Supporting Information

Free ended coaxial cable model with linearly decreasing filament width

Detailed derivation of the modified electrical model discussed in the paper is presented here. Figure S1 shows an schematic diagram of the experimental set-up.

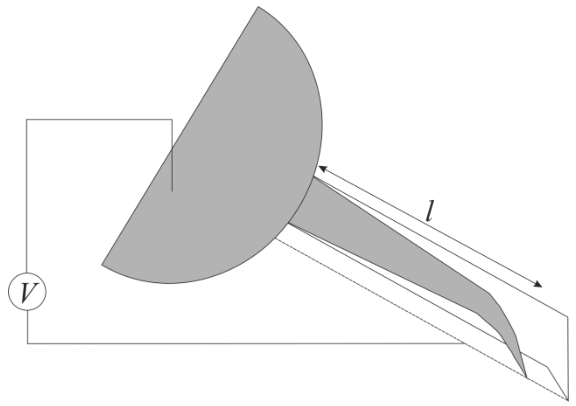

Figure S1. Schematic diagram of the experimental set-up showing a reservoir drop and a drawn liquid filament in a triangular groove. The top electrode was immersed in the reservoir drop and the bottom electrode was connected to underlying conducting silicon substrate.

In the electrical model, a liquid filament in a triangular groove is considered as a free ended coaxial cable and can be modelled as series of resistors and capacitors. Our aim is to find the voltage drop along an electrically conducting liquid filament in a triangular groove of width W and wedge angle ψ. Figure S1(b) shows sketch of a liquid filament of length l in a triangular groove. Thickness and dielectric constant of the insulating layer on triangular groove is T and ε_r, conductivity of the liquid is σ, voltage and frequency of the applied ac signal at one end of the groove ($x=0$) is U_0 and ω. Figure S1(c) shows an equivalent electrical circuit of a free-ended coaxial cable of length l.

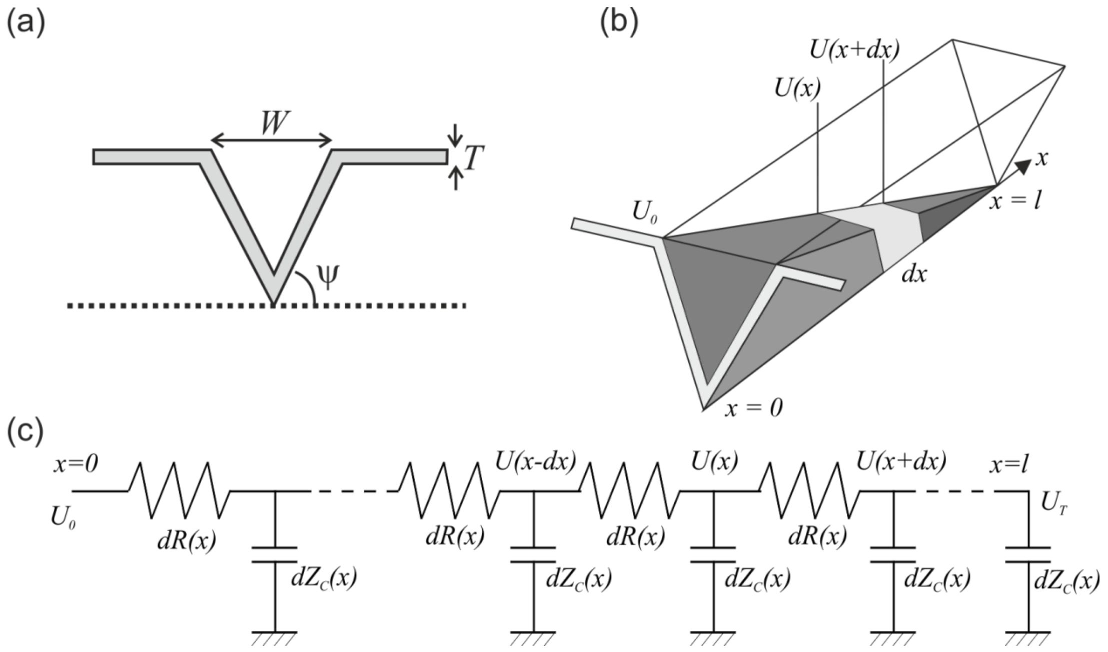

Figure S1. (a) Sketch of a triangular groove showing all parameters, (b) sketch of a liquid filament with linearly decreasing width in a triangular groove and (c) equivalent electrical circuit for a liquid filament in a triangular groove.

Width of the liquid filament along the length decreases linearly as *W(x)=W(1-x/l)*, where *W* is the width at x = 0. Since we are using ac voltage for electrowetting, let *u(x)* be the complex voltage at a distance x of a liquid filament in a triangular groove. Applying Kirchhoff's current law at a node, we obtained

$$u(x+dx)+u(x-dx)=u(x)\left[2+\frac{dR}{dZ_c}\right] \tag{1}$$

where dR and dZ_c are the resistive and capacitive impedance of the liquid filament of length dx which can be written as $dR=\frac{4\,dx}{\sigma\, w^2(1-x/l)^2 tan\psi}$ and $dZ_c=\frac{T\cos\psi}{j\omega\epsilon_r\epsilon_0 w\,(1-\frac{x}{l})dx}$. Now, using the identity

$$u(x+dx)+u(x-dx)-2\,u(x)=\frac{d^2u(x)}{dx^2}\,dx^2 \tag{2}$$

Eq. 1 reduces to

$$\frac{d^2u(x)}{dx^2} = 2j\,\frac{u(x)}{(1-x/l)\lambda^2} \tag{3}$$

where λ is the characteristic length parameter expressed as $\lambda = \sqrt{\frac{2T\sigma}{\omega\varepsilon_0\varepsilon_r}\,\frac{W\,\sin\psi}{4}}$. Equation 3 was solved analytically using "Maple-14" and following solution was obtained

$$u(x) = C_1\sqrt{(x-l)}\,J_1\left(\frac{2\sqrt{2}\sqrt{jl(x-l)}}{\lambda}\right) + C_2\sqrt{(x-l)}\,Y_1\left(\frac{-2j\sqrt{2}\sqrt{jl(x-l)}}{\lambda}\right) \tag{4}$$

where J_1 and Y_1 are the Bessel functions of 1st and 2nd kind. In this equation, $\{C_1, C_2\}$ are the complex constant coefficients of the solution which has to be determined using the appropriate boundary conditions. The solution $u(x)$ is a complex function of real variable x. The physical amplitude of voltage would thus be $U(x) = \|u(x)\|$. To simplify Eq. 4, the Bessel functions were expanded in series and truncated after 3rd term

$$u(x) = \frac{C_1(x-l)\sqrt{2jl}}{\lambda} + C_2\sqrt{2}\,j\,\left\{\lambda^2 - 2jl^2\ln(2) + 2jl\,ln(2)x + 4jl^2\,ln\left(\frac{-2j\sqrt{jl(x-l)}}{\lambda}\right) - 4jl\,ln\left(\frac{-2j\sqrt{jl(x-l)}}{\lambda}\right)x + 4jl^2\gamma - 4jl\gamma x - 2jl^2 + 2jlx\right\}/4\lambda\sqrt{jl} \tag{5}$$

where γ =Euler–Mascheroni constant (Euler's constant).

Boundary condition 1: At equilibrium the current at tip of the liquid filament should be zero i.e. $\left.\frac{du}{dx}\right|_{x=l} = 0$. It is clear from Eq. 3 that the differential equation is unbounded at $x = l$, therefore we need to solve it around $x = l$. Therefore different possible values of x was tried by trial and error method and x=0.966l was found to fit exactly with the experimental data. Therefore the boundary condition 1 was modified to $\left.\frac{du}{dx}\right|_{x=0.966l} = 0$ and the Eq. 5 can be further simplified to

$$u(x) = -\left[2.83 \times 10^{-10}\, jC_2 \left\{2.5 \times 10^9 jl^2 - 2.5 \times 10^9 jlx + 5 \times 10^9 jl^2 ln\left(\frac{-0.37 jl\sqrt{-j}}{\lambda}\right) - 5 \times 10^9 jl\, ln\left(\frac{-0.37 jl\sqrt{-j}}{\lambda}\right) x - 1.25 \times 10^9 \lambda^2 - 5 \times 10^9 jl^2 ln\left(\frac{-2j\sqrt{jl(x-l)}}{\lambda}\right) + 5 \times 10^9 jl\, ln\left(\frac{-2j\sqrt{jl(x-l)}}{\lambda}\right) x\right\}\right] / \left(\sqrt{jl}\, \lambda\right) \quad (6)$$

Boundary condition 2: The voltage at the entrance of the liquid filament is equal to the applied voltage i.e. $u(x=0)=U_0$. So the Eq. 6 could further be simplified as

$$u(x) = -jU_0 \left[2.5 \times 10^9 jl^2 - 2.5 \times 10^9 jl\, x + 5 \times 10^9 jl^2 ln\left(\frac{-0.37 jl\sqrt{-j}}{\lambda}\right) - 5 \times 10^9 jl\, ln\left(\frac{-0.37 jl\sqrt{-j}}{\lambda}\right) x - 1.25 \times 10^9 \lambda^2 - 5 \times 10^9 jl^2\, ln\left(\frac{-2j\sqrt{jl\,(x-l)}}{\lambda}\right) + 5 \times 10^9 jl\, ln\left(\frac{-2j\sqrt{jl(x-l)}}{\lambda}\right) x\right] / \left[2.5 \times 10^9 l^2 + 5 \times 10^9 l^2 ln\left(\frac{-0.37 j}{\lambda}\right) + 1.25 \times 10^9 j\lambda^2 - 5 \times 10^9 jl^2 ln\left(\frac{-2j}{\lambda}\right)\right] \quad (7)$$

Boundary condition 3: The 3rd boundary condition is that the voltage at the tip of a liquid filament equals the threshold voltage, $\|u(x = l)\| = U_T$. This gives the final relation involving the applied voltage and the length as

$$U_T = U_0 \sqrt{\frac{\lambda^4}{\lambda^4 + 22.7\, l^4}} \quad (8)$$

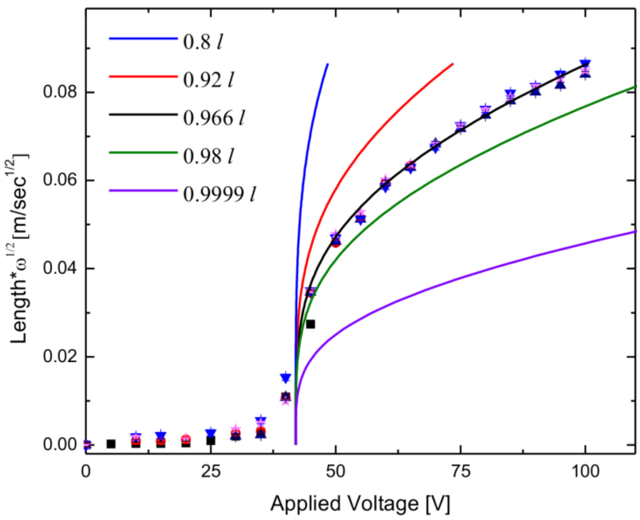

Figure S2. Plot of the scaled experimental data with the solution of linearly decreasing filament model for different values of x around $x = l$ showing that x = 0.966l fits the best.