Transparent sub-diffraction optics: Nanoscale light confinement without metal: supplementary material

SAMAN JAHANI,¹ ZUBIN JACOB,^{1,*}

¹ Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada T6G 2V4

*Corresponding author: zjacob@ualberta.ca

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This document provides supplementary information to: "Transparent sub-diffraction optics: Nanoscale light confinement without metal", <http://dx.doi.org/10.1364/optica.1.000096>. The full analytical mode calculations of 1D, 2D extreme-skin-depth (e-skid) waveguides, and 1D coupled waveguides are reported. The effects of waveguide parameters on power confinement, mode length/area and cross talk have been investigated. The results have been verified with CST Microwave Studio, a full-wave numerical simulation tool, and excellent agreements are observed. © 2014 Optical Society of America

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Analytical calculations

1D Extreme-skin-depth Waveguides. For an infinite slab waveguide in which the wave propagates in the z direction, the conditions to satisfy total internal reflection and fast decaying in the cladding are: $h_z < 1$ and $h_x \gg 1$. Since for the TM modes in slab waveguides only y component of the magnetic field is non-zero, the magnetic field does not feel the anisotropy. Thus we can achieve the momentum transformation with non-magnetic anisotropic metamaterials. The magnetic field is defined as:

$$H_y = \begin{cases} A \cos(k_{x1}x) e^{i\beta z}, & |x| < a \\ B e^{-k_{x2}x} e^{i\beta z}, & |x| > a \end{cases} \quad (S1)$$

where A and B are constants and obtained by matching boundary conditions and excitation, and k_{xi} ($i=1,2$) and β are the wave vector components in the perpendicular and parallel to the propagation direction, respectively. When the second medium is anisotropic dielectric, the electric field components are derived from Maxwell equations as:

$$E_x = \frac{1}{i\omega\epsilon_0\epsilon_x} \frac{\partial H_y}{\partial z} = \frac{\beta}{\omega\epsilon_0} \begin{cases} \frac{A}{\epsilon_1} \cos(k_{x1}x) e^{i\beta z}, & |x| < a \\ \frac{B}{\epsilon_{x2}} e^{-k_{x2}x} e^{i\beta z}, & |x| > a \end{cases} \quad (S2)$$

$$E_z = \frac{-1}{i\omega\epsilon_0\epsilon_z} \frac{\partial H_y}{\partial x} = \frac{1}{i\omega\epsilon_0} \begin{cases} \frac{Ak_{x1}}{\epsilon_1} \sin(k_{x1}x) e^{i\beta z}, & |x| < a \\ \frac{Bk_{x2}}{\epsilon_{z2}} e^{-k_{x2}x} e^{i\beta z}, & |x| > a \end{cases}$$

where ϵ_1 is the first medium permittivity and ϵ_{x2} and ϵ_{z2} are the x and z component of second medium permittivity. Since the structure is invariant in the y direction, other field components are zero. The isofrequency dispersion relations for the two mediums are:

$$\begin{cases} \beta^2 + k_{x1}^2 = k_0^2 \epsilon_1 \\ \beta^2 - \left(\frac{k_{x2}}{\gamma}\right)^2 = k_0^2 \epsilon_{x2} \end{cases} \quad (S3)$$

where $\gamma^2 = \frac{\epsilon_{z2}}{\epsilon_{x2}}$. By applying boundary conditions and

combining the two equations in (S3), we obtain two coupled equations for determining wave vectors:

$$\begin{cases} w = \gamma^2 \frac{\epsilon_{x2}}{\epsilon_1} u \tan(u) \\ u^2 + \left(\frac{w}{\gamma}\right)^2 = (k_0 a)^2 (\epsilon_1 - \epsilon_{x2}) \end{cases} \quad (S4)$$

where $u = k_{x1}a$ and $w = k_{x2}a$. This set of equations can be solved graphically to find components of the wave vectors. Fig. 1 shows the graphical solution for the first even TM mode. The point where two equations in (S4) intersect is the result. The

plot in fig. S1.a and S1.b are for the case in which the core is glass ($\epsilon_1=2.25$) with size of $2a=\lambda_0/10$ and the cladding is air and anisotropic dielectric ($\epsilon_{x2}=1.2$ and $\epsilon_{z2}=20$), respectively. It is clearly observable that the anisotropic cladding leads upscaling of w without a noticeable change in u . An increase in w means faster decaying of the fields in the cladding and consequently more energy confinement inside the core. The electric and magnetic field profiles are plotted in fig. S2 in comparison with the same waveguide without cladding. It is seen that the fields decay fast in the anisotropic cladding.

Fig. S1. The graphical solution of (10) for the glass slab waveguide with a size of $\lambda_0/10$ and (a) air cladding (b) anisotropic cladding ($\epsilon_x=1.2$ and $\epsilon_z=20$). The solution moves to higher w (more confinement) without a large change in u .

2D Extreme-skin-depth Waveguides

a) Dual-anisotropic cladding: Assume an infinitely long isotropic dielectric cylinder oriented in the z direction with relative permittivity and permeability of (ϵ_1, μ_1) surrounded by a dual-anisotropic uniaxial dielectric with optical axis parallel to the cylinder axis and relative permittivity and permeability of $(\epsilon_{\rho 2} = \epsilon_{\varphi 2} = \mu_{\rho 2} = \mu_{\varphi 2} = n_{\perp 2}, \epsilon_{z2} = \mu_{z2} = n_{z2})$. The Helmholtz wave equation for longitudinal field components inside a uniaxial anisotropic magneto-dielectric is

Fig. S2. Electromagnetic field profile for glass slab waveguide with a size of $\lambda_0/10$ and anisotropic profile cladding ($\epsilon_x=1.2$ and $\epsilon_z=15$) in comparison with the same waveguide without cladding. (a) Electric field. (b) Magnetic field. Field amplitudes are normalized to the same input energy. Fields decay faster in the anisotropic cladding.

$$\begin{cases} \nabla_t^2 E_z + \left(k_0^2 \epsilon_z \mu_{\perp} - \frac{\epsilon_z}{\epsilon_{\perp}} \beta^2 \right) E_z = 0 \\ \nabla_t^2 H_z + \left(k_0^2 \epsilon_{\perp} \mu_z - \frac{\mu_z}{\mu_{\perp}} \beta^2 \right) H_z = 0 \end{cases} \quad (S5)$$

where ∇_t^2 is tangential component of Laplacian. The time-harmonic solutions of the above equations in cylindrical coordinate system for the core and cladding are where A , B , C , and D

$$\begin{aligned} E_z &= \begin{cases} A J_n(k_{\rho 1} \rho) \cos(n\varphi) e^{i\beta z}, & |\rho| < a \\ B K_n(k_{\rho 2} \rho) \cos(n\varphi) e^{i\beta z}, & |\rho| > a \end{cases} \\ H_z &= \begin{cases} C J_n(k_{\rho 1} \rho) \sin(n\varphi) e^{i\beta z}, & |\rho| < a \\ D K_n(k_{\rho 2} \rho) \sin(n\varphi) e^{i\beta z}, & |\rho| > a \end{cases} \end{aligned} \quad (S6)$$

are constants and defined from boundary conditions and excitations; J_n and K_n are n^{th} -order of the Bessel function of the first type and the modified Bessel function of the second type, respectively; β is the propagation constant, and $k_{\rho 1}$, $k_{\rho 2}$ are defined as

(S9)

$$\begin{cases} \beta^2 + k_{\rho 1}^2 = k_0^2 \varepsilon_1 \mu_1 \\ \beta^2 - \left(\frac{k_{\rho 2}}{\gamma}\right)^2 = k_0^2 n_{\perp 2}^2, \left(\gamma^2 = \frac{\varepsilon_{z2}}{\varepsilon_{\perp 2}} = \frac{\mu_{z2}}{\mu_{\perp 2}} = \frac{n_{z2}}{n_{\perp 2}}\right) \end{cases} \quad (S7)$$

The other field components are derived from the Maxwell's equations easily:

(S8)

$$E_r = \frac{-1}{i\omega\varepsilon_0\varepsilon_r} \left(\frac{\partial H_z}{r\partial\varphi} - \frac{\partial H_\varphi}{\partial z} \right) = \frac{i}{(k_0^2 \varepsilon_r \mu_\varphi - \beta^2)} \left(\beta \frac{\partial E_z}{\partial r} + \frac{\omega\mu_0\mu_\varphi}{r} \frac{\partial H_z}{\partial\varphi} \right)$$

$$= \cos(n\varphi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A\beta k_{\rho 1} J_n(k_{\rho 1}\rho) + C \frac{\omega\mu_0\mu_1 n}{r} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B\beta k_{\rho 2} K_n(k_{\rho 2}\rho) + D \frac{\omega\mu_0\mu_{\perp 2} n}{r} K_n(k_{\rho 2}\rho) \right], & |\rho| > a \end{cases}$$

$$E_\varphi = \frac{-1}{i\omega\varepsilon_0\varepsilon_\varphi} \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) = \frac{i}{(k_0^2 \varepsilon_\varphi \mu_r - \beta^2)} \left(\frac{\beta}{r} \frac{\partial E_z}{\partial\varphi} - \omega\mu_0\mu_r \frac{\partial H_z}{\partial r} \right)$$

$$= \sin(n\varphi) e^{i\beta z} \times$$

$$\begin{cases} \frac{-i}{k_{\rho 1}^2} \left[A \frac{\beta n}{r} J_n(k_{\rho 1}\rho) + C \omega\mu_0\mu_1 k_{\rho 1} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{i}{(k_{\rho 2}/\gamma)^2} \left[B \frac{\beta n}{r} K_n(k_{\rho 2}\rho) + D \omega\mu_0\mu_{\perp 2} k_{\rho 2} K_n(k_{\rho 2}\rho) \right], & |\rho| > a \end{cases}$$

$$H_r = \frac{1}{i\omega\mu_0\mu_r} \left(\frac{\partial E_z}{r\partial\varphi} - \frac{\partial E_\varphi}{\partial z} \right) = \frac{i}{(k_0^2 \varepsilon_\varphi \mu_r - \beta^2)} \left(\beta \frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon_0\varepsilon_\varphi}{r} \frac{\partial E_z}{\partial\varphi} \right)$$

$$= \sin(n\varphi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A \frac{\omega\varepsilon_0\varepsilon_1 n}{r} J_n(k_{\rho 1}\rho) + C\beta k_{\rho 1} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B \frac{\omega\varepsilon_0\varepsilon_{\perp 2} n}{r} K_n(k_{\rho 2}\rho) + D\beta k_{\rho 2} K_n(k_{\rho 2}\rho) \right], & |\rho| > a \end{cases}$$

$$H_\varphi = \frac{1}{i\omega\mu_0\mu_\varphi} \left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) = \frac{i}{(k_0^2 \varepsilon_r \mu_\varphi - \beta^2)} \left(\frac{\beta}{r} \frac{\partial H_z}{\partial\varphi} + \omega\varepsilon_0\varepsilon_r \frac{\partial E_z}{\partial r} \right)$$

$$= \cos(n\varphi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A\omega\varepsilon_0\varepsilon_1 k_{\rho 1} J_n(k_{\rho 1}\rho) + C \frac{\beta n}{r} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B\omega\varepsilon_0\varepsilon_{\perp 2} k_{\rho 2} K_n(k_{\rho 2}\rho) + D \frac{\beta n}{r} K_n(k_{\rho 2}\rho) \right], & |\rho| > a \end{cases}$$

By applying the boundary conditions and subtracting dispersion relations, two coupled equations are obtained:

$$\begin{cases} \frac{J'_n(u)}{uJ_n(u)} = - \left(\frac{\mu_{\perp 2}}{\mu_1} + \frac{\varepsilon_{\perp 2}}{\varepsilon_1} \right) \frac{\gamma^2 K'_n(w)}{2wK_n(w)} \pm \\ \left[\left(\frac{\mu_{\perp 2}}{\mu_1} - \frac{\varepsilon_{\perp 2}}{\varepsilon_1} \right)^2 \left(\frac{\gamma^2 K'_n(w)}{2wK_n(w)} \right)^2 + \frac{n^2 \beta^2}{\varepsilon_1 \mu_1 k_0^2} \left(\frac{1}{u^2} + \left(\frac{\gamma}{w} \right)^2 \right)^2 \right]^{\frac{1}{2}} \\ u^2 + \left(\frac{w}{\gamma} \right)^2 = (k_0 a)^2 (\varepsilon_1 \mu_1 - \varepsilon_{\perp 2} \mu_{\perp 2}) \end{cases}$$

where $u = k_{\rho 1} a$ and $w = k_{\rho 2} a$. This set of equations can yield two independent solutions according to the sign of the second term in the first equation. For $n > 1$, the solutions are conventionally known as HE and EH modes for negative and positive signs respectively, for the conventional optical fibers. The first HE mode can propagate without any cut-off.

b) non-magnetic anisotropic cladding. To verify that the transformed momentum of light can indeed lead to sub-diffraction confinement even without using dual anisotropic cladding, we solved Maxwell's equations in an arbitrary shaped glass core surrounded by all-dielectric nonmagnetic anisotropic metamaterials that transform momentum. Assume that the core and anisotropic cladding is nonmagnetic ($\mu_1 = \mu_2 = 1$) in (S5). The longitudinal field components in the core and cladding are:

$$E_z = \begin{cases} A J_n(k_{\rho 1}\rho) \cos(n\varphi) e^{i\beta z}, & |\rho| < a \\ B K_n(k_{\rho 2}\rho) \cos(n\varphi) e^{i\beta z}, & |\rho| > a \end{cases} \quad (S10)$$

$$H_z = \begin{cases} C J_n(k_{\rho 1}\rho) \sin(n\varphi) e^{i\beta z}, & |\rho| < a \\ D K_n\left(\frac{k_{\rho 2}}{\gamma}\rho\right) \sin(n\varphi) e^{i\beta z}, & |\rho| > a \end{cases}$$

where $k_{\rho 1}, k_{\rho 2}$ are defined as

$$\begin{cases} \beta^2 + k_{\rho 1}^2 = k_0^2 \varepsilon_1 \\ \beta^2 - \left(\frac{k_{\rho 2}}{\gamma}\right)^2 = k_0^2 \varepsilon_{\perp 2}, \left(\gamma^2 = \frac{\varepsilon_{z2}}{\varepsilon_{\perp 2}}\right) \end{cases} \quad (S11)$$

The other field components are derived in the same way:

(S12)

$$E_r = \frac{-1}{i\omega\varepsilon_0\varepsilon_r} \left(\frac{\partial H_z}{r\partial\varphi} - \frac{\partial H_\varphi}{\partial z} \right) = \frac{i}{(k_0^2 \varepsilon_r - \beta^2)} \left(\beta \frac{\partial E_z}{\partial r} + \frac{\omega\mu_0}{r} \frac{\partial H_z}{\partial\varphi} \right)$$

$$= \cos(n\varphi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A\beta k_{\rho 1} J_n(k_{\rho 1}\rho) + C \frac{\omega\mu_0 n}{r} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B\beta k_{\rho 2} K_n(k_{\rho 2}\rho) + D \frac{\omega\mu_0 n}{r} K_n\left(\frac{k_{\rho 2}}{\gamma}\rho\right) \right], & |\rho| > a \end{cases}$$

$$E_\phi = \frac{-1}{i\omega\epsilon_0\epsilon_\phi} \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) = \frac{i}{(k_0^2\epsilon_\phi - \beta^2)} \left(\frac{\beta}{r} \frac{\partial E_z}{\partial \phi} - \omega\mu_0 \frac{\partial H_z}{\partial r} \right)$$

$$= \sin(n\phi) e^{i\beta z} \times$$

$$\begin{cases} \frac{-i}{k_{\rho 1}^2} \left[A \frac{\beta n}{r} J_n(k_{\rho 1}\rho) + C\omega\mu_0 k_{\rho 1} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{i}{(k_{\rho 2}/\gamma)^2} \left[B \frac{\beta n}{r} K_n(k_{\rho 2}\rho) + D\omega\mu_0 \frac{k_{\rho 2}}{\gamma} K_n\left(\frac{k_{\rho 2}}{\gamma}\rho\right) \right], & |\rho| > a \end{cases}$$

$$H_r = \frac{1}{i\omega\mu_0} \left(\frac{\partial E_z}{r\partial\phi} - \frac{\partial E_\phi}{\partial z} \right) = \frac{i}{(k_0^2\epsilon_\phi - \beta^2)} \left(\beta \frac{\partial H_z}{\partial r} - \frac{\omega\epsilon_0\epsilon_\phi}{r} \frac{\partial E_z}{\partial \phi} \right)$$

$$= \sin(n\phi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A \frac{\omega\epsilon_0\epsilon_1 n}{r} J_n(k_{\rho 1}\rho) + C\beta k_{\rho 1} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B \frac{\omega\epsilon_0\epsilon_{12} n}{r} K_n(k_{\rho 2}\rho) + D\beta \frac{k_{\rho 2}}{\gamma} K_n\left(\frac{k_{\rho 2}}{\gamma}\rho\right) \right], & |\rho| > a \end{cases}$$

$$H_\phi = \frac{1}{i\omega\mu_0} \left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) = \frac{i}{(k_0^2\epsilon_r - \beta^2)} \left(\frac{\beta}{r} \frac{\partial H_z}{\partial \phi} + \omega\epsilon_0\epsilon_r \frac{\partial E_z}{\partial r} \right)$$

$$= \cos(n\phi) e^{i\beta z} \times$$

$$\begin{cases} \frac{i}{k_{\rho 1}^2} \left[A\omega\epsilon_0\epsilon_1 k_{\rho 1} J_n(k_{\rho 1}\rho) + C \frac{\beta n}{r} J_n(k_{\rho 1}\rho) \right], & |\rho| < a \\ \frac{-i}{(k_{\rho 2}/\gamma)^2} \left[B\omega\epsilon_0\epsilon_{12} k_{\rho 2} K_n(k_{\rho 2}\rho) + D \frac{\beta n}{r} K_n\left(\frac{k_{\rho 2}}{\gamma}\rho\right) \right], & |\rho| > a \end{cases}$$

Similar to the dual-anisotropic case, by applying the boundary conditions, two coupled equations are obtained:

(S13)

$$\left\{ \frac{J'_n(u)}{uJ_n(u)} = - \left(\frac{\gamma K'_n\left(\frac{w}{\gamma}\right)}{2wK_n\left(\frac{w}{\gamma}\right)} + \frac{\epsilon_{12}\gamma^2 K'_n(w)}{2\epsilon_1 w K_n(w)} \right) \pm \left[\left(\frac{\gamma K'_n\left(\frac{w}{\gamma}\right)}{2wK_n\left(\frac{w}{\gamma}\right)} - \frac{\epsilon_{12}\gamma^2 K'_n(w)}{2\epsilon_1 w K_n(w)} \right)^2 + \frac{n^2\beta^2}{\epsilon_1 k_0^2} \left(\frac{1}{u^2} + \left(\frac{\gamma}{w} \right)^2 \right)^2 \right]^{\frac{1}{2}} \right.$$

$$\left. u^2 + \left(\frac{w}{\gamma} \right)^2 = (k_0 a)^2 (\epsilon_1 - \epsilon_{12}) \right\}$$

Fig. S3. The graphical solution by intersecting the left and right hand side of (S9) and (S13) for a cylindrical waveguide with glass core whose radius is $\lambda_0/10$ and the cladding is (a) air (b) anisotropic non-magnetic dielectric ($\epsilon_\perp = 1$ and $\epsilon_z = 20$) (c) dual-anisotropic dielectric ($\epsilon_\perp = \mu_\perp = 20$ and $\epsilon_z = \mu_z = 20$). It is seen that the waveguide mode shifts to lower values of u (higher w) denoting faster decay in the cladding.

Fig. S3 displays the graphical solution of the equation (S9) and (S13) for a cylindrical glass core with radius of $\lambda_0/10$ for air, non-magnetic anisotropic, and dual anisotropic claddings. The plots show the right and left side of the first equation in (S9) and (S13) versus u when w is replaced from the second equation in (S9) and (S13). The asymptote (shown by the dashed line) is the maximum value for the u where w becomes zero (which means that although the mode propagates without cut-off, the wave does not decay in the cladding and its size must be infinite). It can be seen that anisotropic cladding shifts the intersection to the lower values for u (higher w) without changing the asymptote position, so, anisotropic cladding leads to the faster decaying and consequently more confinement.

Fig. S4. The propagation constant of (a) 1D and (b) 2D nonmagnetic extreme-skin-depth (e-skid) waveguides in comparison with a conventional slab waveguide and an optical fiber with cladding permittivity of $\epsilon = 1.1$. The core size/radius is $\lambda_0/10$; $\epsilon_x = 1.1$ ($\epsilon_{\perp} = 1.1$) and $\epsilon_z = 20$. The light line in the core and the cladding is also plotted, which shows the wave propagates inside the core due to the total internal reflection and decays away outside the core. The propagation constant in anisotropic case is larger since the power confinement is more.

Propagation constant dispersion for both 1D and 2D extreme-skin-depth (e-skid) waveguides are displayed in fig. S4. Propagation constant is always between the light lines in the core and the cladding. It guarantees that the guided mode bounces off by total internal reflection inside the core and decays away in the cladding. At low frequencies the propagation constant is close to the light line in the cladding. This means that the mode is poorly confined inside the core, but as the frequency increases, more power is confined inside the core and the propagation constant asymptotically meets the light line inside the core. Since the power confinement is better for extreme-skin-depth (e-skid) waveguides in comparison with conventional dielectric waveguides, the propagation constant is larger in the entire spectrum.

Fig. S5. (a) Schematic representation of a practical extreme-skin-depth (e-skid) metamaterial fiber at optical telecommunication wavelength ($\lambda = 1550\text{nm}$). The metamaterial cladding consisting of germanium nanorods embedded in porous silica surrounding a silicon rod core. (b-g) Normalized simulated distribution of the electric (top plots) and magnetic (bottom plots) energy density of the waveguide when the core diameter is 0.15λ and germanium fill fraction is 62.5%. (b) & (c) Electric and magnetic energy density of the bare waveguide. The fraction of power inside the core to the total power (η) is 3% and the mode area is $43(\lambda/2n_{\text{core}})^2$. (d) & (e) Electric and magnetic energy density of the waveguide with anisotropic cladding ($\epsilon_x = \epsilon_y = 3.7$ and $\epsilon_z = 11.8$). In the homogenized limit, the calculated η , mode area, and effective index are 35%, $1.85(\lambda/2n_{\text{core}})^2$, and 2.13, respectively. (f) & (g) Electric and magnetic energy density of the practical waveguide surrounded by nanowires which achieves the required anisotropy. According to effective theory, it achieves the effective permittivity of the cladding of part D & E. The simulated results of η , mode area, and effective index are 40%, $1.61(\lambda/2n_{\text{core}})^2$, and 2.09, respectively in agreement with the effective medium calculations. Significantly better performance can be achieved by using higher index rods in the nanowire design.

Metamaterial fiber. We present a practical metamaterial fiber that utilizes extreme-skin-depth (e-skid) waveguides for enhanced confinement at the telecommunication wavelength (1550 nm). We consider a cylindrical core of silicon ($n=3.47$) with a diameter of 0.15λ and the cladding is composed of germanium ($n=4.3$) nanowires surrounded by ultra-low index porous silica [1] ($n=1.05$). The cladding size is three-times bigger than the core (Fig. S5.a). The germanium nanowire axis is parallel to the fiber axis and their periodicity as well as diameter is much smaller than the wavelength ($\Lambda = 20\text{nm}, D = 18\text{nm}$). Maxwell-Garnett method can be used to approximate the effective permittivity of the cladding. For the germanium filling fraction of $\rho = 0.625$, we can obtain strong anisotropy ($\epsilon_x = \epsilon_y = 3.7$ and $\epsilon_z = 11.8$). It is important to note that the dielectric constant along the propagation direction is large however the transverse dielectric constant is lesser than silicon preserving total internal reflection. A detailed theory of how the fundamental HE_{11} mode of any cylindrical structure is affected by the momentum transformation is presented in the supplementary information.

Fig S5. b & c show the simulation results of the electric and magnetic energy density, respectively, for the bare waveguide. Only 3% of the total power is inside the core and the mode area is $43A_0$. Fig. S5 d & e show the results when the anisotropic cladding is added. In these plots, we consider a homogenous effective medium cladding. In this case, the fraction of power inside the core to the total power (η) increases to 35% and the

mode area decreases to $1.85A_0$. The results for the practical nanorod realization ($\Lambda = 20\text{nm}, D = 18\text{nm}$) are plotted in fig. S5 f & g. In this case, $\eta = 40\%$ and the mode area is $1.61A_0$, which is in excellent agreement with the homogenized case.

Fig. S6. Electric field and energy density distribution of a extreme-skin-depth (e-skid) waveguide composed a cylindrical silicon core with a diameter of $0.15\lambda_0$ covered by a nonmagnetic anisotropic cladding ($\epsilon_{\perp 2} = 3.7$ and $\epsilon_z = 11.8$). (a) Analytical calculation. (b) Numerical calculation. The analytical calculation of η , mode area, and effective index are 34.6% , $1.94(\lambda / 2n_{\text{core}})^2$, and 2.126 , respectively. The simulated results are 26% , $2.58(\lambda / 2n_{\text{core}})^2$, and 2.15 , respectively. The slight deviations between analytical and simulated results arise due to a finite sized simulation domain.

We note that the nanowire design does not attain deeply subwavelength performance at 1.55 micron wavelength due to limitations in available high index media for the extreme-skin-depth (e-skid). However, we note the excellent agreement between the effective medium theory of 2D anisotropic claddings and the practical nanowire design. Therefore the cross-talk reduction over conventional designs presented in the earlier sections will hold true for the 2D design as well. We emphasize that at lower frequencies readily available high index media [2] can lead to deep subwavelength performance using both 1D multilayer and 2D nanowire designs. Comparison between analytical calculations explained above and the simulation results shown in fig. 6. There is a good agreement between the exact solution and numerical calculations. Fig. S7 shows the effect of anisotropic cladding on decaying of evanescent waves in the cladding for the structure in fig. S6.

Fig. S7. Normalized electric field distribution of the waveguide in fig. S6 in comparison with the case without cladding. The cladding causes fields to decay fast outside in the cladding signifying the transformation of optical momentum.

Figures of Merit

a) Mode Length/Area: To confirm the sub-diffraction nature of waveguide we calculated the mode area for waveguides, a common figure of merit for sub-diffraction devices [3]:

$$A_m = \frac{W_m}{\max\{W(r)\}} = \frac{1}{\max\{W(r)\}} \iint_{-\infty}^{\infty} W(r) d^2r \quad (\text{S14})$$

where W_m and $W(r)$ are electromagnetic energy and energy density, respectively, and the integration is calculated on the surface normal to the propagation direction. In fig. S8 and S9. a we compare the mode length/area of the 1D and 2D extreme-skin-depth (e-skid) waveguides with conventional waveguides. The mode size reduction using anisotropic cladding is clearly evident.

Fig. S8. Deep sub-diffraction performance: Irrespective of core size made of glass, one can always achieve sub-diffraction mode lengths with access to higher index media. This is difficult at optical frequencies but can be achieved by anisotropic metamaterials at THz and microwave frequencies.

Fig. S9. Comparison of (a) normalized mode area and (b) power inside the core fraction for the glass 2D waveguide with and without cladding versus core radius. The cladding permittivity in tangential and longitudinal direction is 1 and 20 , respectively. It is seen that the metamaterial cladding decreases

the mode area and also causes the power in the core to increase.

b) Power in the core: Note that the above definition of mode area essentially elucidates the peak power carried for a given input power in the waveguide. However, due to the inhomogeneous profile of the fields it becomes imperative to define the power carried within this mode area. For example, in the case of slot waveguides the evanescent tail carries a significant fraction of the power though the peak power in the slot is quite high. In fig. S9, we have evaluated both the mode area and power in the core for TCWs.

Fig. S10. The normalized magnetic field amplitude of the first (a) even (b) odd TM mode of coupled silicon slab waveguides with the slab size of 0.1λ and center-to-center separation of 0.5λ for silica (dashed red lines) and germanium-silica multilayer (solid blue lines) cladding with effective permittivity of $\epsilon_x = 4.8$ and $\epsilon_z = 11.9$.

c) Cross-talk: For dielectric waveguides it also becomes necessary to evaluate the cross talk once the size of the core decreases. This is important for understanding the potential for dense photonic integration. The major advantage of practical TCWs is the reduced cross talk.

Magnetic field of even and odd TM mode for a pair of coupled dielectric slab waveguides with size of $2a$ and separation of s surrounded by an anisotropic dielectric can be written as:

$$H_y^e = \begin{cases} A^e \cosh(k_{x2}^e x) e^{i\beta^e z} & |x| \leq s \\ \left[B^e \cos(k_{x1}^e x) + C^e \sin(k_{x1}^e x) \right] e^{i\beta^e z} & s < |x| \leq s + 2a \\ D^e e^{-k_{x2}^e x} e^{i\beta^e z} & s + 2a < |x| \end{cases} \quad (\text{S15})$$

$$H_y^o = \begin{cases} A^o \sinh(k_{x2}^o x) e^{i\beta^o z} & |x| \leq s \\ \left[B^o \cos(k_{x1}^o x) + C^o \sin(k_{x1}^o x) \right] e^{i\beta^o z} & s < |x| \leq s + 2a \\ D^o e^{-k_{x2}^o x} e^{i\beta^o z} & s + 2a < |x| \end{cases}$$

where momentums are defined similar to the single dielectric slab waveguide case shown in (S3). By matching boundary conditions and using dispersion relations, propagation constant for the even and odd modes can be calculated. We also analytically calculated propagation constant of the even and odd mode of coupled slab waveguides for half space and using symmetric and antisymmetric boundary conditions, respectively.

Fig. S11. (a) Simulation results of the tangential electric field for even (up) and odd (bottom) modes of the coupled silicon waveguides. The core size is 0.1λ and center-to-center separation of 0.5λ . The required anisotropic all-dielectric response can be obtained using a germanium/silica multilayer with germanium fill fraction of 0.6. The inset shows the fields when the cladding is bulk silica. Note that the fields at the midpoint of the waveguide is significantly decreased with the TCW, a necessary condition for low cross talk. The coupling length of the structure with silica cladding is only 6λ , but it becomes more than 132λ with the metamaterial cladding. The power fraction inside the core also increases from 25% to 45%. (b) The simulation results of symmetric (up) and antisymmetric (bottom) modes for the practical structure with germanium/silica multilayer cladding and a unit cell size of 40 nm. The fields show good correspondence with the effective medium approximation and the coupling length is 119λ .

Fig. S10 compares the normalized magnetic field of the even and odd mode of two coupled silicon slab waveguides with a size of $\lambda/10$ and the separation of $\lambda/2$ when the cladding is air (dashed lines) and when it is anisotropic (solid lines). In this case, the anisotropic dielectric permittivity is derived from effective medium theory (EMT) for silica-germanium multilayer at optical telecommunication wavelength (1550 nm) when the germanium fill fraction is 0.6. The EMT formulations for multilayers and nanorods are presented in the next section. The calculated propagation constant for the even and odd mode is $1.73k_0$ and $1.64k_0$, so the coupling length ($L_c \equiv \pi / |\beta^e - \beta^o|$) for silica cladding is less than 6λ , but when the waveguides are covered by the anisotropic cladding, the propagation constant of the two modes become closer ($\beta^e = 2.5490k_0$, $\beta^o = 2.5448k_0$) and the coupling length increases dramatically and becomes 119λ . For the practical multilayer cladding, $\beta^e = 2.5646k_0$ and $\beta^o = 2.5605k_0$ ascertained through full wave numerical simulations shown in fig. S10.

In fact, for large coupling length, the propagation constant and consequently fields amplitude of the even and odd modes

should be very close. Note that the magnetic field for odd mode is zero at the center. The coupling length increases if the wave decays fast outside the cores and become negligible at the center for the even mode. As we can see in fig. 10 and 11, the momentum transformation helps to reduce field amplitude at the center and decrease crosstalk.

We have also simulated the coupling length versus the cladding size shown in fig. S12. It is seen that the performance approaches the ideal infinite cladding for sizes approximately 3 times the core. The effect of core index on power confinement and crosstalk is displayed in fig. S13. As the core index increases, the effect of the anisotropic cladding on power confinement is modest but the cross talk in extreme-skin-depth (e-skid) waveguides is almost two orders of magnitude less than that in conventional waveguides.

Fig. S12. Normalized coupling length versus the cladding size. The core is silicon ($n=3.47$) with a size of 0.1λ and center to center separation of 0.5λ . When the cladding is anisotropic ($\epsilon_x = 4.8$ and $\epsilon_z = 11.9$) the coupling length is much larger than that when the cladding is isotropic ($\epsilon = 4.8$). The waveguides are surrounded by glass.

Practical realization 1D and 2D. We discuss how to practically achieve these momentum transformations. Firstly, we argue that no naturally occurring medium has a strong anisotropy and the the maximum contrast between permittivity tensor elements is low for natural dielectrics (e.g. TiO_2) as well as artificial polymers [8]. Thus we cannot use natural dielectrics to preserve total internal reflection with a glass or silicon waveguide core interface while simultaneously increasing the momentum of evanescent waves. However, we can realize this extreme anisotropy by artificially structured media using available lossless dielectrics.

One practical way for realization of extreme anisotropic metamaterials is embedding periodic thin high index nanorods in a low index host dielectric such that the periodicity is much lower than the operating wavelength to ensure that the structure behaves as a homogenous material and it is far away from its band-gap. Effective relative permittivity normal ($\epsilon_\perp$) and parallel ($\epsilon_\parallel$) to nanorods axis for subwavelength conditions is independent to the periodicity and is calculated using Maxwell-Garnett approximation as [4]:

$$\epsilon_\parallel = \rho\epsilon_d + (1-\rho)\epsilon_h \quad (\text{S16})$$

$$\epsilon_\perp = \frac{(1+\rho)\epsilon_d\epsilon_h + (1-\rho)\epsilon_h^2}{(1-\rho)\epsilon_d + (1+\rho)\epsilon_h}$$

where ϵ_d and ϵ_h are permittivity of the dielectric nanorods and host, respectively, and ρ is the fill-fraction of a nanorods in its unit-cell. According to (S16), thin high index nanorods in a low-index host behave effectively as extreme anisotropic

material to transform the desired momentum transformation in the cladding of the proposed cylindrical waveguides.

Another structure for realization of extreme anisotropic metamaterials is multilayer combination of a high index and low index dielectric. In this case, effective relative permittivity normal and parallel to the layers are calculated as [4]

$$\epsilon_\parallel = \rho\epsilon_d + (1-\rho)\epsilon_h \quad (\text{S17})$$

$$\epsilon_\perp = \frac{\epsilon_d\epsilon_h}{(1-\rho)\epsilon_d + \rho\epsilon_h}$$

This structure can be used for the momentum transformation in single and coupled slab waveguides as well as silicon strip waveguides.

Fig. S13. Role of core index, core size, cladding anisotropy and cladding size. (a) Comparing power confinement in the slab waveguide with anisotropic cladding ($\epsilon_x = 1.1$ and $\epsilon_z = 20$) as opposed to isotropic cladding. The core size is $a=0.1\lambda$ and the cladding size is three-times bigger than the core. It is seen that the power confinement in the core can be larger for the TCW even for high index cores. (b) Coupling length in the slab waveguide with anisotropic cladding ($\epsilon_x = 1.1$ and $\epsilon_z = 20$) is two orders of magnitude larger when compared to the isotropic cladding. The center-to-center separation of the coupled waveguides is $s=0.5\lambda$. The cladding size is three-times bigger than the core surrounded by air. The core size is $a=0.1\lambda$.

Comparison of different waveguide classes. The main figures of merit for a nano-waveguide is the length of propagation, confinement (mode area), power in the waveguide core (or within the mode area) and cross-talk between waveguides to

assess the possibility of dense integration. Now we compare these figures of merit for the most well-known optical waveguides:

Optical fibers: It is almost loss-less and is appropriate for long distances communications, but it is diffraction-limited so energy cannot be confined for sub-diffraction sizes. Moreover, the wave decays slowly inside the cladding thus it is not suitable for photonic integration.

Photonic crystal fibers: It works based on Bragg reflection not total internal reflection. Thus the power does not scatter at bends and power can be confined inside a low index core. However, when the core size and period numbers becomes very small, power leaks outside so photonic integration density and power confinement is limited [5].

Plasmonic waveguides: Metals, in contrast to dielectrics, can confine energy below diffraction limit of light, but metals are too lossy at optical frequencies and even very low-loss metallic waveguides [3] cannot propagate light more than several wavelengths.

Slot waveguides: In this type of dielectric waveguides [6], light can be confined in a low-index sub-diffraction slot surrounded by a high-index dielectric. This kind of waveguide is suitable for nonlinear applications because of its small mode area, but since only a small fraction of power is inside the core (in comparison with optical fibers with similar integrated density) cross-talk for slot-waveguides is higher [7], so they are not suitable for confined photonic circuits.

Extreme-skin-depth waveguides: On the other hand, by transforming the space around a low-index dielectric, it is possible to confine energy inside a sub-diffraction core. Since both peak of energy density inside the core and decaying in cladding increases, photonic integration density increases and it is also suitable for nonlinear processes.

Methods. We have used CST Microwave Studio [9], full-wave commercial software based on a finite integration technique (FIT), to obtain simulation results for the coupled slab waveguides reported in fig. S4, for the waveguides with arbitrary cross section in figs. S5, and for the metamaterial fibers in supplementary information. Hexahedral meshes with 40 lines per wavelength with lower mesh limit of 10 have been assigned for plots in fig. S4, and S5. For plots in fig. S5 the time-domain solver has been used. The simulation area in transverse plane has been $2\lambda \times 2\lambda$ for the waveguides with anisotropic cladding and $10\lambda \times 10\lambda$ for the bare waveguides. The mode area and power ratio have been estimated based on the template-based post-processing 2D integral calculations. For plots in fig. S4, we have used the frequency domain solver. The simulation domain for plots in fig. S4 length in y direction is only $\lambda/40$, terminated to magnetic boundary conditions to model infinite width for the slab waveguides. The open boundaries in x directions have been assigned one λ away from slab waveguides. The propagation constants have been derived from port information 1D results, and are in good agreement with analytical calculations. The simulations were made to converge with a maximum residual energy inside the calculation domain of 10^{-6} and 10^{-4} for time domain and frequency domain simulations, respectively.

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