Comprehensive Rate Equation Analysis of Upconversion Luminescence Enhancement Due to $BaCl_2$ Nanocrystals in Neodymium Doped Fluorozirconate-based Glass Ceramics

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\begin{aligned}
\partial_t N_1 =& - W_{\mathrm{PE},1} N_1 + W_{\mathrm{PE},2} N_2 + W_{\mathrm{PE},1} \nu_1 N_0 - W_{\mathrm{PE},2} \nu_2 N_1 \\
& - A_{\mathrm{SPE},1} N_1 + \sum_{i=2}^{15} A_{\mathrm{SPE,i}} \beta_{i,1} N_i \\
& + A_{\mathrm{ETU3a}} \left(N_4^2 - N_1 N_{12} \right) + A_{\mathrm{ETU3b}} \left(N_4^2 - N_1 N_{13} \right) \\
\partial_t N_2 =& - W_{\mathrm{PE},2} N_2 + W_{\mathrm{PE},3} N_3 + W_{\mathrm{PE},2} \nu_2 N_1 - W_{\mathrm{PE},3} \nu_3 N_2 \\
& - A_{\mathrm{SPE},2} N_2 + \sum_{i=3}^{15} A_{\mathrm{SPE,i}} \beta_{i,2} N_i \\
& + A_{\mathrm{ETU2a}} \left(N_4^2 - N_{10} N_2 \right) + A_{\mathrm{ETU2b}} \left(N_4^2 - N_{11} N_2 \right) \\
\partial_t N_3 =& - W_{\mathrm{PE},3} N_3 + W_{\mathrm{PE},4} N_4 + W_{\mathrm{PE},3} \nu_3 N_2 - W_{\mathrm{PE},4} \nu_4 N_3 \\
& - A_{\mathrm{SPE},3} N_3 + \sum_{i=4}^{15} A_{\mathrm{SPE,i}} \beta_{i,3} N_i \\
& + A_{\mathrm{ETU1}} \left(N_4^2 - N_3 N_9 \right) \\
& + 2 A_{\mathrm{CR}} \left(N_0 N_4 - N_3^2 \right) \\
\partial_t N_4 =& - W_{\mathrm{PE},4} N_4 + W_{\mathrm{PE},5} N_5 + W_{\mathrm{PE},4} \nu_4 N_3 - W_{\mathrm{PE},5} \nu_5 N_4 \\
& - A_{\mathrm{SPE},4} N_4 + \sum_{i=5}^{15} A_{\mathrm{SPE,i}} \beta_{i,4} N_i \\
& - \eta R_{\mathrm{p}} \sigma_{\mathrm{ESA}} (N_4 - N_{15}) \\
& - A_{\mathrm{CR}} \left(N_0 N_4 - N_3^2 \right) \\
& - 2 A_{\mathrm{ETU1}} \left(N_4^2 - N_3 N_9 \right) \\
& - 2 \left(A_{\mathrm{ETU3a}} \left(N_4^2 - N_1 N_{12} \right) + A_{\mathrm{ETU3b}} \left(N_4^2 - N_1 N_{13} \right) \right) \\
& - 2 \left(A_{\mathrm{ETU4a}} \left(N_4^2 - N_0 N_{14} \right) + A_{\mathrm{ETU4b}} \left(N_4^2 - N_0 N_{15} \right) \right) \\
& - 2 \left(A_{\mathrm{ETU2a}} \left(N_4^2 - N_{10} N_2 \right) + A_{\mathrm{ETU2b}} \left(N_4^2 - N_{11} N_2 \right) \right) \\
\partial_t N_5 =& + (1 - \eta) R_{\mathrm{p}} \sigma_{\mathrm{GSA}} (N_0 - N_5) \\
& - W_{\mathrm{PE},5} N_5 + W_{\mathrm{PE},6} N_6 + W_{\mathrm{PE},5} \nu_5 N_4 - W_{\mathrm{PE},6} \nu_6 N_5 \\
& - A_{\mathrm{SPE},5} N_5 + \sum_{i=6}^{15} A_{\mathrm{SPE,i}} \beta_{i,5} N_i \\
& - R_{\mathrm{p}} \sigma_{\mathrm{STE},50} N_5 \\
\partial_t N_6 =& - W_{\mathrm{PE},6} N_6 + W_{\mathrm{PE},7} N_7 + \nu_6 N_5 W_{\mathrm{PE},6} - W_{\mathrm{PE},7} \nu_7 N_6 \\
& - A_{\mathrm{SPE},6} N_6 + \sum_{i=7}^{15} A_{\mathrm{SPE,i}} \beta_{i,6} N_i \\
\partial_t N_7 =& - W_{\mathrm{PE},7} N_7 + W_{\mathrm{PE},8} N_8 + \nu_7 N_6 W_{\mathrm{PE},7} - W_{\mathrm{PE},8} \nu_8 N_7 \\
& - A_{\mathrm{SPE},7} N_7 + \sum_{i=8}^{15} A_{\mathrm{SPE,i}} \beta_{i,7} N_i
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\partial_t N_8 =& - W_{\mathrm{PE},8} N_8 + W_{\mathrm{PE},9} N_9 + \nu_8 N_7 W_{\mathrm{PE},8} - W_{\mathrm{PE},9} \nu_9 N_8 \\
& - A_{\mathrm{SPE},8} N_8 + \sum_{i=9}^{15} A_{\mathrm{SPE,i}} \beta_{i,8} N_i \\
\partial_t N_9 =& - W_{\mathrm{PE},9} N_9 + W_{\mathrm{PE},10} N_{10} + W_{\mathrm{PE},9} \nu_9 N_8 - W_{\mathrm{PE},10} N_9 \nu_{10} \\
& - A_{\mathrm{SPE},9} N_9 + \sum_{i=10}^{15} A_{\mathrm{SPE,i}} \beta_{i,9} N_i \\
& + A_{\mathrm{ETU1}} \left(N_4^2 - N_3 N_9 \right) \\
\partial_t N_{10} =& - W_{\mathrm{PE},10} N_{10} + W_{\mathrm{PE},11} N_{11} + W_{\mathrm{PE},10} N_9 \nu_{10} - W_{\mathrm{PE},11} \nu_{11} N_{10} \\
& - A_{\mathrm{SPE},10} N_{10} + \sum_{i=11}^{15} A_{\mathrm{SPE,i}} \beta_{i,10} N_i \\
& + A_{\mathrm{ETU2a}} \left(N_4^2 - N_{10} N_2 \right) \\
\partial_t N_{11} =& - W_{\mathrm{PE},11} N_{11} + W_{\mathrm{PE},12} N_{12} + W_{\mathrm{PE},11} \nu_{11} N_{10} - W_{\mathrm{PE},12} \nu_{12} N_{11} \\
& - A_{\mathrm{SPE},11} N_{11} + \sum_{i=12}^{15} A_{\mathrm{SPE,i}} \beta_{i,11} N_i \\
& + A_{\mathrm{ETU2b}} \left(N_4^2 - N_{11} N_2 \right) \\
\partial_t N_{12} =& - W_{\mathrm{PE},12} N_{12} + W_{\mathrm{PE},13} N_{13} + W_{\mathrm{PE},12} \nu_{12} N_{11} - W_{\mathrm{PE},13} \nu_{13} N_{12} \\
& - A_{\mathrm{SPE},12} N_{12} + \sum_{i=13}^{15} A_{\mathrm{SPE,i}} \beta_{i,12} N_i \\
& + A_{\mathrm{ETU3a}} \left(N_4^2 - N_1 N_{12} \right) \\
\partial_t N_{13} =& - W_{\mathrm{PE},13} N_{13} + W_{\mathrm{PE},14} N_{14} + W_{\mathrm{PE},13} \nu_{13} N_{12} - W_{\mathrm{PE},14} \nu_{14} N_{13} \\
& - A_{\mathrm{SPE},13} N_{13} + \sum_{i=14}^{15} A_{\mathrm{SPE,i}} \beta_{i,13} N_i \\
& + A_{\mathrm{ETU3b}} \left(N_4^2 - N_1 N_{13} \right) \\
\partial_t N_{14} =& - W_{\mathrm{PE},14} N_{14} + W_{\mathrm{PE},15} N_{15} + W_{\mathrm{PE},14} \nu_{14} N_{13} - W_{\mathrm{PE},15} \nu_{15} N_{14} \\
& - A_{\mathrm{SPE},14} N_{14} + A_{\mathrm{SPE},15} \beta_{15,14} N_{15} \\
& + A_{\mathrm{ETU4a}} \left(N_4^2 - N_0 N_{14} \right) \\
\partial_t N_{15} =& - W_{\mathrm{PE},15} N_{15} + W_{\mathrm{PE},15} \nu_{15} N_{14} \\
& - A_{\mathrm{SPE},15} N_{15} \\
& + \eta R_{\mathrm{p}} \sigma_{\mathrm{ESA}} (N_4 - N_{15}) \\
& + A_{\mathrm{ETU4b}} \left(N_4^2 - N_0 N_{15} \right)
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