Optical magnetic mirrors without metals: supplementary material

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S1- Spectral locations of the magnetic and electric resonances. Transmission, reflection and loss of an OMM

Figure S1-1 | Simulated transmission, reflection and loss spectra of an OMM. Loss is calculated as: $\text{Loss} = 1 - \text{transmission} - \text{reflection}$ [1].

Figure S1-2 | Electric intensity (a) and magnetic intensity (b) as a function of wavelength and location inside the cube (along the z axis shown in (c)) showing the strongest electric and magnetic intensity located at $z = -0.06 \mu\text{m}$ and $-0.18 \mu\text{m}$ (z is the axis passing through the center of the truncated pyramid shape resonator, $z=0$ is the height center). Therefore, we locate the effective reflection plane near the center of the cube. The intensity maxima are located slightly below this center plane because of the substrate and the truncated pyramid shape. Simulations show that both electric and magnetic field maxima are located at the center when the resonator geometry is that of a perfect cubic and no substrate is present. This figure also shows the strongest electric and magnetic dipole resonances at $\lambda = 8.95 \mu\text{m}$ and $7.08 \mu\text{m}$.

S2- Mid-IR time domain spectroscopy setup in detail.

Figure S2 | Schematic of the difference frequency generation (DFG) and time-domain spectroscopy (TDS) set-up [2].

We used a phase-locked TDS in the mid-IR to directly measure the phase shift of the reflected electric field from the OMM. Mid-IR pulses of 250 fs duration, produced by difference frequency mixing between $\sim 1.35 \mu\text{m}$ and $\sim 1.55 \mu\text{m}$ pulses, were focused by a ZnSe lens onto the OMM or gold surface. Another ZnSe lens was used to collect the reflected mid-IR beam. Gate pulses with a wavelength of $1.05 \mu\text{m}$ and duration of 15 fs were combined with the mid-IR pulses using a dichroic mirror. Both the gate and the mid-IR pulses were focused into a 0.5 mm thick GaSe crystal, taking care to match the focal widths of the beams. The two polarizations are separated using a quarter wave plate and a Wollaston prism, and detected by a balanced detector. AOM, acousto-optic modulator; DM, dichroic mirror; GS, GaSe nonlinear crystal; LP, long-wavelength pass filter; G, diffraction grating; BD, balanced detector; $\lambda/2$, half-wave plate; $\lambda/4$, quarter-wave plate; WP, Wollaston prism.

S3- Three ways of confirming that switching between the gold surface and the OMM did not cause spurious delay change.

The reflection time-domain-spectroscopy system was aligned to make sure no spurious delay change occurs when the reflection spot is switched from the OMM to the gold surface. As illustrated in Figure S2, the red dots labeled “1”, “2” and “3” represent three equally spaced focal spots of the fs mid-IR beam on the OMM and the gold surface. Before translating between spots “1” and “2”, we first move the beam between spots “2” and “3” (both on gold). If the sample is not perpendicular to the plane of incidence, translation of the sample will cause a spurious delay change. We developed three methods for correcting this spurious delay change: 1) Adjusting the sample’s angle until TDS signals of the reflected electric fields from “2” and “3” temporally overlap with each other; 2) Using the spurious delay change between “2” and “3” to correct the translation between “1” and “2”; and 3) Using the reflected electric field phase difference (calculated using the Fourier transform) between spots “2” and “3” to calculate the spurious delay change.

Moreover, our TDS experiment used a digital oscilloscope which records the signal at 103–106 delay positions and averaged hundreds of scans as the delay stage moved back and forth. Several hundreds of scans can be averaged in 2-3 minutes.

Figure S3 | Schematic showing the positions of the mid-IR focal spots used to correct the spurious delay changes caused by translation.

S4-Theoretical explanation of the observed reflection phases employing two-dimensional periodic dyadic Green’s functions.

In this section we provide a theoretical basis for the expected phase difference between the magnetic mirror and the electric mirror behaviors of the OMM, and show that the “proper” location for the placement of the reference mirror is the center of the cubes. For convenience, we consider perfect cubic shape resonators with a side dimension of $1.45 \mu\text{m}$ in the analyses performed in this section. The electric and magnetic polarizabilities of an isolated cube (possessing the experimentally determined permittivity of Te) are computed following the procedure described in Ref. 3-4 and are shown in Fig. S4-1. Similar results have been recently reported in Ref 5. In Fig. S4-1, electric and magnetic resonances are clearly visible at wavelengths of about $6.9 \mu\text{m}$ and $9 \mu\text{m}$, respectively, in agreement with what was reported in Fig. 1c.

Figure S4-1 | Electric (blue) and magnetic (red) cube polarizabilities. Solid lines are real parts, dashed lines imaginary parts.

According to the result in Fig. S4-1, in the vicinity of the electric resonance we can approximate the scattering response of each cube as an electric dipole response. Similarly, near the magnetic resonance, we can approximate the scattering response of each cube as a magnetic dipole response.

We now consider a 2D array of cubes (assuming for simplicity the resonators are in free space without a substrate). Using the polarizabilities in Fig. S4-1, we can build the two independent systems of equations describing an array of either electric or magnetic dipoles only as (a more complete formulation of the problem that includes coupling between the electric and magnetic dipoles will be given below)

$$\left(\mathbf{I} - \alpha_{ee} \tilde{\mathbf{G}}_{ee}^{\infty}\right) \cdot \mathbf{p} = \alpha_{ee} \mathbf{E}^{\text{inc}} \quad \left(\mathbf{I} - \alpha_{mm} \tilde{\mathbf{G}}_{mm}^{\infty}\right) \cdot \mathbf{m} = \alpha_{mm} \mathbf{H}^{\text{inc}} \quad (\text{S1})$$

where $\mathbf{p}$ and $\mathbf{m}$ are the induced electric and magnetic dipole moments of the reference dipoles in the array, α_{ee} and α_{mm} are the cube's electric and magnetic polarizabilities reported in Fig. S4-1, $\tilde{\mathbf{G}}_{ee}^{\infty}$ and $\tilde{\mathbf{G}}_{mm}^{\infty}$ are the regularized periodic dyadic Green's functions for the phased array of electric and magnetic dipoles [6,7], $\mathbf{E}^{\text{inc}}$ and $\mathbf{H}^{\text{inc}}$ are the incident electric and magnetic fields, and $\mathbf{I}$ is the unit dyad. By solving the two systems in Eq. (S1), we are able to evaluate $\mathbf{p}$ and $\mathbf{m}$ including field interactions among all the (electric or magnetic) dipoles in the array. The knowledge of $\mathbf{p}$ and $\mathbf{m}$ is essential for the estimation of reflection coefficients and scattered fields. In particular, limiting the analysis only to the propagating fundamental plane wave with longitudinal wavenumber $k_z = k_0$ radiated by the dipoles, with k_0 the free space wavenumber, an array of magnetic dipoles radiates a magnetic field as

$$\mathbf{H}_m = \mathbf{G}_{mm} \cdot \mathbf{m}, \quad \mathbf{G}_{mm} = \frac{e^{ik_z(z-z_0)}}{-2iabk_z} \left[k_0^2 \mathbf{I} - \mathbf{k}\mathbf{k} \right] \quad (\text{S2})$$

whereas an array of electric dipoles radiates an electric field as

$$\mathbf{E}_e = \mathbf{G}_{ee} \cdot \mathbf{p}, \quad \mathbf{G}_{ee} = \frac{e^{ik_z(z-z_0)}}{-2iab\epsilon_0 k_z} \left[k_0^2 \mathbf{I} - \mathbf{k}\mathbf{k} \right]. \quad (\text{S3})$$

In Eqs. (S2) and (S3), a and b indicate the periods in the x and y directions (equal to $3.45 \mu\text{m}$ in the full-wave simulation), $\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} \pm k_z \hat{\mathbf{z}}$ is the wavevector (+/- is used when

the observation point z is above/below the source point z_0), and $\mathbf{p}$ and $\mathbf{m}$ are the induced electric and magnetic dipole moments calculated solving Eq. (S1). The dyadic Green's functions $\mathbf{G}_{ee}$ and $\mathbf{G}_{mm}$ in Eqs. (S2) and (S3) are computed using a spectral approach [7,8] and model only the fundamental propagating plane wave — the only wave responsible for specular reflection. In accordance with the experiment, we further assume normal plane wave excitation with $\mathbf{E}^{\text{inc}} = E_0 e^{-ik_0 d_0} \hat{\mathbf{y}}$, where $z = d_0$ represents the zero-phase distance from the array plane $z_0 = 0$. For comparison with the experimental measurements, we define the electric field reflection coefficient at the electric and magnetic resonances as Γ_e and Γ_m respectively. These reflection coefficients are defined as the ratio of the transverse electric field components at the array plane and are given by

$$\Gamma_e = \frac{\mathbf{E}_e \cdot \hat{\mathbf{y}}}{\mathbf{E}^{\text{inc}} \cdot \hat{\mathbf{y}}} \propto -\frac{\alpha_{ee}}{i} \quad \Gamma_m = -\frac{\mathbf{H}_m \cdot \hat{\mathbf{x}}}{\mathbf{H}^{\text{inc}} \cdot \hat{\mathbf{x}}} \propto \frac{\alpha_{mm}}{i} \quad (\text{S4})$$

Note that for Γ_m we have expressed the electric field reflection coefficient in terms of the incident and reflected magnetic fields, which requires the inclusion of the minus sign in this expression.

Closer inspection of Eq. (S4) shows the phases of both α_{ee} and α_{mm} at resonance are 90 degrees, and are compensated by the imaginary term at the denominator of the two expressions in Eq.

(S4). Therefore, the 180 and 0 degrees phase shifts for Γ_e and Γ_m for the respective resonances inherently originate from the presence and absence of a minus sign in their corresponding equations in Eq. (S4). In general, there could be an additional phase shift in these expressions due to the $\left(\mathbf{I} - \alpha_{ee} \tilde{\mathbf{G}}_{ee}^{\infty}\right)^{-1}$ and $\left(\mathbf{I} - \alpha_{mm} \tilde{\mathbf{G}}_{mm}^{\infty}\right)^{-1}$ factors in the solution of Eq. (S1), however this phase shift is expected to be small. A plot of the two reflection coefficients evaluated at the array plane $z = z_0$ is reported in Fig. S4-2 (this plane is assumed to be at the cubes' center). We observe that the phase of the reflection coefficient Γ_e at the electric resonance frequency is about 180 degrees, signature of a perfect electric conductor behavior. We further observe about 0 degrees phase for the reflection coefficient Γ_m at the magnetic resonance frequency, signature of a perfect magnetic conductor behavior. This result is in agreement with the experimental observations of the OMM behavior in the main text of this paper.

Figure S4-2 | Plots of the two reflection coefficients from the array of cubes modeled as an array of either electric (solid blue) or magnetic (dashed red) dipoles.

For resonators that are very small compared to the operating wavelength it is obvious that the correct location of the reference mirror should be at the center of the resonators. This assumption has been implicitly made in the dipole formulations given earlier in this section. However, as the resonators become larger it is not clear whether the incident field will experience any additional phase delay as it propagates from the top surface to the center of the array of high permittivity cubes. While the neglect of coupling between the electric and magnetic dipoles was allowable when considering the relative phase shift between the reflected electric

field at the two resonances, such a description cannot be used to determine the correct location of the reference mirror to which the experimental phases are referenced, nor for a more accurate estimation of the (total) electric field reflection coefficient measured experimentally. Such an extra phase shift might require that the mirror reference plane be displaced from the cubes' center. To determine whether such an additional phase shift is required, we compute the scattered electric and magnetic fields at $z = d_0$ (i.e., at the location of the plane wave injection in full-wave simulations, considered to be in the far-field of the metasurface with $d_0 = 9.275 \mu\text{m}$) using both a dipolar method and full-wave simulations. For the dipolar method, we now consider coupled electric and magnetic dipoles adapting the formulation in Ref. 5&9 to the case of 2D periodic arrays, as for example recently shown in Ref. 10. In other words, we find $\mathbf{P}$ and $\mathbf{m}$ by solving the 6 by 6 system

$$\begin{bmatrix} \alpha_{ee}^{-1} & \mathbf{0} \\ \mathbf{0} & \alpha_{mm}^{-1} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{G}}_{ee}^{\infty} & \tilde{\mathbf{G}}_{em}^{\infty} \\ \tilde{\mathbf{G}}_{me}^{\infty} & \tilde{\mathbf{G}}_{mm}^{\infty} \end{bmatrix} \cdot \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \end{pmatrix} = \begin{pmatrix} \mathbf{E}^{\text{inc}} \\ \mathbf{H}^{\text{inc}} \end{pmatrix} \quad (\text{S5})$$

where the dyads $\tilde{\mathbf{G}}_{em}^{\infty}$ and $\tilde{\mathbf{G}}_{me}^{\infty}$ introduce electromagnetic coupling between different dipoles [5,9,10]. In full-wave simulations, we consider a 2D periodic array of cubes in free space under normal incidence. Fig. S4-3 shows the magnitude and phase of the reflection coefficient referred at the array plane using both the dipole theory and the full-wave simulations. We observe a remarkable agreement between the theoretical and full-wave results over the entire frequency range, given the approximate nature of the dipolar methods (discrepancies may still be due to multipolar contributions and/or spatial dispersion). In particular, when looking at the phase result, we do not observe a significant additional phase shift. This result confirms that any extra phase shift due to the presence of the real cubes is small and the approximation of locating the mirror reference plane at the cubes' center is a proper one. For comparison, we also plot the experimentally determined reflection phase in Fig. S4-3. Surprisingly good agreement with the theory and simulation is observed, considering that the theory and simulation utilized perfect cubes rather than the truncated pyramids measured in the experiment, and since the theory and simulation did not include the effect of the substrate.

Figure S4-3 | Magnitude and phase of the reflection coefficient at the array plane computed with full-wave simulations (blue solid) and dual (electric and magnetic) dipolar approximation (red dashed). The experimental result is reported for completeness as a green dotted curve.

S5-Electric field fringes reflected from a gold mirror and the OMM closely match each other at the electric resonance wavelength of the OMM

Figure S5 | Simulated electric field transients reflected from a gold mirror (blue curve) and the OMM (red curve) at the electric dipole resonance.

S6-Phase different between magnetic and electric resonances for another OMM sample

We also measured the phase of the reflected electric field using our TDS system for a different OMM sample which exhibited a smaller spectral separation between the electric and magnetic resonances compared with the OMM shown in Fig. S3. The experimental results show that within a narrow wavelength range of electric and magnetic resonances, the phase difference between the resonances is almost the same as for the sample described in the main text.

Figure S6 | Experimental results of another OMM sample with smaller spectral difference between electric and magnetic dipole resonances. Therefore, our mid-IR fs pulses are spectrally broad enough to cover both resonances without tuning. Despite the difference between this OMM sample and the sample discussed in the main text, we observe close to 180 degree phase difference between electric and magnetic resonances in both cases. This unambiguously demonstrates an OMM formed by Te dielectric resonators at magnetic dipole resonance. The inset shows the SEM image of this sample.

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