a photon on one of two bases with a projection measurement using an MZI (1/2). With this equation, the triple coincidence rate for the 100-km teleportation experiment is calculated to be ~ 0.027 [cps], which agrees well with the experimental result (0.028 [cps]).

In Fig. 2 (b) and (c) in the main text, we can observe a difference in the triple coincidence rates at the peak of the fringes, which corresponds to a loss difference of around 1.6 dB between the projection measurements with SNSPD3 and 4. The difference is partly due to the detection efficiency difference between SNSPD3 and 4, which corresponds to 0.3 dB. Other possible origins of the difference could be the loss difference of the 10-m fiber patch cords that connected the outputs of MZIs and SNSPDs, and the variation of the losses at the fiber connectors. This loss imbalance might have been an additional reason for the fidelity reduction.

To establish the reference frames between the superposition basis measurements by MZI1 and 2 for the quantum state tomography, we used the fringes shown in Fig. 2 (b) and (c) in the main text. Since the unitary transformation that accompanies the quantum teleportation is known as already described in Section 2, the “zero phase” of MZI2 relative to that of MZI1

Table S2. List of symbols in Eq. (S3)

Symbols	Descriptions	Experimental values
f_c	Pump repetition frequency	35.53 MHz
μ_{pp}	Average entangled photon pair per qubit	0.016
μ_{in}	Average input photon per qubit	0.016
η_x	System detection efficiency of SNSPDx	see Table S1
c_s	Signal photon collection efficiency	0.13
c_i	Idler photon collection efficiency	0.13
α_c	3-dB fiber coupler excess loss	0.5 dB
α_{pol}	Polarizer insertion loss	1.0 dB
α_{pc}	Polarization controller insertion loss	0.5 dB
α_{mzi}	MZI2 insertion loss	2.0 dB
α_f	Fiber loss	21.8 dB (102 km)

can be defined as the point where the fringe hits the bottom at an output port of MZI2.

5. FIDELITY ERROR ANALYSIS

Based on the scheme reported in [9], we can derive the density matrix of a single time-bin qubit. We define the number of triple coincidences obtained as a result of maximum likelihood estimation for the projection measurements $|+\rangle\langle+|$, $|-\rangle\langle-|$, $|L\rangle\langle L|$, $|R\rangle\langle R|$, $|1\rangle\langle 1|$ and $|2\rangle\langle 2|$ as n_+ , n_- , n_L , n_R , n_1 , and n_2 , respectively. Using those numbers, we obtain the Stokes parameters as follows.

$$S_0 = n_1 + n_2 \quad (\text{S4})$$

$$S_1 = n_+ - n_- \quad (\text{S5})$$

$$S_2 = n_L - n_R \quad (\text{S6})$$

$$S_3 = n_1 - n_2 \quad (\text{S7})$$

We define $N_i (i = 0, 1, 2, 3)$ as

$$N_i = \frac{S_i}{S_0}. \quad (\text{S8})$$

Then, the density matrix is obtained as

$$\rho = \frac{1}{2} (N_0 M_0 + N_1 M_1 + N_2 M_2 + N_3 M_3), \quad (\text{S9})$$

where $M_i (i = \{0, 1, 2, 3\})$ denotes the Pauli matrix. The fidelity for a target state $|\psi\rangle$ is obtained as

$$F = \langle \psi | \rho | \psi \rangle \quad (\text{S10})$$

In our measurement, we use a time-bin qubit with temporal interval of 1 ns, while the timing jitter of the detectors is about

90 ps. Therefore, we can reasonably assume that there is no error caused by the inter-symbol interference. This means the errors observed in the time basis measurement are simply caused by statistical fluctuations. On the other hand, the uncertainty in the interferometer phase (hereafter denoted by $\Delta\theta$) can induce the errors in superposition basis measurement, and so it should be taken into consideration. In our measurement, the phase uncertainty was smaller than $\pi/72$ (which corresponds to a 0.005 deg. temperature fluctuation of the interferometer). Hence, we set $\Delta\theta = \pi/72$ in the error calculation.

The projected state for the superposition basis is given by

$$|\psi(\theta)\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\theta}|2\rangle) \quad (\text{S11})$$

where θ is the interferometer phase that takes values of $0, \pi, \pi/2$ for $|+\rangle, |-\rangle, |L\rangle$, respectively. Then, the average value of the superposition basis measurement can be given by

$$\langle n(\theta) \rangle = C \langle \psi(\theta) | \rho | \psi(\theta) \rangle \quad (\text{S12})$$

where C is a constant that is dependent on the experimental setup. Note that $n_p = \langle n(0) \rangle$, $n_m = \langle n(\pi) \rangle$, and $n_L = \langle n(\pi/2) \rangle$. Using Eq. (S9), we obtain

$$\frac{\partial n(\theta)}{\partial \theta} = \frac{C}{2} \sum_{i=0}^3 N_i \left[\left\{ \frac{\partial}{\partial \theta} \langle \psi(\theta) | \right\} M_i | \psi(\theta) \rangle + \langle \psi(\theta) | M_i \left\{ \frac{\partial}{\partial \theta} | \psi(\theta) \rangle \right\} \right] \quad (\text{S13})$$

Assuming that the number distribution of the measurement data is Poissonian, we can obtain the errors in the measured data as

$$(\Delta n_j)^2 = \left(\frac{\partial n_j}{\partial \theta} \Delta \theta \right)^2 + n_j, \quad (\text{S14})$$

where $j \in \{+, -, L, R, 1, 2\}$. The errors in the Stokes parameters are obtained as

$$(\Delta S_0)^2 = (\Delta n_1)^2 + (\Delta n_2)^2 \quad (\text{S15})$$

$$(\Delta S_1)^2 = (\Delta n_+)^2 + (\Delta n_-)^2 \quad (\text{S16})$$

$$(\Delta S_2)^2 = (\Delta n_L)^2 + (\Delta n_R)^2 \quad (\text{S17})$$

$$(\Delta S_3)^2 = (\Delta n_1)^2 + (\Delta n_2)^2 \quad (\text{S18})$$

Using the above equations, we can obtain the errors in N_i as

$$(\Delta N_i)^2 = \left(\Delta S_0 \frac{S_i}{S_0^2} \right)^2 + \left(\frac{\Delta S_i}{S_0} \right)^2. \quad (\text{S19})$$

Using Eqs. (S10) and (S19), we can calculate the errors in fidelities.

6. SUPPLEMENTARY EXPERIMENTAL RESULTS

Here we show experimental results for quantum teleportation that supplement the data shown in the main text. Figure S3 shows the density matrices obtained by quantum state tomography for the target states $|-\rangle, |L\rangle, |R\rangle$ and $|1\rangle$. Table S3 summarizes the fidelity measurement results for six input states.

7. DISCUSSION: POSSIBILITY OF QUANTUM TELEPORTATION OVER INSTALLED FIBER

An important future challenge is to perform quantum teleportation over an installed fiber, which poses certain additional difficulties that we need to overcome. First, the temperature change

Table S3. Summary of fidelity measurements

Input state	Target state	Fidelity [%]
$ -\rangle$	$ +\rangle$	80.1 ± 4.2
$ +\rangle$	$ -\rangle$	78.6 ± 5.2
$ R\rangle$	$ R\rangle$	73.5 ± 4.4
$ L\rangle$	$ L\rangle$	83.1 ± 4.4
$ 2\rangle$	$ 1\rangle$	94.1 ± 5.6
$ 1\rangle$	$ 2\rangle$	93.1 ± 4.9
	Average	83.7 ± 2.0

of the fiber will be much larger than in the laboratory experiment, but this problem may be obviated with the drift monitor scheme mentioned above. Moreover, the loss of an installed fiber tends to be larger than that of a fiber wound in coils, because of connection losses and bending losses. As a result, we will need more integration time than in the experiment with fiber coils. Another possible problem is increased background counts caused by the crosstalk from optical communication signals in other fibers in the cable that contains the fiber for quantum transmission [10]. Therefore, we will need to improve the signal to noise ratio of our system.

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Fig. S3. Quantum state tomography results for target states (input states) (a) $|+\rangle$ ($|-\rangle$), (b) $|-\rangle$ ($|+\rangle$), (c) $|L\rangle$ ($|L\rangle$), (d) $|R\rangle$ ($|R\rangle$), (e) $|1\rangle$ ($|2\rangle$), and (f) $|2\rangle$ ($|1\rangle$)

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