Another mathematical process

In a magnetic field, the force working on a particle, $\boldsymbol{F}_p$, is expressed by the following equation[1],

$$\boldsymbol{F}_p = (\chi_p / \mu_0) V \boldsymbol{B}\cdot\nabla\boldsymbol{B} \quad (1)$$

And the x-component of $\boldsymbol{B}\cdot\nabla\boldsymbol{B}$ can be written by,

$$(\boldsymbol{B}\cdot\nabla\boldsymbol{B})_x = B_x(\partial B_x/\partial x) + B_y(\partial B_x/\partial y) + B_z(\partial B_x/\partial z) \quad (2)$$

Magnetic field property is given by Maxwell's equation as follows,

$$\mathrm{rot}\boldsymbol{H} = \boldsymbol{j} + \partial\boldsymbol{D}/\partial t, \quad (3)$$

where $\boldsymbol{H}$ is magnetic field. In our experimental system, the electric current density, $\boldsymbol{j}$, and the time derivative of electric flux density, $\partial\boldsymbol{D}/\partial t$, are zero. Hence, the upper equation is written as,

$$\mathrm{rot}\boldsymbol{H} = (\partial H_z/\partial y - \partial H_y/\partial z, \partial H_x/\partial z - \partial H_z/\partial x, \partial H_y/\partial x - \partial H_x/\partial y) = \boldsymbol{0}. \quad (4)$$

Here, notice y and z-component of rot$\boldsymbol{H}$. Equation 4 can be rewritten as follows,

$$\partial B_x/\partial z = \partial B_z/\partial x, \partial B_x/\partial y = \partial B_y/\partial x, \quad (5)$$

since the magnetic flux density, $\boldsymbol{B}$, is given by,

$$\boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{M} = \mu_0(1 + \chi)\boldsymbol{H} \quad (6)$$

where μ_0 and χ are the vacuum magnetic permeability and the magnetic susceptibility, respectively, and these have no anisotropy. Therefore, substitution of eq.5 into eq.2 yeilds

$$(\boldsymbol{B}\cdot\nabla\boldsymbol{B})_x = B_x(\partial B_x/\partial x) + B_y(\partial B_y/\partial x) + B_z(\partial B_z/\partial x) \quad (7)$$

The terms $B_y(\partial B_y/\partial x)$ and $B_z(\partial B_z/\partial x)$ appear in $(\boldsymbol{B}\cdot\nabla\boldsymbol{B})_x$. Figure 1 shows the comparison between the value of $B_y(\partial B_y/\partial x)$ and $B_z(\partial B_z/\partial x)$ calculated for the magnetic flux density simulated by SUPER MOMENT soft ware. The system of this simulation was the same one expressed in the text. The solid line shows $B_y(\partial B_y/\partial x)$ and the circle dots represent $B_z(\partial B_z/\partial x)$. They completely agreed with each other.

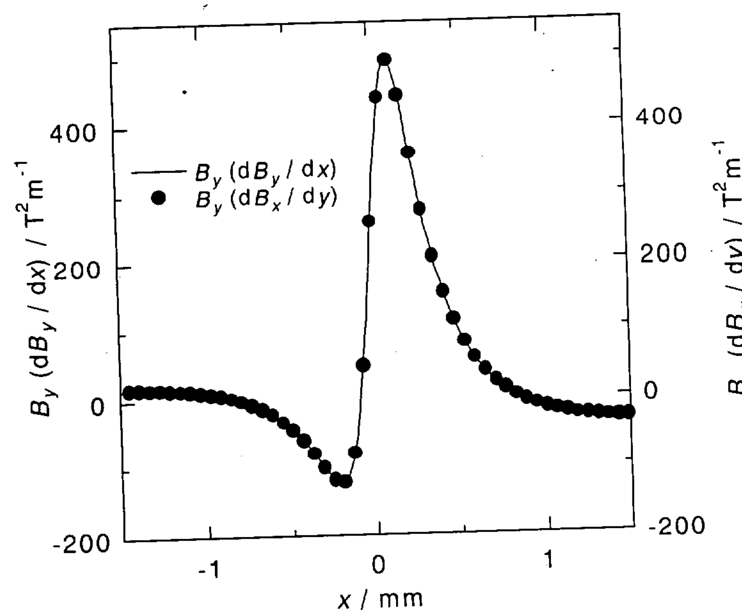

Figure 1 The comparison between $B_y(\partial B_y/\partial x)$ and $B_z(\partial B_z/\partial x)$ around the edges of the pair of magnets whose gap was kept 400 μm. They completely agreed with each other.

Reference

1. Pohl, H. A.; *DIELECTROPHORESIS*; Cambridge University Press: Cambridge, London, New York, Melbourne, 1978