Mitochondrial Transport in Axons with Multiple Branching Junctions: A Computational Study

Ivan A. Kuznetsov[a), (b] and Andrey V. Kuznetsov[c]

[a]Perelman School of Medicine, University of Pennsylvania, Philadelphia, PA 19104, USA

[b]Department of Bioengineering, University of Pennsylvania, Philadelphia, PA 19104, USA

[c]Dept. of Mechanical and Aerospace Engineering, North Carolina State University,

Raleigh, NC 27695-7910, USA; e-mail: avkuznet@ncsu.edu

Supplemental Materials

S1. Governing equations for the case of an axon with two asymmetric branches (Figs. 1b and 2b)

S1.1. Equations that express the conservation of the total length of mitochondria in various compartments for an axon with two asymmetric branches (Figs. 1b and 2b)

Equations that are analogous to Eqs. (1)-(42) but now for the asymmetric axon displayed in Fig. 1b and 2b are

$$L_1 \frac{dl_1^s}{dt} = p_s \left(j_{soma \to 1} + j_{2 \to 1} \right) - L_1 k_w l_1^s , \tag{S1}$$

$$L_1 \frac{dl_1^a}{dt} = j_{soma \to 1} - p_s j_{soma \to 1} - j_{1 \to 2} + \varepsilon L_1 k_w l_1^s , \tag{S2}$$

$$L_1 \frac{dl_1^r}{dt} = j_{2 \to 1} - p_s j_{2 \to 1} - j_{1 \to soma} + \left(1 - \varepsilon\right) L_1 k_w l_1^s , \tag{S3}$$

$$L_2 \frac{dl_2^s}{dt} = p_s \left(j_{1 \to 2} + j_{3 \to 2} + j_{9 \to 2} \right) - L_2 k_w l_2^s , \tag{S4}$$

$$L_2 \frac{dl_2^a}{dt} = j_{1 \to 2} - p_s j_{1 \to 2} - j_{2 \to 3} + \varepsilon L_2 k_w l_2^s , \tag{S5}$$

$$L_2 \frac{dl_2^r}{dt} = j_{3 \to 2} + j_{9 \to 2} - p_s \left(j_{3 \to 2} + j_{9 \to 2} \right) - j_{2 \to 1} + \left(1 - \varepsilon\right) L_2 k_w l_2^s , \tag{S6}$$

$$L_3 \frac{dl_3^s}{dt} = p_s \left(\varpi_1 j_{2 \to 3} + j_{4 \to 3} \right) - L_3 k_w l_3^s , \tag{S7}$$

$$L_3 \frac{dl_3^a}{dt} = \varpi_1 j_{2\to3} - \varpi_1 p_s j_{2\to3} - j_{3\to4} + \varepsilon L_3 k_w l_3^s , \tag{S8}$$

$$L_3 \frac{dl_3^r}{dt} = j_{4\to3} - p_s j_{4\to3} - j_{3\to2} + (1-\varepsilon) L_3 k_w l_3^s , \tag{S9}$$

$$L_4 \frac{dl_4^s}{dt} = p_s \left(j_{3\to4} + j_{5\to4} + j_{7\to4} \right) - L_4 k_w l_4^s , \tag{S10}$$

$$L_4 \frac{dl_4^a}{dt} = j_{3\to4} - p_s j_{3\to4} - j_{4\to5} + \varepsilon L_4 k_w l_4^s , \tag{S11}$$

$$L_4 \frac{dl_4^r}{dt} = j_{5\to4} + j_{7\to4} - p_s \left(j_{5\to4} + j_{7\to4} \right) - j_{4\to3} + (1-\varepsilon) L_4 k_w l_4^s , \tag{S12}$$

$$L_5 \frac{dl_5^s}{dt} = p_s \left(\varpi_2 j_{4\to5} + j_{6\to5} \right) - L_5 k_w l_5^s , \tag{S13}$$

$$L_5 \frac{dl_5^a}{dt} = \varpi_2 j_{4\to5} - \varpi_2 p_s j_{4\to5} - j_{5\to6} + \varepsilon L_5 k_w l_5^s , \tag{S14}$$

$$L_5 \frac{dl_5^r}{dt} = j_{6\to5} - p_s j_{6\to5} - j_{5\to4} + (1-\varepsilon) L_5 k_w l_5^s , \tag{S15}$$

$$L_6 \frac{dl_6^s}{dt} = p_s \left(j_{5\to6} + j_{6a\to6r} \right) - L_6 k_w l_6^s , \tag{S16}$$

$$L_6 \frac{dl_6^a}{dt} = j_{5\to6} - p_s j_{5\to6} - j_{6a\to6r} + \varepsilon L_6 k_w l_6^s , \tag{S17}$$

$$L_6 \frac{dl_6^r}{dt} = j_{6a\to6r} - p_s j_{6a\to6r} - j_{6\to5} + (1-\varepsilon) L_6 k_w l_6^s , \tag{S18}$$

$$L_7 \frac{dl_7^s}{dt} = p_s \left((1-\varpi_2) j_{4\to5} + j_{8\to7} \right) - L_7 k_w l_7^s , \tag{S19}$$

$$L_7 \frac{dl_7^a}{dt} = (1-\varpi_2) j_{4\to5} - p_s (1-\varpi_2) j_{4\to5} - j_{7\to8} + \varepsilon L_7 k_w l_7^s , \tag{S20}$$

$$L_7 \frac{dl_7^r}{dt} = j_{8\to7} - p_s j_{8\to7} - j_{7\to4} + (1-\varepsilon) L_7 k_w l_7^s , \tag{S21}$$

$$L_8 \frac{dl_8^s}{dt} = p_s \left(j_{7\to8} + j_{8a\to8r} \right) - L_8 k_w l_8^s , \tag{S22}$$

$$L_8 \frac{dl_8^a}{dt} = j_{7\to 8} - p_s j_{7\to 8} - j_{8a\to 8r} + \varepsilon L_8 k_w l_8^s \,, \tag{S23}$$

$$L_8 \frac{dl_8^r}{dt} = j_{8a\to 8r} - p_s j_{8a\to 8r} - j_{8\to 7} + \left(1-\varepsilon\right) L_8 k_w l_8^s \,, \tag{S24}$$

$$L_9 \frac{dl_9^s}{dt} = \left(1-\varpi_1\right) p_s j_{2\to 3} + p_s j_{10\to 9} - L_9 k_w l_9^s \,, \tag{S25}$$

$$L_9 \frac{dl_9^a}{dt} = \left(1-\varpi_1\right) j_{2\to 3} - p_s \left(1-\varpi_1\right) j_{2\to 3} - j_{9\to 10} + \varepsilon L_9 k_w l_9^s \,, \tag{S26}$$

$$L_9 \frac{dl_9^r}{dt} = j_{10\to 9} - p_s j_{10\to 9} - j_{9\to 2} + \left(1-\varepsilon\right) L_9 k_w l_9^s \,, \tag{S27}$$

$$L_{10} \frac{dl_{10}^s}{dt} = p_s j_{9\to 10} + p_s j_{10a\to 10r} - L_{10} k_w l_{10}^s \,, \tag{S28}$$

$$L_{10} \frac{dl_{10}^a}{dt} = j_{9\to 10} - p_s j_{9\to 10} - j_{10a\to 10r} + \varepsilon L_{10} k_w l_{10}^s \,, \tag{S29}$$

$$L_{10} \frac{dl_{10}^r}{dt} = j_{10a\to 10r} - p_s j_{10a\to 10r} - j_{10\to 9} + \left(1-\varepsilon\right) L_{10} k_w l_{10}^s \,. \tag{S30}$$

Equations that are analogous to Eqs. (43)-(70) but now for the asymmetric axon displayed in Fig. 1b are as follows. Equations for anterograde fluxes (Fig. 2b) now are

$$j_{1\to 2} = v_a l_1^a \,, \tag{S31}$$

$$j_{2\to 3} = v_a l_2^a \,, \tag{S32}$$

$$j_{3\to 4} = v_a l_3^a \,, \tag{S33}$$

$$j_{4\to 5} = v_a l_4^a \,, \tag{S34}$$

$$j_{5\to 6} = v_a l_5^a \,, \tag{S35}$$

$$j_{7\to 8} = v_a l_7^a \,, \tag{S36}$$

$$j_{9\to 10} = v_a l_9^a \,. \tag{S37}$$

Equations for retrograde fluxes (Fig. 2b) now are

$$j_{1\to soma} = v_r l_1^r \,, \tag{S38}$$

$$j_{2\to1} = v_r l_2^r \,,\tag{S39}$$

$$j_{3\to2} = v_r l_3^r \,,\tag{S40}$$

$$j_{4\to3} = v_r l_4^r \,,\tag{S41}$$

$$j_{5\to4} = v_r l_5^r \,,\tag{S42}$$

$$j_{6\to5} = v_r l_6^r \,,\tag{S43}$$

$$j_{7\to4} = v_r l_7^r \,,\tag{S44}$$

$$j_{8\to7} = v_r l_8^r \,,\tag{S45}$$

$$j_{9\to2} = v_r l_9^r \,,\tag{S46}$$

$$j_{10\to9} = v_r l_{10}^r \,.\tag{S47}$$

The turn-around fluxes (Fig. 2b) now are simulated as follows:

$$j_{6a\to6r} = v_a l_6^a \,,\tag{S48}$$

$$j_{8a\to8r} = v_a l_8^a \,,\tag{S49}$$

$$j_{10a\to10r} = v_a l_{10}^a \,.\tag{S50}$$

S1.2. Model of mean age and age density distributions of mitochondria in the demand sites for the asymmetric axon displayed in Figs. 1b and 2b

Equations that are analogous to Eqs. (74)-(201) but now for the asymmetric axon displayed in Figs. 1b and 2b are (obtained using the method described in Anderson (1983))

$$b_{1,1} = -L_1 k_w l_1^s / \left(L_1 l_1^s \right),\tag{S51}$$

$$b_{N+1,N+1} = -\left(p_s j_{soma\to1} + j_{1\to2} \right) / \left(L_1 l_1^a \right),\tag{S52}$$

$$b_{2N+1,2N+1} = -\left(p_s j_{2\to1} + j_{1\to soma} \right) / \left(L_1 l_1^r \right),\tag{S53}$$

$$b_{N+1,1} = \varepsilon L_1 k_w l_1^s / \left(L_1 l_1^s \right),\tag{S54}$$

$$b_{1,N+1} = p_s j_{soma\to 1} / \left(L_1 l_1^a \right), \tag{S55}$$

$$b_{2N+1,1} = \left(1-\varepsilon\right) L_1 k_w l_1^s / \left(L_1 l_1^s \right), \tag{S56}$$

$$b_{1,2N+1} = p_s j_{2\to 1} / \left(L_1 l_1^r \right), \tag{S57}$$

$$b_{N+2,N+1} = j_{1\to 2} / \left(L_1 l_1^a \right), \tag{S58}$$

$$b_{2N+1,2N+2} = j_{2\to 1} / \left(L_2 l_2^r \right), \tag{S59}$$

$$b_{2,2} = -L_2 k_w l_2^s / \left(L_2 l_2^s \right), \tag{S60}$$

$$b_{N+2,N+2} = -\left(p_s j_{1\to 2} + j_{2\to 3} \right) / \left(L_2 l_2^a \right), \tag{S61}$$

$$b_{2N+2,2N+2} = -\left(p_s j_{3\to 2} + p_s j_{9\to 2} + j_{2\to 1} \right) / \left(L_2 l_2^r \right), \tag{S62}$$

$$b_{N+2,2} = \varepsilon L_2 k_w l_2^s / \left(L_2 l_2^s \right), \tag{S63}$$

$$b_{2,N+2} = p_s j_{1\to 2} / \left(L_2 l_2^a \right), \tag{S64}$$

$$b_{2N+2,2} = \left(1-\varepsilon\right) L_2 k_w l_2^s / \left(L_2 l_2^s \right), \tag{S65}$$

$$b_{2,2N+2} = p_s \left(j_{3\to 2} + j_{9\to 2} \right) / \left(L_2 l_2^r \right), \tag{S66}$$

$$b_{N+9,N+2} = \left(1-\varpi_1\right) j_{2\to 3} / \left(L_2 l_2^a \right), \tag{S67}$$

$$b_{N+3,N+2} = \varpi_1 j_{2\to 3} / \left(L_2 l_2^a \right), \tag{S68}$$

$$b_{3,3} = -L_3 k_w l_3^s / \left(L_3 l_3^s \right), \tag{S69}$$

$$b_{N+3,N+3} = -\left(\varpi_1 p_s j_{2\to 3} + j_{3\to 4} \right) / \left(L_3 l_3^a \right), \tag{S70}$$

$$b_{2N+3,2N+3} = -\left(p_s j_{4\to 3} + j_{3\to 2} \right) / \left(L_3 l_3^r \right), \tag{S71}$$

$$b_{N+3,3} = \varepsilon L_3 k_w l_3^s / \left(L_3 l_3^s \right), \tag{S72}$$

$$b_{3,N+3} = \varpi_1 p_s j_{2\to 3} / \left(L_3 l_3^a \right), \tag{S73}$$

$$b_{2N+3,3} = (1-\varepsilon) L_3 k_w l_3^s / \left(L_3 l_3^s \right), \tag{S74}$$

$$b_{3,2N+3} = p_s j_{4\to3} / \left(L_3 l_3^r \right), \tag{S75}$$

$$b_{N+4,N+3} = j_{3\to4} / \left(L_3 l_3^a \right), \tag{S76}$$

$$b_{4,N+4} = p_s j_{3\to4} / \left(L_4 l_4^a \right), \tag{S77}$$

$$b_{4,4} = -L_4 k_w l_4^s / \left(L_4 l_4^s \right), \tag{S78}$$

$$b_{N+4,N+4} = -\left(p_s j_{3\to4} + j_{4\to5} \right) / \left(L_4 l_4^a \right), \tag{S79}$$

$$b_{N+5,N+4} = \varpi_2 j_{4\to5} / \left(L_4 l_4^a \right), \tag{S80}$$

$$b_{N+7,N+4} = (1-\varpi_2) j_{4\to5} / \left(L_4 l_4^a \right), \tag{S81}$$

$$b_{2N+4,2N+4} = -\left(p_s j_{5\to4} + p_s j_{7\to4} + j_{4\to3} \right) / \left(L_4 l_4^r \right), \tag{S82}$$

$$b_{N+4,4} = \varepsilon L_4 k_w l_4^s / \left(L_4 l_4^s \right), \tag{S83}$$

$$b_{2N+4,4} = (1-\varepsilon) L_4 k_w l_4^s / \left(L_4 l_4^s \right), \tag{S84}$$

$$b_{4,2N+4} = p_s \left(j_{5\to4} + j_{7\to4} \right) / \left(L_4 l_4^r \right), \tag{S85}$$

$$b_{2N+3,2N+4} = j_{4\to3} / \left(L_4 l_4^r \right), \tag{S86}$$

$$b_{2N+2,2N+3} = j_{3\to2} / \left(L_3 l_3^r \right), \tag{S87}$$

$$b_{N+6,N+5} = j_{5\to6} / \left(L_5 l_5^a \right), \tag{S88}$$

$$b_{5,5} = -L_5 k_w l_5^s / \left(L_5 l_5^s \right), \tag{S89}$$

$$b_{N+5,N+5} = -\left(\varpi_2 p_s j_{4\to5} + j_{5\to6} \right) / \left(L_5 l_5^a \right), \tag{S90}$$

$$b_{2N+5,2N+5} = -\left(p_s j_{6\to5} + j_{5\to4} \right) / \left(L_5 l_5^r \right), \tag{S91}$$

$$b_{2N+4,2N+5} = j_{5\to4} / \left(L_5 l_5^r \right), \tag{S92}$$

$$b_{N+5,5} = \varepsilon L_5 k_w l_5^s / \left(L_5 l_5^s \right), \tag{S93}$$

$$b_{5,N+5} = \varpi_2 p_s j_{4\to5} / \left(L_5 l_5^a \right), \tag{S94}$$

$$b_{2N+5,5} = \left(1-\varepsilon\right) L_5 k_w l_5^s / \left(L_5 l_5^s \right), \tag{S95}$$

$$b_{5,2N+5} = p_s j_{6\to5} / \left(L_5 l_5^r \right), \tag{S96}$$

$$b_{6,N+6} = p_s j_{5\to6} / \left(L_6 l_6^a \right), \tag{S97}$$

$$b_{6,6} = -L_6 k_w l_6^s / \left(L_6 l_6^s \right), \tag{S98}$$

$$b_{N+6,N+6} = -\left(p_s j_{5\to6} + j_{6a\to6r} \right) / \left(L_6 l_6^a \right), \tag{S99}$$

$$b_{2N+6,2N+6} = -\left(p_s j_{6a\to6r} + j_{6\to5} \right) / \left(L_6 l_6^r \right), \tag{S100}$$

$$b_{N+6,6} = \varepsilon L_6 k_w l_6^s / \left(L_6 l_6^s \right), \tag{S101}$$

$$b_{6,2N+6} = p_s j_{6a\to6r} / \left(L_6 l_6^r \right), \tag{S102}$$

$$b_{2N+6,6} = \left(1-\varepsilon\right) L_6 k_w l_6^s / \left(L_6 l_6^s \right), \tag{S103}$$

$$b_{2N+5,2N+6} = j_{6\to5} / \left(L_6 l_6^r \right), \tag{S104}$$

$$b_{2N+6,N+6} = j_{6a\to6r} / \left(L_6 l_6^a \right), \tag{S105}$$

$$b_{7,N+7} = \left(1-\varpi_2\right) p_s j_{4\to5} / \left(L_7 l_7^a \right), \tag{S106}$$

$$b_{7,7} = -L_7 k_w l_7^s / \left(L_7 l_7^s \right), \tag{S107}$$

$$b_{N+7,N+7} = -\left(\left(1-\varpi_2\right) p_s j_{4\to5} + j_{7\to8} \right) / \left(L_7 l_7^a \right), \tag{S108}$$

$$b_{N+8,N+7} = j_{7\to8} / \left(L_7 l_7^a \right), \tag{S109}$$

$$b_{2N+7,2N+7} = -\left(p_s j_{8\to7} + j_{7\to4} \right) / \left(L_7 l_7^r \right), \tag{S110}$$

$$b_{7,2N+7} = p_s j_{8\to7} / \left(L_7 l_7^r \right), \tag{S111}$$

$$b_{N+7,7} = \varepsilon L_7 k_w l_7^s / \left(L_7 l_7^s \right), \tag{S112}$$

$$b_{2N+7,7} = \left(1-\varepsilon\right) L_7 k_w l_7^s / \left(L_7 l_7^s \right), \tag{S113}$$

$$b_{2N+4,2N+7} = j_{7\to 4} / \left(L_7 l_7^r \right), \tag{S114}$$

$$b_{8,N+8} = p_s j_{7\to 8} / \left(L_8 l_8^a \right), \tag{S115}$$

$$b_{8,8} = -L_8 k_w l_8^s / \left(L_8 l_8^s \right), \tag{S116}$$

$$b_{N+8,N+8} = -\left(p_s j_{7\to 8} + j_{8a\to 8r} \right) / \left(L_8 l_8^a \right), \tag{S117}$$

$$b_{2N+8,2N+8} = -\left(p_s j_{8a\to 8r} + j_{8\to 7} \right) / \left(L_8 l_8^r \right), \tag{S118}$$

$$b_{8,2N+8} = p_s j_{8a\to 8r} / \left(L_8 l_8^r \right), \tag{S119}$$

$$b_{2N+7,2N+8} = j_{8\to 7} / \left(L_8 l_8^r \right), \tag{S120}$$

$$b_{N+8,8} = \varepsilon L_8 k_w l_8^s / \left(L_8 l_8^s \right), \tag{S121}$$

$$b_{2N+8,8} = \left(1-\varepsilon\right) L_8 k_w l_8^s / \left(L_8 l_8^s \right), \tag{S122}$$

$$b_{2N+8,N+8} = j_{8a\to 8r} / \left(L_8 l_8^a \right), \tag{S123}$$

$$b_{9,N+9} = \left(1-\varpi_1\right) p_s j_{2\to 3} / \left(L_9 l_9^a \right), \tag{S124}$$

$$b_{9,9} = -L_9 k_w l_9^s / \left(L_9 l_9^s \right), \tag{S125}$$

$$b_{N+9,N+9} = -\left(\left(1-\varpi_1\right) p_s j_{2\to 3} + j_{9\to 10} \right) / \left(L_9 l_9^a \right), \tag{S126}$$

$$b_{2N+9,2N+9} = -\left(p_s j_{10\to 9} + j_{9\to 2} \right) / \left(L_9 l_9^r \right), \tag{S127}$$

$$b_{2N+2,2N+9} = j_{9\to 2} / \left(L_9 l_9^r \right), \tag{S128}$$

$$b_{N+9,9} = \varepsilon L_9 k_w l_9^s / \left(L_9 l_9^s \right), \tag{S129}$$

$$b_{2N+9,9} = \left(1-\varepsilon\right) L_9 k_w l_9^s / \left(L_9 l_9^s \right), \tag{S130}$$

$$b_{9,2N+9} = p_s j_{10\to 9} / \left(L_9 l_9^r \right), \tag{S131}$$

$$b_{N+10,N+9} = j_{9\to 10} / \left(L_9 l_9^a \right), \tag{S132}$$

$$b_{10,N+10} = p_s j_{9\to 10} / \left(L_{10} l_{10}^a \right), \tag{S133}$$

$$b_{10,10} = -L_{10} k_w l_{10}^s / \left(L_{10} l_{10}^s \right), \tag{S134}$$

$$b_{N+10,N+10} = -\left(p_s j_{9\to 10} + j_{10a\to 10r} \right) / \left(L_{10} l_{10}^a \right), \tag{S135}$$

$$b_{2N+10,2N+10} = -\left(p_s j_{10a\to 10r} + j_{10\to 9} \right) / \left(L_{10} l_{10}^r \right), \tag{S136}$$

$$b_{2N+9,2N+10} = j_{10\to 9} / \left(L_{10} l_{10}^r \right), \tag{S137}$$

$$b_{N+10,10} = \varepsilon L_{10} k_w l_{10}^s / \left(L_{10} l_{10}^s \right), \tag{S138}$$

$$b_{2N+10,10} = \left(1-\varepsilon \right) L_{10} k_w l_{10}^s / \left(L_{10} l_{10}^s \right), \tag{S139}$$

$$b_{10,2N+10} = p_s j_{10a\to 10r} / \left(L_{10} l_{10}^r \right), \tag{S140}$$

$$b_{2N+10,N+10} = j_{10a\to 10r} / \left(L_{10} l_{10}^a \right), \tag{S141}$$

All other matrix B elements, except for those specified in Eqs. (S51)-(S141), are set to zero.

The only flux of mitochondria entering the axon is the anterograde flux from the soma to the compartment with anterograde mitochondria by the most proximal demand site, $j_{soma\to 1}$.

Our model assumes that all mitochondria that leave the axon (their flux is $j_{1\to soma}$) return to the soma for degradation, and none re-enter the axon. The mitochondrial age in our model is thus understood as the amount of time that has passed since the mitochondria entered the axon.

The $(N+1)^{\text{th}}$ element of vector $\mathbf{u}$ is obtained from the following equation:

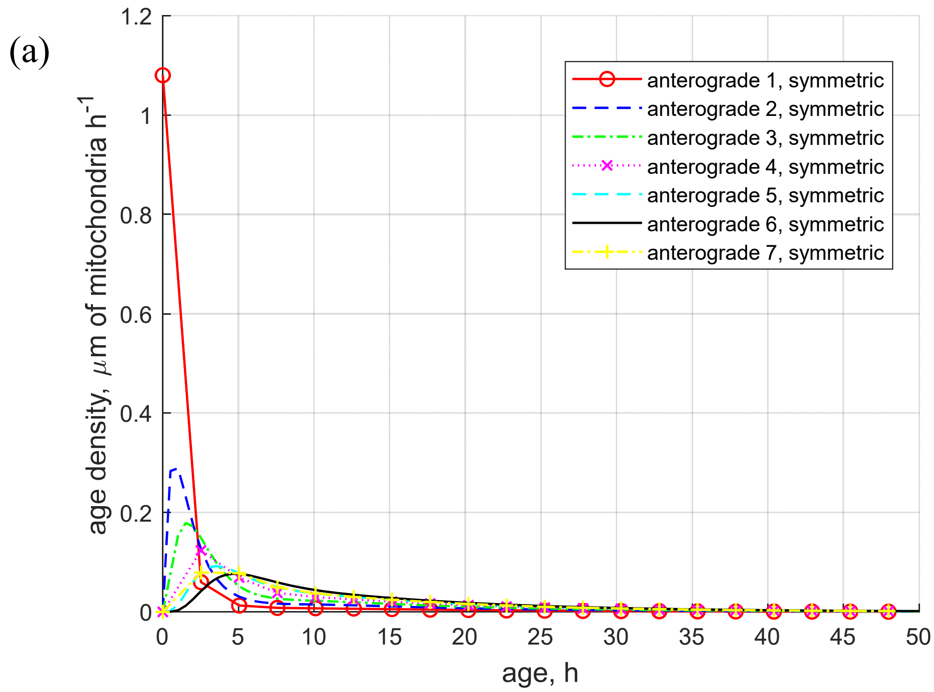
$$u_{N+1} = j_{soma\to 1}. \tag{S142}$$

As no other external fluxes enter the terminal,

$$u_i = 0 \quad (i=1,\ldots,N,N+2,\ldots,N+6,2N+1,\ldots,2N+10). \tag{S143}$$

S2. Supplementary figures

S2.1. Symmetric branched axon displayed in Figs. 1a and 2a



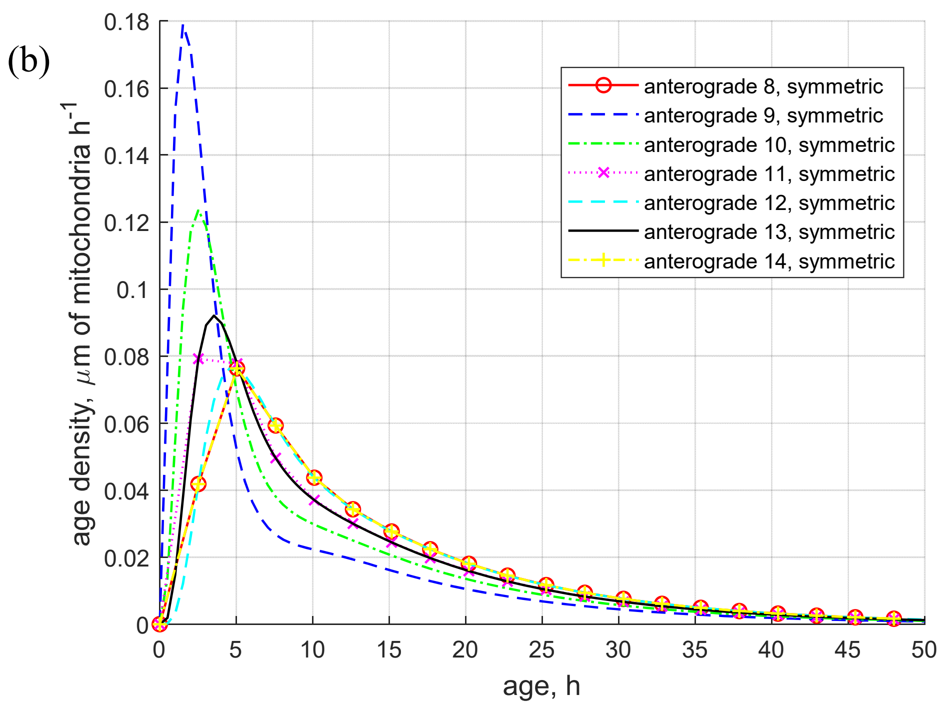
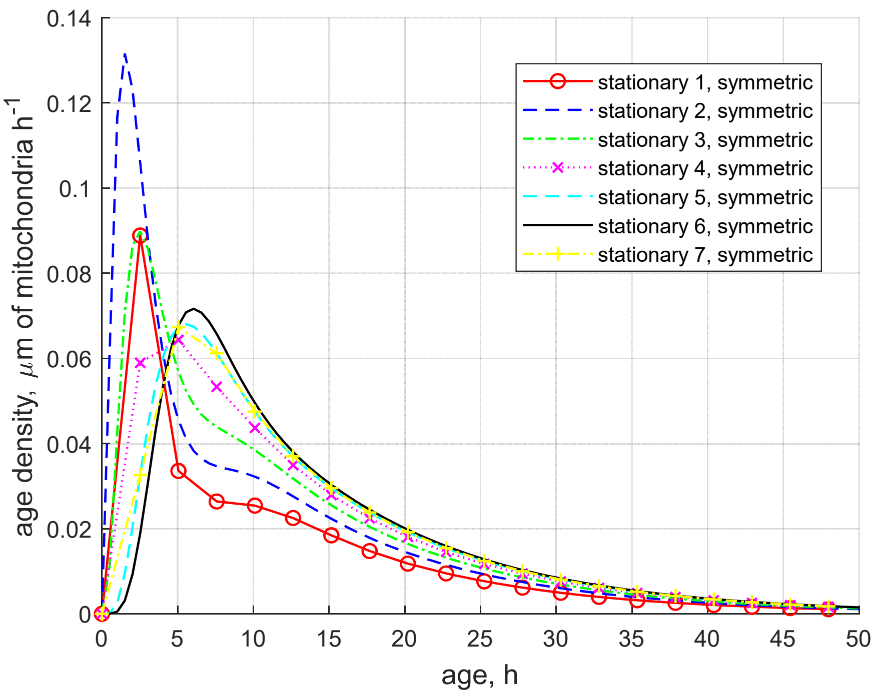


Fig. S1. Age density distributions of anterogradely-moving mitochondria in a symmetric branched axon across various demand sites. (a) Demand sites 1 to 7; (b) Demand sites 8 to 14. The existence of elongated tails in the positive direction in the age density distributions implies that there are mitochondria in each demand site that are considerably older than the mean age.

(a)



(b)

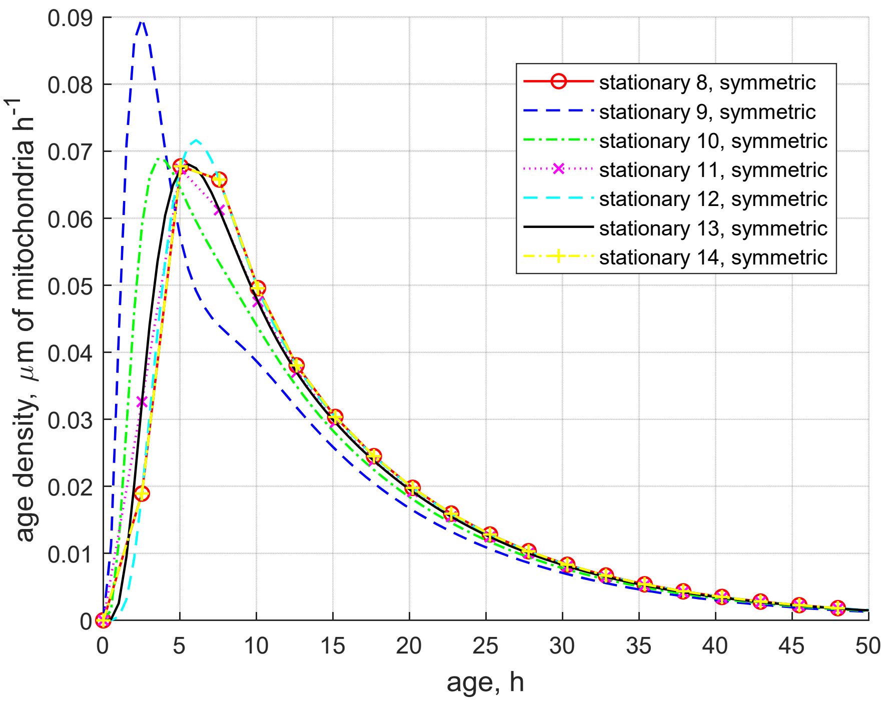
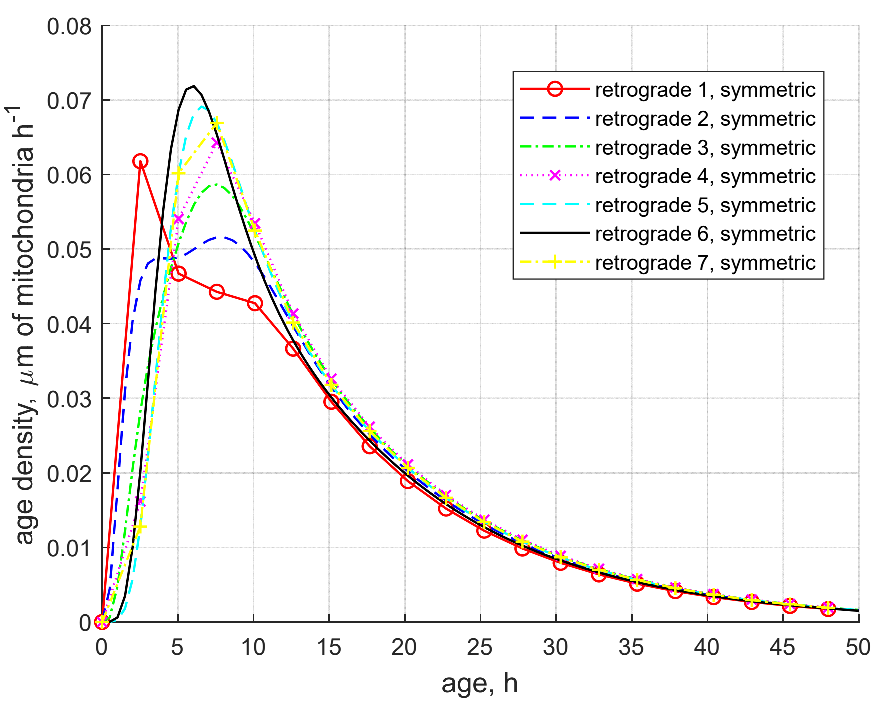


Fig. S2. Age density distributions of stationary mitochondria in a symmetric branched axon across various demand sites. (a) Demand sites 1 to 7; (b) Demand sites 8 to 14.

(a)



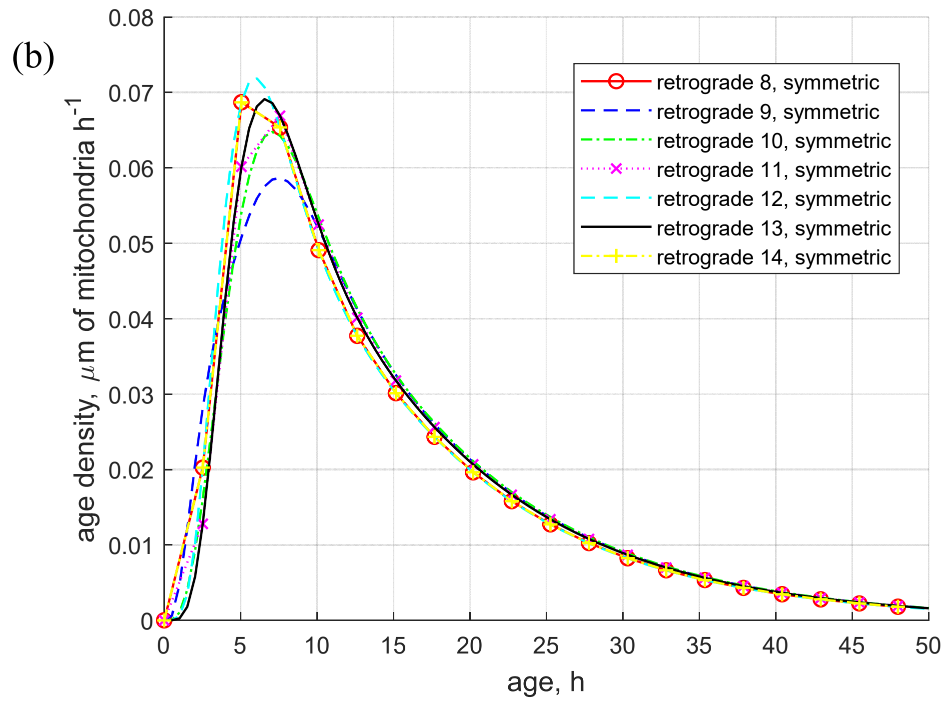
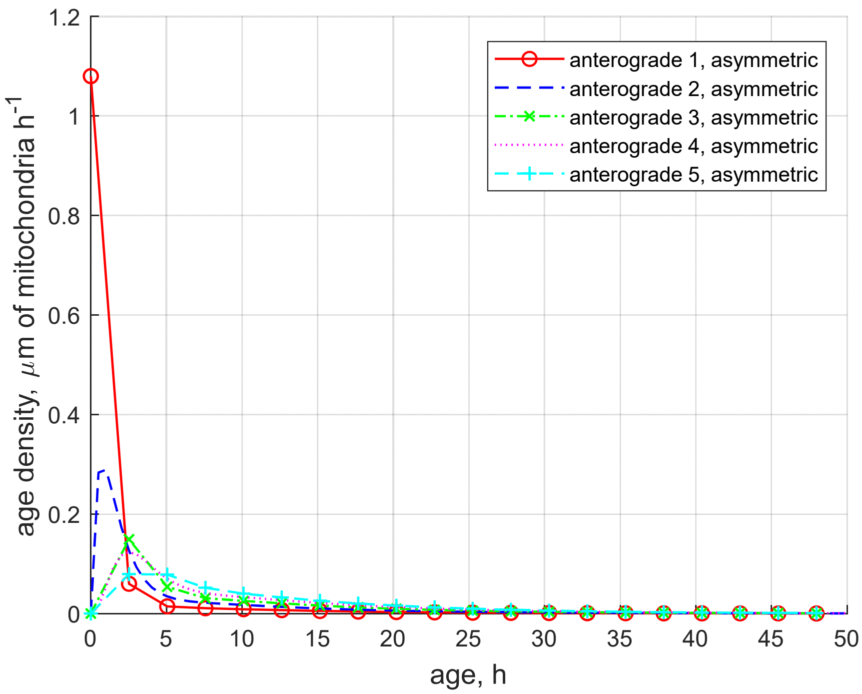


Fig. S3. Age density distributions of retrogradely-moving mitochondria in a symmetric branched axon across various demand sites. (a) Demand sites 1 to 7; (b) Demand sites 8 to 14.

S2.2. Asymmetric branched axon displayed in Figs. 1b and 2b

(a)



(b)

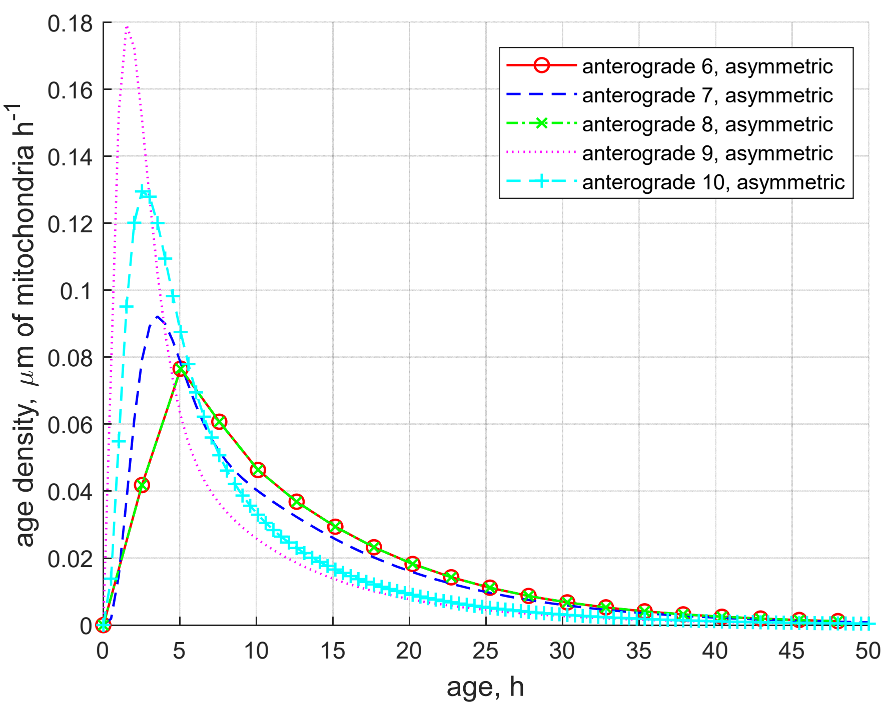
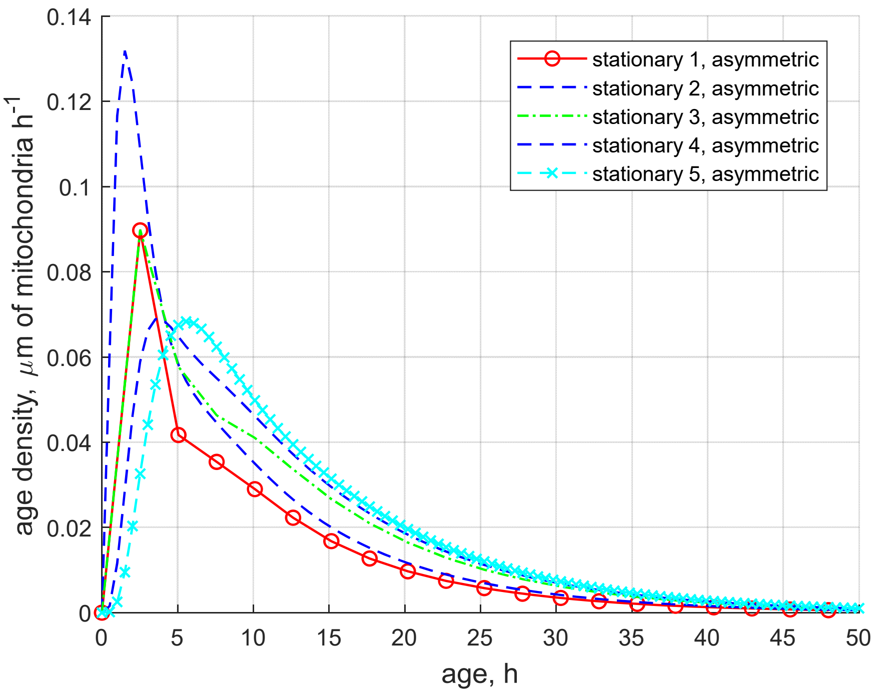


Fig. S4. Age density distributions of anterogradely-moving mitochondria in an asymmetric branched axon across various demand sites. (a) Demand sites 1 to 5; (b) Demand sites 6 to 10.

(a)


stationary 1, asymmetric
stationary 2, asymmetric
stationary 3, asymmetric
stationary 4, asymmetric
stationary 5, asymmetric
age density, μm of mitochondria h^{-1}
age, h

(b)

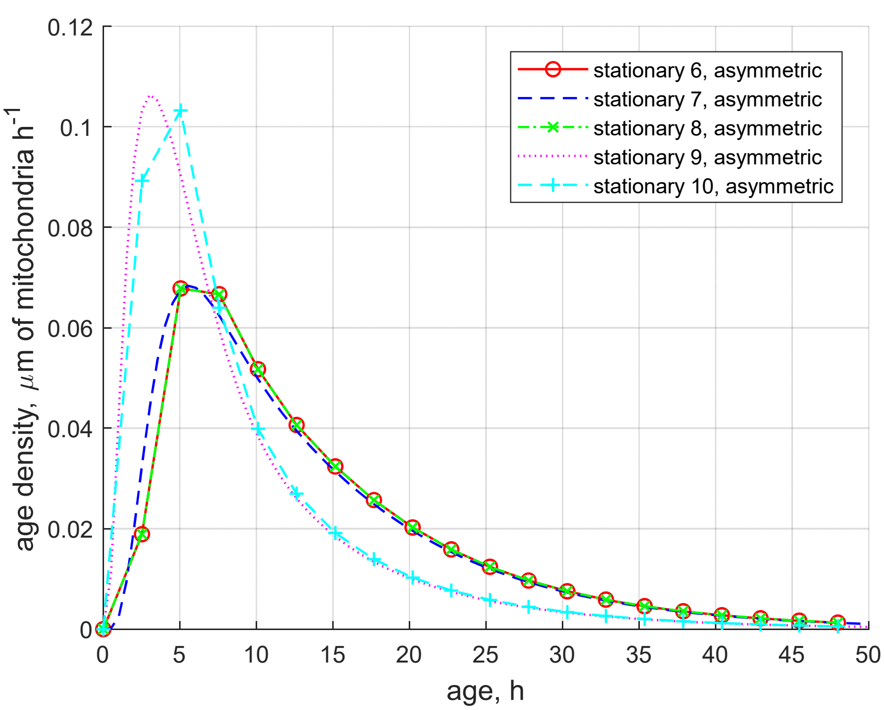
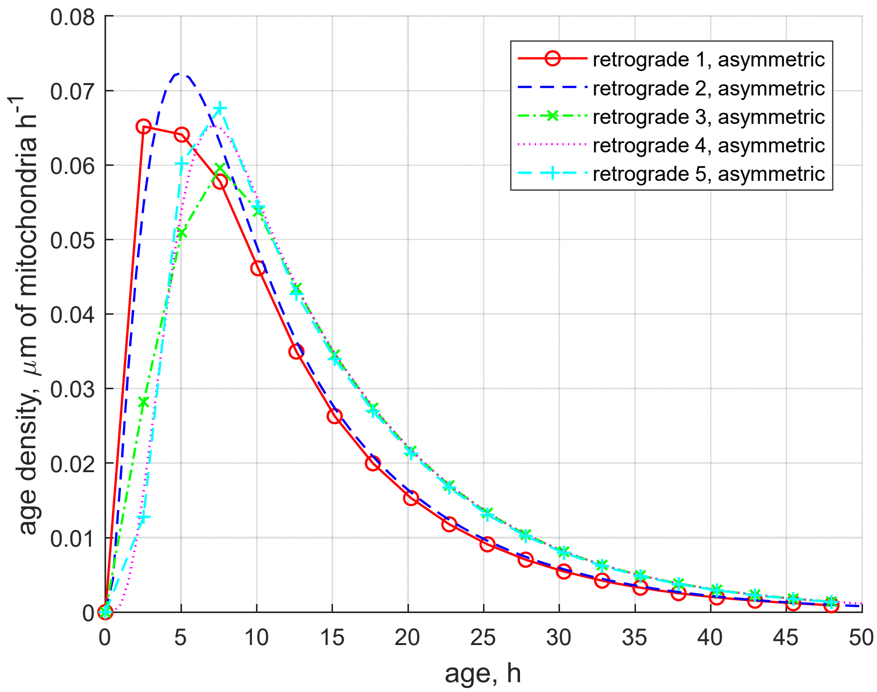

stationary 6, asymmetric
stationary 7, asymmetric
stationary 8, asymmetric
stationary 9, asymmetric
stationary 10, asymmetric
age density, μm of mitochondria h^{-1}
age, h

Fig. S5. Age density distributions of stationary mitochondria in an asymmetric branched axon across various demand sites. (a) Demand sites 1 to 5; (b) Demand sites 6 to 10.

(a)



(b)

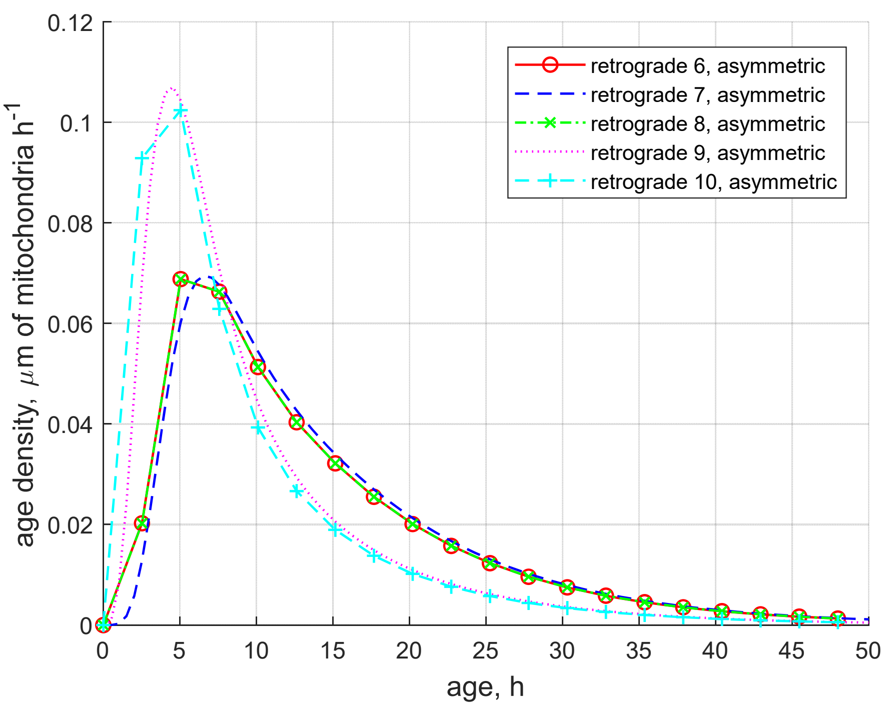


Fig. S6. Age density distributions of retrogradely-moving mitochondria in an asymmetric branched axon across various demand sites. (a) Demand sites 1 to 5; (b) Demand sites 6 to 10.

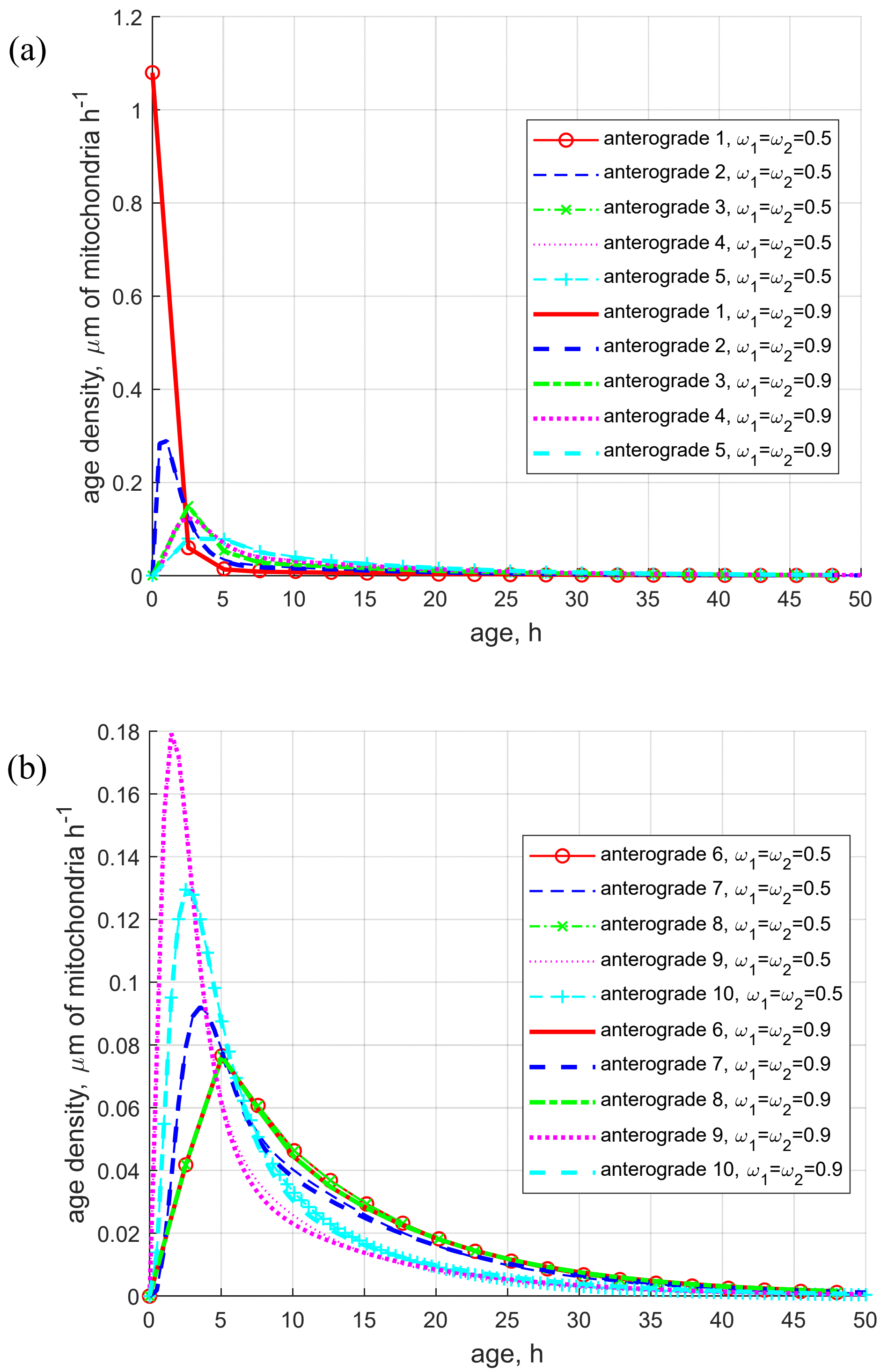
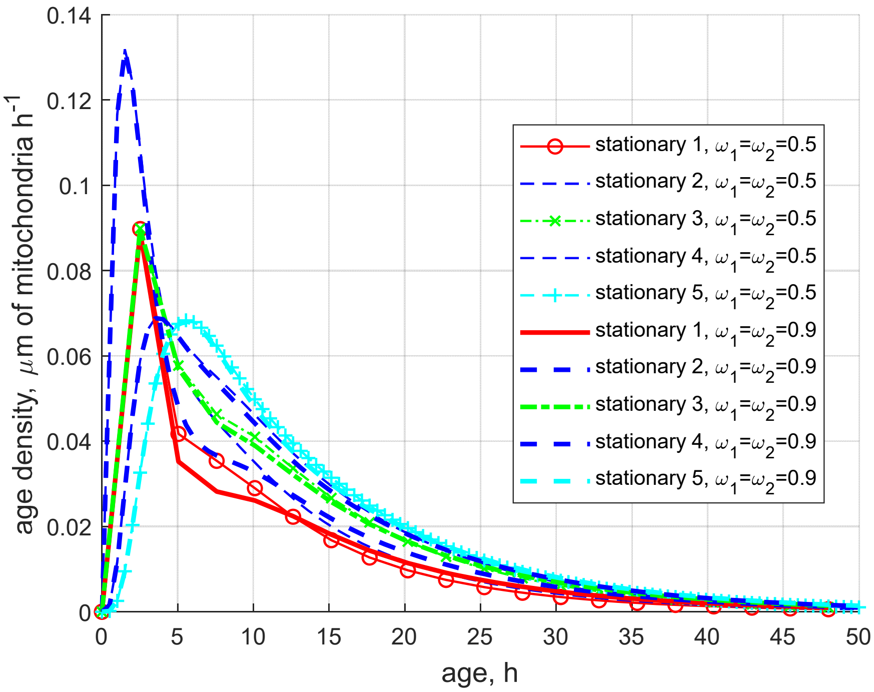


Fig. S7. Age density distributions of anterogradely-moving mitochondria in an asymmetric branched axon across various demand sites. Comparison of the cases $\omega_1 = \omega_2 = 0.5$ and $\omega_1 = \omega_2 = 0.9$ for (a) Demand sites 1 to 5; (b) Demand sites 6 to 10. In demand sites 9 and 10, the age density of older mitochondria is visibly higher in the case of $\omega_1 = \omega_2 = 0.9$ compared to $\omega_1 = \omega_2 = 0.5$. This is because in the case of $\omega_1 = \omega_2 = 0.9$, a greater number of mitochondria enter the upper (longer) branch.

(a)



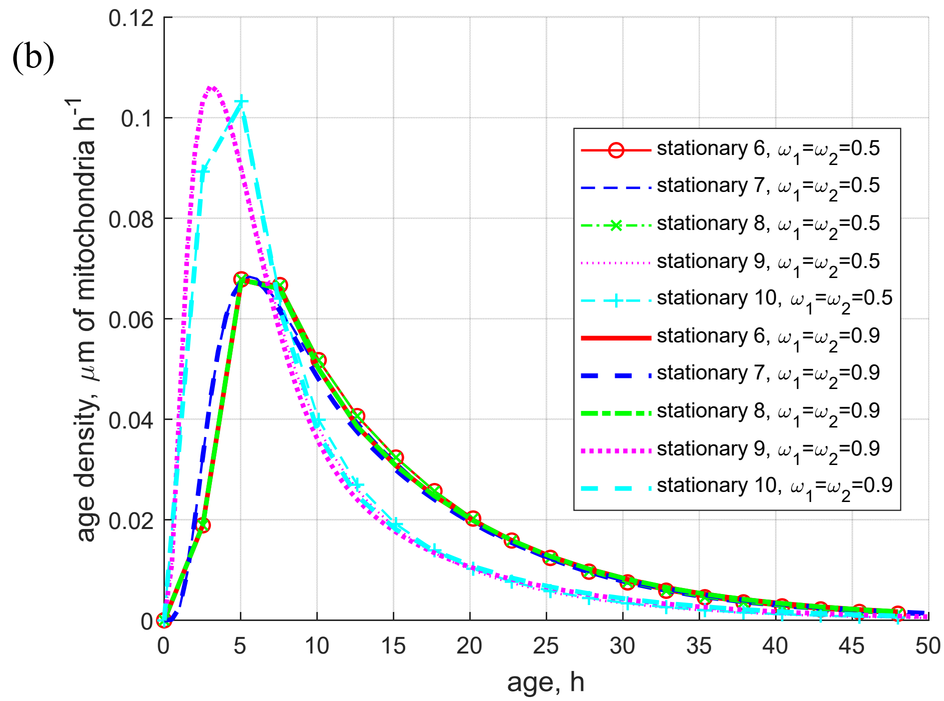


Fig. S8. Age density distributions of stationary mitochondria in an asymmetric branched axon across various demand sites. Comparison of the cases $\omega_1 = \omega_2 = 0.5$ and $\omega_1 = \omega_2 = 0.9$ for (a) Demand sites 1 to 5; (b) Demand sites 6 to 10.

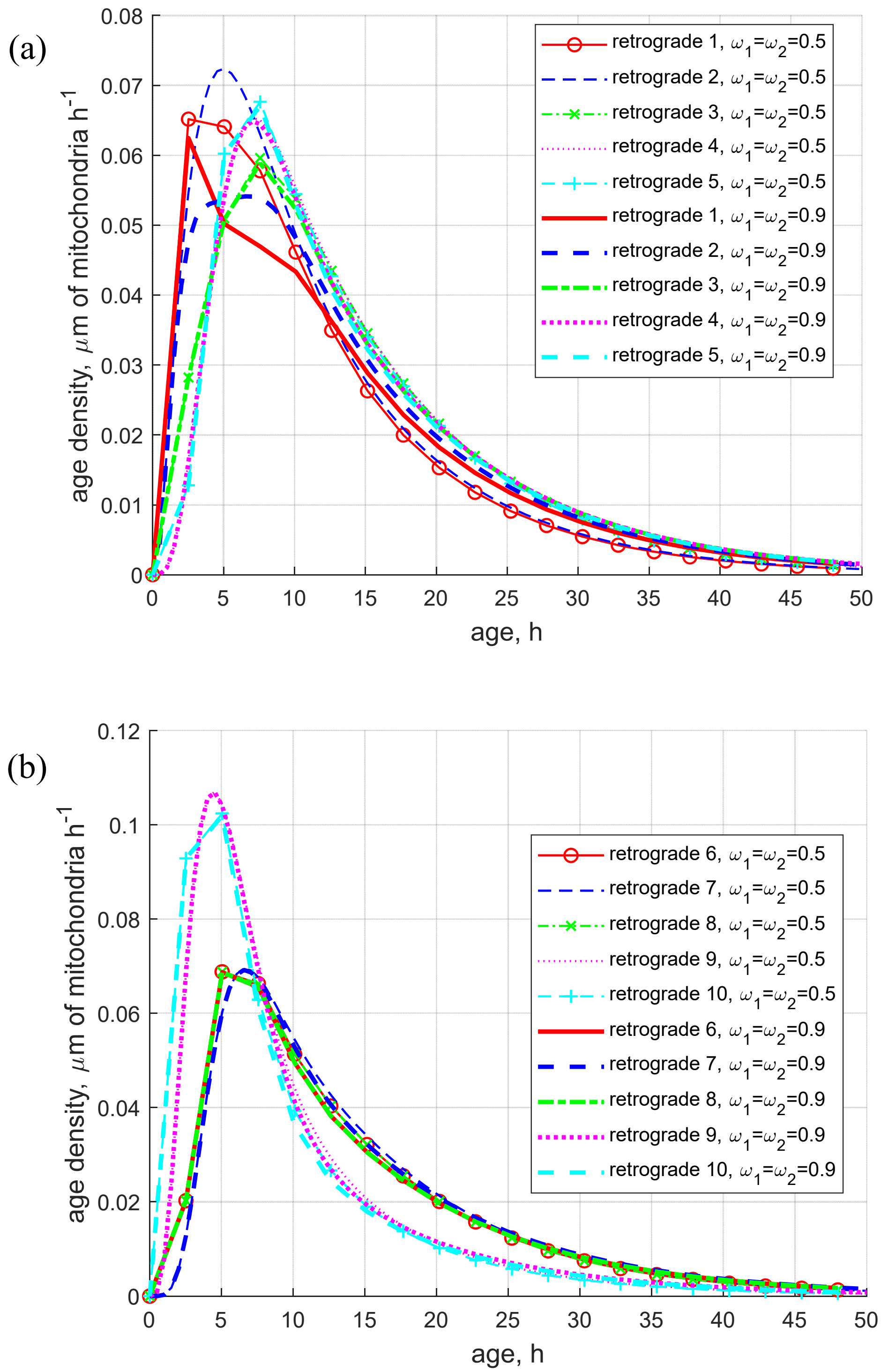

(a)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
age density, μm of mitochondria h-1
0
5
10
15
20
25
30
35
40
45
50
age, h
retrograde 1, ω1=ω2=0.5
retrograde 2, ω1=ω2=0.5
retrograde 3, ω1=ω2=0.5
retrograde 4, ω1=ω2=0.5
retrograde 5, ω1=ω2=0.5
retrograde 1, ω1=ω2=0.9
retrograde 2, ω1=ω2=0.9
retrograde 3, ω1=ω2=0.9
retrograde 4, ω1=ω2=0.9
retrograde 5, ω1=ω2=0.9
(b)
0.12
0.1
0.08
0.06
0.04
0.02
0
age density, μm of mitochondria h-1
0
5
10
15
20
25
30
35
40
45
50
age, h
retrograde 6, ω1=ω2=0.5
retrograde 7, ω1=ω2=0.5
retrograde 8, ω1=ω2=0.5
retrograde 9, ω1=ω2=0.5
retrograde 10, ω1=ω2=0.5
retrograde 6, ω1=ω2=0.9
retrograde 7, ω1=ω2=0.9
retrograde 8, ω1=ω2=0.9
retrograde 9, ω1=ω2=0.9
retrograde 10, ω1=ω2=0.9

Fig. S9. Age density distributions of retrogradely-moving mitochondria in an asymmetric branched axon across various demand sites. Comparison of the cases $\omega_1 = \omega_2 = 0.5$ and $\omega_1 = \omega_2 = 0.9$ for (a) Demand sites 1 to 5; (b) Demand sites 6 to 10.