Intermodal nonlinear mixing with Bessel beams in optical fiber: supplementary material

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This document provides supplementary information to: "Intermodal nonlinear mixing with Bessel beams in optical fiber," <http://dx.doi.org/10.1364/optica.2.000014>. Materials and methods for the experiments are further documented including the specifications of the source laser, and methods for inscribing long-period gratings, measuring spectra, and capturing mode images. The design of the experimental fiber is discussed as are the properties of the Bessel-like modes we excite. Finally, we discuss further features within the measured spectra, and discuss the phase matching methodology in full. © 2015 Optical Society of America

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S.1 MATERIALS AND METHODS

Source Laser

We used a high peak power source with relatively long pulses so that temporal walk off due to GVD is not a consideration. Our source is based on a micro-chip Nd:YAG laser (Teem Photonics SNP-20F-100, $\lambda = 1064$ nm, 0.5 ns, 19.56 kHz repetition rate) master oscillator which is amplified in a single pass through a double-clad Ytterbium-doped gain fiber (Nufern LMA-YDF-25/250-XIII) pumped by multimode CW pump diodes (JDSU L4-9897510-100E, $\lambda = 975$ nm) to achieve up to 86.9 kW of peak power. The collimated output passes through a free-space isolator (EOT 110-10182-0002) and a narrow bandpass filter (Semrock LL01-1064-12.5, 4 nm FWHM) which eliminates out-of-band light before it is coupled into the test fiber for nonlinear investigations.

Long Period Grating Inscription

Long period gratings (LPGs) were inscribed in the hydrogen-loaded fiber point-by-point using a UV writing system comprised of an Aerotech ANT180-360-L translation stage and a Coherent MotoFRED UV laser. Cylindrical lenses were used to focus the UV beam on to the fiber with a small spot size (8 μ m) along the translation axis. The UV laser power was 100 mW. To fabricate

each grating, the fiber was stripped of its polymer jacket by immersion in an acid solution (90:10 ratio sulfuric to nitric acid). The fiber was then mounted on the stage and translated the length of the grating with a particular speed to generate the right UV dosage. The UV laser was shuttered on and off to create the desired periodicity of index perturbations. The length of the grating was 5 mm, the period was 181.5 μ m, and the total exposure time was 215 seconds. The conversion efficiency from the incoming Gaussian-like LP_{0,1} mode to the Bessel-like LP_{0,6} mode at the pump wavelength was 96.5%, and the 10-dB bandwidth was 16 nm.

In this work, we used an LPG which converts to a single mode over a narrow wavelength band. Alternatively, techniques have been demonstrated to develop gratings with bandwidths as large as 500 nm [1,2]. Cascading multiple LPGs in a fiber has also been demonstrated [3,4] as a means to excite multiple fiber modes simultaneously. In addition, free-space coupling methods using spatial light modulators or MEMS mirror arrays can be used to couple into desired HOMs and dynamically switch between them over broad wavelength bands [5,6]. Reciprocally, the gratings and free-space devices would also work for converting the generated mode back to Gaussian beam.

Fig. S1. Characteristics of the higher order mode fiber. (a) Facet image of the double-clad fiber used in the experiments and (b) refractive index profile, r_a is the inner cladding radius (24.6 μm), r_b is the core radius (8.4 μm); (c) Effective index data; (d) Group velocity dispersion; (e) Effective area data, for the first eight $LP_{0,m}$ modes; modes $LP_{0,2}$ - $LP_{0,8}$ all have effective area greater than 400 μm² (yellow shaded region).

Measurement of Spectra and Mode Imaging

Spectral data was measured with an optical spectrum analyzer (Ando AQ6317B, sensitive from 600-1700 nm), and a silicon CCD spectrometer (Avantes AvaSpec-2048, sensitive from 350-1100 nm). Each spectrum was captured by stitching data from the devices at a wavelength where the sensitivities of each device overlapped. To isolate different spectral windows, we used a set of bandpass filters (Thorlabs, 10 nm full-width-half-maximum). For wavelengths between 400 and 1200 nm, we used a Silicon CCD camera (Point Grey CMLN-13S2C-CS) to take images. For wavelengths between 1200 nm and 1700 nm we used a Vidicon camera (Electrophysics Micronviewer 7290A) to take images.

S.2 FIBER DESIGN

We investigate multimode four-wave mixing interactions with a double-clad fiber, comprising a segmented core, inner-cladding, and an outer-cladding [Fig. S1(a),(b)], designed by us and fabricated by Nufern, Inc. The core is segmented into two regions: an inner portion co-doped with germanium and fluorine, and an outer ring doped with phosphorus. We designed this core to be single-moded and have a large effective area for the fundamental mode to reduce unwanted nonlinearity prior to mode conversion by the long period grating. Due to fabrication difficulties, the core additionally guided the $LP_{1,1}$ mode, and care needed to be exercised in coupling the pump light into only the desired fundamental ($LP_{0,1}$) mode of the fiber.

Mode conversion is realized by using LPGs, which are length-wise periodic index perturbations in the fiber that offer phase-matched coupling between two orthogonal co-propagating modes. The efficiency of these gratings depends on the overlap integral between the transverse mode-fields over the induced (index) perturbations. Since Ge-doped silica is UV-sensitive and fluorine and phosphorus are not, our design maximizes grating efficiency in the following manner: Co-doping with germanium and fluorine in the inner core ($r = 4.1$ μm) allows increasing photosensitivity while still matching the refractive index of the phosphorus-doped outer core region ($r_a = 8.4$ μm); Controlling the size of the inner core (with respect to the entire core) helps maximize the overlap integral that governs grating coupling efficiency. Thus, we obtain independent control over the core size and grating perturbation strength, and experimental conversion efficiencies > 95%, and as high as 99.9%, are readily

achieved. Such intricate design for the core is not needed to write mode converting gratings in fiber – indeed, strong LPGs are often obtained in fibers with simple step-index cores – this complex core, instead allowed achieving very high efficiencies in very compact devices (our grating lengths were only 5 mm).

The inner-cladding ($r_b = 24.6$ μm) is a pure silica region where the majority of the higher order modes' energy resides and the outer-cladding is down-doped with fluorine to facilitate highly multimode guidance. The inner-cladding guides approximately 600 modes at the pump wavelength of 1064 nm (highest $LP_{0,m}$ mode guided at this wavelength is $LP_{0,11}$) and approximately 3300 modes at 453 nm (the shortest wavelength at which coherent light generation was observed), where the highest $LP_{0,m}$ mode guided is $LP_{0,27}$.

The refractive index profile, shown in Fig. S1(b), was measured with a commercial fiber index profiler (Interfiber Analysis IFA-100, [7]) The profile was then used to model the effective indices (c), group velocity dispersion (d), and effective areas (e) for the $LP_{0,m}$ modes in the fiber. The zero dispersion wavelength decreases with mode order such that $LP_{0,5}$ is still within the normal dispersion regime at the pump wavelength, and $LP_{0,6}$ and all higher order $LP_{0,m}$ modes have anomalous dispersion.

S.3 PROPERTIES OF BESSEL-LIKE FIBER MODES

In this work, we study nonlinear interactions within the basis set of $LP_{0,m}$ modes in the fiber. These azimuthally symmetric modes are reminiscent of Bessel beams in free space – both in intensity pattern and diffractive properties – though they are truncated and thus not true solutions of Bessel's equation (hence 'Bessel-like'). These modes are particularly useful for nonlinear fiber optics given that they can propagate over long lengths of fiber (50 m demonstrated, [8]) without coupling to the nearest neighbor antisymmetric $LP_{1,m}$ modes. The stability of these modes, along with some of their other exotic properties, is discussed herein.

Mode coupling in fibers due to refractive index or geometric perturbations requires a substantial amount of spatial overlap and phase coherence between the coupled modes. The spatial overlap term depends on the azimuthal symmetry of the perturbation and that of the modes considered. For example, fiber bends can be conformally mapped to a slanted index

profile, which primarily couples modes whose azimuthal order differs by one – for our purposes, bends cause $LP_{0,m}$ modes to primarily couple to $LP_{1,m}$ modes. Phase coherence implies that mode coupling increases as the effective index difference, Δn_{eff} , between the modes decreases. While this rule is somewhat phenomenological, $\Delta n_{\text{eff}} > 10^{-4}$ has been accepted as a decent proxy for mode stability as polarization-maintaining fibers function well with mode separation of this order (further treatment of this argument in the context of fiber vortex modes can be found in [9]).

In large mode area fibers, separation between the fundamental mode and first higher-order mode, $LP_{1,1}$, can easily fall below this limit of stability as the mode area is increased. But, in a somewhat counter-intuitive fashion, it has recently been shown that $LP_{0,m}$ modes separate from their antisymmetric $LP_{1,m}$ counterparts in effective index as the radial order ('m') is increased [10]. This makes the $LP_{0,m>1}$ modes ideal candidates for stable propagation – indeed, in a highly multimode fiber, a high 'm' state can be more stable than the fundamental mode. Furthermore, these modes have intrinsically large mode areas [8] ($> 6000 \mu\text{m}^2$ demonstrated [11]) making them ideal candidates for high power laser applications. Bessel-like fiber modes have some other exotic diffractive properties outside the waveguide, such as increased Rayleigh range with respect to a Gaussian mode, and self-healing behavior [12]. It is also noteworthy that the far-field pattern for these modes primarily takes the form of a ring with radius proportional to the transverse wavevector, which increases with radial mode order [13]. Due to the different Bessel-like modes and colors being created within the fiber, it is consistent that our far-field image (inset in Fig. 2 in the main text) is a superposition of rings with different radii and different colors.

S.4 FULL ANALYSIS OF SPECTRAL CONTENT AND PHASE MATCHING

Spectral content

The primary observations from our experiment are as follows: the pump laser at 1064 nm generates a flat continuum from the interplay between monomode FWM interactions and spontaneous Raman scattering. As the short wavelength shoulder of the continuum grows more powerful for longer lengths of fiber, we note the emergence of a single discrete peak at 678 nm, outside the bandwidth for which we would anticipate monomode continuum to exist. At even longer fiber lengths, more peaks begin to arise. Mode imaging of these peaks confirms that they are a set of $LP_{0,m}$ modes, increasing monotonically in 'm' from $LP_{0,7}$ at 678 nm to $LP_{0,16}$ at 453 nm.

In addition to these discrete peaks, we observed other $LP_{0,m}$ modes at visible wavelengths in between the dominant peaks. On the long wavelength side of the pump at approximately 1450 nm (for the longest length – 16.3 m – of fiber we tested) the mode transitions from $LP_{0,6}$ into the $LP_{1,5}$ mode. This transition wavelength manifests as a slight dip in the spectrum, and decreases in wavelength as a function of increasing length and pump power. With higher pump power (~ 86 kW), another transition occurs at roughly 1525 nm from the $LP_{1,5}$ mode to the $LP_{0,5}$ mode. Understanding the processes that lead to the generation of these other modes will be the subject of future research, while here, we concentrate on analyzing the dominant nonlinear interactions we observed.

Phase matching along effective index lines

The condition for phase-matching (ignoring the small nonlinear phase accumulation due to self-phase modulation of the pump) is given by:

$$\beta_1(\lambda_1) + \beta_2(\lambda_2) - \beta_3(\lambda_3) - \beta_4(\lambda_4) = 0; \beta = 2\pi n_{\text{eff}} / \lambda \quad (\text{S1})$$

If the effective index at each of the spectral positions for the four photons falls on a line

$$n_{\text{eff}}(\lambda_k) = a\lambda_k + b \quad (\text{S2})$$

where a and b are arbitrary constants for the line, and k denotes the wavelength, then we can combine Eqs. (S1) and (S2) to get:

$$b \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3} - \frac{1}{\lambda_4} \right) = 0 \quad (\text{S3})$$

This relationship is identically the energy conservation condition. Therefore, any four photons whose effective indices are linear with wavelength are necessarily phase matched, provided they are energy matched.

If each of the four photons belongs to the same mode, a straight-line n_{eff} is essentially the zero-dispersion condition. If they are not in the same mode, then there is no dispersion requirement – only that the n_{eff} 's for these 4 photons, *regardless of which mode they are in*, fit a straight line, such as the fit we have extended in Fig. 2(a).

Phase matching analysis

There are three necessary conditions for coherent processes like four-wave mixing: phase matching, energy matching and spatial overlap. In conventional monomode experiments, the first two are trivial to calculate, and the third is generally described by an effective area calculation (since the fundamental mode at different wavelengths generally overlaps well with itself). Multimode systems are more complicated due to the numerous combinations of interacting modes. To determine the transitions that give rise to each discrete peak, we first assume that the peak is the anti-Stokes photon in an intermodal FWM process. Each process can only include the modes that were experimentally observed ($LP_{0,6} - LP_{0,16}$), and we assume that each mode only exists within ± 16.5 THz of where it was measured (accounting for the slight discrepancy between experiment and simulation, as shown in Fig. 4(a) in the main text). The $LP_{0,6}$ mode is assumed to extend from 700 – 1600 nm, because it was observed over this entire bandwidth. We consider every combination of the modes, at the wavelengths where they are allowed to exist, and find all solutions where energy and phase matching are upheld. Phase matching is examined by calculating the wave vector mismatch between all four photons, $\Delta\beta$:

$$\Delta\beta = \beta^{P1} + \beta^{P2} - \beta^{AS} - \beta^S \quad (\text{S4})$$

The superscripts 'P1' and 'P2' above denote the FWM pumps, 'AS' is the anti-Stokes, and 'S' is the Stokes. We define phase-matched solutions as cases where $|\Delta\beta| \leq 0.001 \mu\text{m}^{-1}$. All cases where energy matching and phase matching are simultaneously upheld are therefore valid FWM transitions.

Accounting for all the possible modes and wavelengths leads to thousands of valid processes, so further constraints are needed to truncate the data set. The spontaneous growth of the anti-Stokes photon is described by the following equation [14]

where each subscripted A is the amplitude of a given wave, z is the propagation coordinate, and A_{FWM} is the effective area for FWM [defined in Eq. (1) of the main text]:

$$\frac{\partial A_m}{\partial z} \propto \frac{2}{A_{\text{FWM}}} A_j A_k A_l^* e^{iA\beta z} \quad (\text{S5})$$

Since the process is directly proportional to the amplitude of the FWM pump waves, any transition involving a FWM pump at 1064 nm in the $\text{LP}_{0,6}$ mode will dominate as our laser pump is the strongest spectral feature we observe. The efficiency of the process is inversely proportional to A_{FWM} – thus the smallest effective area processes will dominate. With these considerations, we can distill the valid transitions down to the rule shown in the main text in Eq. (2):

$$\beta_{0,6}^{P1} \big|_{\lambda=1064\text{nm}} + \beta_{0,m-1}^{P2} = \beta_{0,m}^{AS} + \beta_{0,6}^S \quad (\text{S6})$$

The processes described by this rule only dominate because we have enforced constraints through our choice of pump wavelength and mode. In principle, we are able to pump with any mode at arbitrary wavelengths, and utilize the multitude of momentum conservation paths that exist in this medium. Even with the 11 distinct stable spatial modes we observed, given the fact that FWM involves 4 photons, the number of *potential* momentum conservation paths could be as large as $11^4 = 14,641$.

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