Searching for Common Principles of Relativity & Quantum Mechanics

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Abstract The article reviews an approach or way of exploration for the unification theory of physics. Step by step, the exploration encompasses two postulates as principles which underpin both relativity and quantum mechanics: (I) deflection occurs between any two coordinates with motion relative to one another; (II) the space-time area (product of length and time) is invariant under deflection. Based on these postulates, the article interprets the conception of Einstein's theory for length contraction and time dilation in a simple manner; it presents a cause for stochastic phenomena in the quantum world and derives Schrödinger's equation from a complex function in which the deflection of space reduces to an amplitude probability distribution. In the last part, a new precession formula for the planets is presented and the results calculated using the new equation are detailed in a table.

Keywords Relativity, Lorentz factor, deflection, precession, Schrödinger Equation

Introduction

For nearly a century, people have unceasingly searched for a complete physical theory that unifies relativity and quantum mechanics (QM). Many unification theories have emerged in recent decades, such as string and M-theory, but in general we have always hoped to find a simple way to explain the physical world in a way that is understandable to ordinary people.

Some scientists and philosophers believe that there should be one set of simple and basic principles that governs physics, which may hide behind relativity and QM; however, because such principles have not been found for so long that they have begun to doubt whether a set of common principles (CP) even exist for relativity and QM.

We know that it is so hard to find the CP of relativity and quantum mechanics for a unification theory of physics while undergoing a century of searching by physicists. Is it possible to find other avenues outside habitual ways of thinking to establish or innovate CP?

The article introduces a method used in searching for common principles behind relativity and QM that is focused on simplicity, symmetry and the existence of a common foundation. This approach may be preliminary to a new basic theory, and it has not led to the unification of the four fundamental interactions of the universe. But, as an approach, it is presented herein.

A way of searching for CP via simplicity

1) **Raising the question.** Is there a CP for both relativity and QM or not?

2) **Determining the nature of the CP.** Supposing there is a CP of relativity and QM, then it follows that the existing primary principles of both relativity and QM can be derived from it (and are thus both no longer principles because they are inferred from the CP). In other words, both relativity and QM can be described with one set of principles—they share a common essence. The characteristics of CP, since it is a broad principle at a fundamental level of physics, is thought to be:

   - Simple with a physical meaning at its core
   - Mathematically expressible
   - With shared physical quantities hidden in the primary principles or results of relativity and QM respectively
   - Acceptable depending on whether the inferences are proven correct in experiments

3) **Orientation and scope of exploration.** Relativity consists of the special theory of relativity (STR) and general relativity (GR). First, according to the equivalence principle of GR, an infinitesimal inertial system may be established at each point of Riemannian space-time[1], and the exploration is mainly for a space of zero-curvature, which is the space specified in the STR. Second, Schrödinger's equation is the primary principle of QM. Therefore, the search for a CP focuses only on the STR and Schrödinger's equation.

4) **Searching for clues for CP.** What do the STR and Schrödinger's equation have in common and what could lie behind these principles or their results? The approach seeking simplicity starts by studying the contraction factor $\sqrt{1-u^2/c^2}$, i.e., $1/\gamma$ (γ is the Lorentz factor). As for Schrödinger's equation, it relates to the wave function $\psi e^{-i\theta}$ ($\theta = \omega t$)[2] in a complex coordinate system. Then, a link between the contraction factor $\sqrt{1-u^2/c^2}$ and θ in $\psi e^{-i\theta}$ is sought.

5) **Postulating.** Let $\sin\theta = u/c$ (based on Earth's aberration[3]) or $\sin\theta = \beta$ (β is the speed parameter), then $\sqrt{1-u^2/c^2} = \cos\theta$. The contraction factor becomes a deflection factor and the length contraction $l = l_0\sqrt{1-u^2/c^2}$ ($l = l_0/\gamma$) of the STR becomes $l = l_0 \cos\theta$. Therefore, it is postulated that the deflection of space-time occurs in a two coordinate system with an angle θ, where $\theta = \sin^{-1}\beta$, and that there may be a link between the deflection angle θ and the θ in $\psi e^{-i\theta}$. It is important that the postulate $\sin\theta = \beta$ is verified by Earth's aberration[3], which is a universal phenomenon in astronomical observations. Furthermore, another postulate derives from the product of length contraction in the equation $l = l_0/\gamma$ and time dilation $t = \gamma t_0$ (t and t_0 represent time interval Δt and Δt_0, respectively), i.e., $l \times t = l_0 \times t_0$, which indicates that there is an equality of the space-time area. Until now, these two postulates were described as candidates for a CP, but they are not yet qualified to be the CP. It is still not known what the interrelation might be between the deflection angle θ and the angle θ in $\psi e^{-i\theta}$ and how Schrödinger's equation could be derived mathematically and conceptually from these two postulates with concepts such as the deflection of space.

6) **Inferring and proving.** The two postulates stated above can be illustrated as in Fig. 1. (S') is a moving coordinate with proper length l' and proper time t'; (S) is an observing coordinate where l and t are results of the measurement in (S).

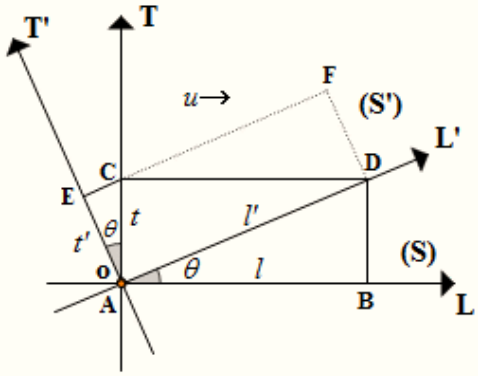

Fig.1 moving coordinate (S′) and observing coordinate (S) in deflection

In Fig. 1, the two postulates illustrated are annotated as follows:

(I) deflection of space-time ($\sin\theta = u/c$,);

(II) equality of space-time area (S_{ABDC} equals to S'_{ADFE}, $l \times t = l' \times t'$).

Then, we get $$l = l' \cos\theta \quad (1)$$

$$t = t'/\cos\theta \quad (2)$$

As per **(I):** $\sin\theta = u/c$, Eq.(1) and Eq.(2) can be rewritten as

$$l = l' \sqrt{1-u^2/c^2} \quad (3)$$

$$t = t'/\sqrt{1-u^2/c^2} \quad (4)$$

It is therefore found that Eq. (3) and Eq. (4) give the same results for length contraction and time dilation as the STR without referencing the basic principle of the STR.

In the following, we will see how the speed expression for (S') and (S) are expressed in two dimensions. According to the orthogonal coordinate transformations of (S') and (S) we have

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

Differentiating the above two equations with respect to t (i.e., dt), keeping in mind that $dt'/dt = \cos\theta$, gives

$$v_x = v'_x \cos^2\theta - v'_y \sin\theta\cos\theta \qquad (5)$$

$$v_y = v'_x \sin\theta\cos\theta + v'_y \cos^2\theta \qquad (6)$$

We will discuss the red shift of the sun in a gravitational field as deduced by general relativity. We set the observing system (S) on Earth and the moving system (S') on an imaginary planet that moves around the surface of the sun.

For ($v'_x = 0$), Eq. (6) gives us $v_y = v'_y \cos^2\theta$, and

further $v_y = v'_y(1-u^2/c^2)$ (7)

$$\frac{v'_y - v_y}{v'_y} = \frac{u^2}{c^2}$$

Let $v_y' - v_y = \Delta v$ then

$$\frac{\Delta v}{v_y} = \frac{u^2}{c^2} \qquad (8)$$

The revolution speed of planets around the sun in the solar system can be deduced from the classical equation of gravity and Newton's second law, i.e.

$$v = \sqrt{\frac{GM}{R}}$$

If this is substituted for u in Eq. (8) we find

$$\frac{\Delta v}{v_y} = \frac{GM}{Rc^2} \qquad (9)$$

From Eq. (9), we can see that the red shift of the sun in general relativity can be deduced from the deflection of space-time; other conclusions of general relativity can also be related to deflection, such as the precession of the planets, which is discussed later in this paper.

In general, from a modern viewpoint however, the theory of special relativity implies that space and time are not independent and separate, but parts of a four-dimensional spacetime[4], when deflection between two systems with relative motion occurs, the observer in one system may measure the changes the other system undergoes (not only length and time, but also velocity and acceleration, as well as force and energy); generally, of course, all these parameters change, but discussing how this formulism treats those changes is outside the scope of this article.

Inspired by the geometrical optics of the moving dispersionless media detailed in Ref[5], we will discuss complex planes in two-dimensional space.

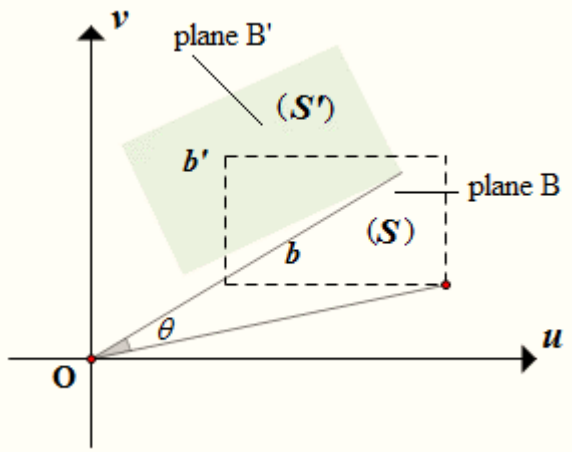

Fig. 2 shows how each point in (S') and (S) is expressed as b' ∈ (S') and b ∈ (S), while plane B' ⊆ (S') and plane B ⊆ (S). If (S') and (S) are introduced in complex planes, then B' is a preimage and B is an image of B'.

Because a deflection of (S') and (S) occurs, there is a functional relation (mapping) w=f (z) between each pair of points in (S') and (S); accordingly, the shape of B may differ from that of B'. Based on the above-mentioned concept, deflection of space can be treated as a complex function, which we will discuss. Before that, however, it is necessary to validate the concept of deflection by studying the Compton Effect.

Compton Effect

The Compton Effect describes the inelastic scattering of a photon, which was discovered in the early 20th century by Arthur Holly Compton. In the microworld, evidence for the "deflection of space-time" principle is demonstrated by showing that conventional results can be reproduced using mathematics based on the deflection of space-time and the equation for the wavelength shift in the Compton Effect.

In Eq. (1), $l = l' \cos\theta$, let $\triangle l = l' - l$ then

$$\triangle l = l'(1 - \cos\theta) \qquad (10)$$

We now introduce Compton's wavelength λ_c ($\lambda_c = h/mc = 2.426 \times 10^{-12}$ m) and the difference between incidence wavelength λ_1 and scattering wavelength λ_2 is $\triangle\lambda$.

Let $\triangle\lambda = \triangle l$ and $\lambda_c = l'$, substitutes $\triangle\lambda$, λ_c for $\triangle l$ and l' respectively in Eq. (10)

which gives $\triangle\lambda = \lambda_c(1 - \cos\theta)$

$$= \frac{h}{mc}(1 - \cos\theta) \qquad (11)$$

Eq. (11) is the equation for the wavelength shift of the Compton Effect for dispersed X-rays, and the dispersion angle θ is simply the deflective angle θ in postulate **(I)**. Both angles match mathematically and conceptually in the Compton Effect, which is not treated as a coincidence.

Hence, with the principle of space-time deflection, there is no boundary between the micro- and macro-world because Eq. (1) and Eq. (11) are on very different spatial scales. Further, the reason of the observed probabilistic nature and uncertainty in the quantum field is sought. It is interesting to understand what happens in the article with regards to time and how that ties in with probability. This article draws some preliminary conclusions by deriving Schrödinger's equation from the deflection of space using a complex function, which is described below.

Derivation of Schrödinger's equation from the deflection of space using a complex function

As stated above, there is a relationship between each pair point in (S') and (S). Based on that, let $w = f(z)$, and thus

$$w = r(\cos\theta + \boldsymbol{i}\sin\theta) = re^{i\theta} \qquad (12)$$

With reference to Fig. 1, it can be seen in Fig. 3 that **L** is on the **X**-axis and **L'** on the **X'**-axis and that the deflective angle θ is an argument of the complex function. The real part of Eq. (12) is $w = r\cos\theta$ (13)

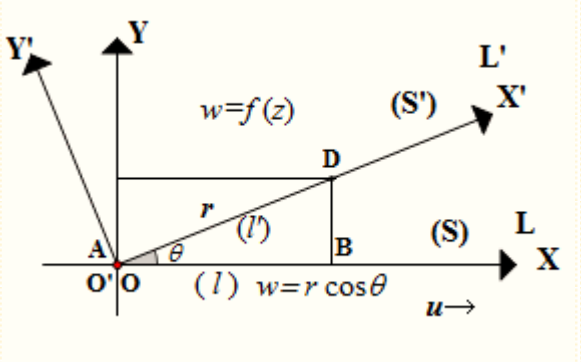

Fig. 3 deflection of (S') and (S) in complex planes

From Fig.3 and Eq. (1), we can say that $r = l'$, $w = l$.

Mathematically, when θ is a constant, the set of point D in three-dimension space is a circle around **X**-axis, see Fig.4 below.

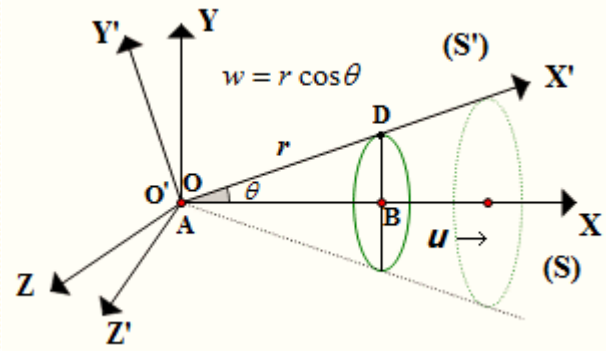

Fig. 4 deflection of (S') and (S) in complex space

When (S') moves along the **X**-axis with speed u, according to **(I),** it deflects from (S) with angle θ. Point D is any points on the circumference at time t, the circle is a set of *probability distributions* of point D in three-dimensional space. The probability density function of w is given by its first derivative, i.e., by w'. If θ is a variable given by $\theta = \omega t$, then the probability density function is the second derivative of w, i.e., w''.

Now, to derive the equation of the amplitude probability density from complex function:

According to Eq.(12) $w = re^{i\theta}$ if we let $\theta = \omega t$

$$w = re^{i\omega t}$$

with $\omega = 2\pi/T$, $t = x/u$ and $uT = \lambda$, replacing w with $\psi(x)$,

we get $\psi(x) = re^{i\frac{2\pi x}{\lambda}}$ (14)

Differentiating x twice in Eq.(14)

then the equation below derived from complex function that reduces to amplitude probability density is Schrödinger Equation in one-dimensional motion

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-U)\psi = 0 \qquad (15)$$

From the derivation process above, please note that the key point is that the position of point D is uncertain in Fig. 4 and its "trajectories" are a set of probability distributions on the circumference of a circle in three-dimensional space, although angle θ is certain. If a particle changes its velocity, then θ changes accordingly, and differentiating twice with respect to t is needed to determine the amplitude probability density. That is the root cause of probability.

So far, the CP may lift the veil on the probability in the microscopic world and lead to explanations for uncertainty as well as other weird phenomena in the quantum world.

Precession formula for a particle moving round the centroid

Given two coordinates, See Fig. 5, **O-XYZ** as a moving system and **o-xyz** as an observing system with origins overlapping at O, θ is an angle (Chapter angle) between **Z** and **z**, and precession angle ψ between OX and ON. A particle goes round **z**-axle at an angular $\dot{\varphi}$ which is an observed value. ON is an intersecting line of plane **O-XY** and **o-xy**, and point A is on ON.

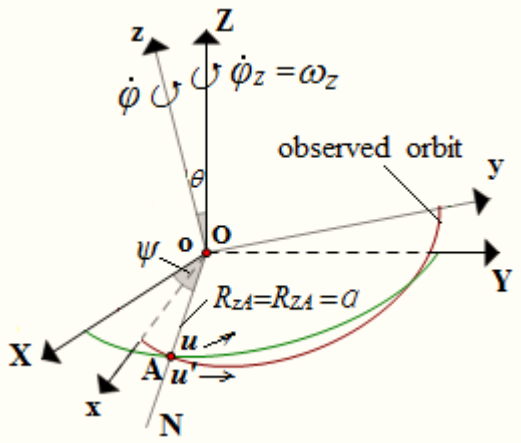

Fig. 5 Precession of a particle moving round the centroid

With **(I)**, the space-time of a particle is deflected due to its motion round the centroid, now we introduce this conception into Euler equation of motion

$$\begin{cases} \omega_x = \dot{\psi}\sin\theta\sin\varphi + \dot{\theta}\cos\varphi \\ \omega_y = \dot{\psi}\sin\theta\cos\varphi - \dot{\theta}\sin\varphi \\ \omega_z = \dot{\psi}\cos\theta + \dot{\varphi} \end{cases}$$

Thus, a formula for the average precession angular speed is as follows

$$\Delta\bar{\dot{\psi}} = \frac{\pi\bar{v}^3}{ac^2} \quad \text{(r/s)} \qquad (16)$$

$\Delta\bar{\dot{\psi}}$: Average precession angular speed
$\bar{v}$: Average speed of orbit
a: Semi-major radius of orbit

Table.1 below shows the calculated precession of the planets results from GR, astronomically observed data, and this formula (16).

Formula (16) can be cross-referenced with other astronomical data for other planets, asteroids, and similar.

Table 1 Precession of Planets

Planets in solar system			Mercury	Venus	The Earth
Parameter / Result	$\bar{v}$	km/s	47.89	35.03	29.79
	a	AU	0.387	0.723	1
	T	day	87.97	244.7	365.26
G. Relativity	"/century		43.03	8.63	3.84
$\Delta\bar{\psi}$	"/century		42.96	8.94	4.00
Observed	"/century		42.6±1	8.4±4.8	5±1.2

Conclusions

1) Length contraction, mass change, and time dilation in the STR are expressed via the Lorentz factor. With postulate **(I)**, it is defined to be $1/\cos\theta$, which provides a new perspective for transforming measurements between reference frames.

2) With the derivation of the Schrödinger Equation from a complex function with deflection of space-time, both relativity and quantum mechanics are found to be based on common principles, which means that they were compatible ever since they were established.

3) The new formula for the average precessional angular speed indicates that there may be another interpretation of space-time besides Riemannian space-time.

4) For hundreds of years, from Newtonian theory to relativity and quantum mechanics, the basic theory of physics may be summarized in terms of the domain of the space-time variable θ, as shown in Table 2.

Domain of space-time variable θ		
0 $L = L_0$	Absolute space-time	Newtonian theory
$[0, \pi/2]$ $L = L_0 \cos\theta$	Relativistic space-time	Relativity
$[0, +\infty)$ $L = L_0 \cos\omega t$	Quantum space-time	Quantum Mechanics

Table 2 Veriable of deflection

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