Understanding Energy Transfer Mechanisms for Tunable Emission of Yb³⁺-Er³⁺ Codoped GdF₃ Nanoparticles: Concentration-Dependent Luminescence by Near-Infrared and Violet Excitation

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Figure S1 XRD patterns of GdF_3 with (a) different Er^{3+} dose and (b) different Yb^{3+} dose. (c) Magnification of (111) diffraction peak of samples with different Yb^{3+} dose.

The structure of all samples are typical orthorhombic phase with Pnma space group. From Fig. 1(a), the structures of the samples doped with different Er^{3+} concentration are almost unchanged, due to the small radii difference of Gd^{3+} (0.935 Å) ion and Er^{3+} (0.89 Å) ion and small amount of doping. However, the structures of samples with different Yb³⁺ (0.868 Å) dose change obviously, as revealed in Fig. 1 (c), where all diffraction peaks move to higher angle as Yb³⁺ concentration increase. Moreover, according to Scherrer formula, $D = \frac{k\lambda}{\beta \cos \theta}$, full width at half maximum of all diffraction peaks are practically unchanged, revealing the particle sizes are nearly same for all samples.



Figure S2 EDX spectra of $GdF_3:2\% Er^{3+}$ with different nominal Yb³⁺ dose: (a) 10 mol%, (b) 20 mol%, (c) 40 mol% and (d) 60 mol%.

To testify whether all the Yb³⁺ ions are doped into the crystal structure, the energy disperse X-ray spectra of all samples with different Yb³⁺ concentration are performed. The molar ratio of Yb³⁺ ions with respect to all lanthanide ions (Mol_{Yb} : Mol_{Ln}) are calculated as at[Yb]/(at[Gd] + at[Yb]) (Er^{3+} ions are ignored for its low doping concentration.): (a) 15.09%, (b) 23.51%, (c) 39.88%, (d) 58.90%. These results match quite well with nominal Yb³⁺ concentration, respectively, indicating that all Yb³⁺ ions can be considered to be completely doped into the crystal structure.

Mechanisms	Rate Equation (General Theory for High Yb ³⁺)	Steady State RGR (N ₃ /N ₄) ^a
ET1	$\begin{aligned} \frac{dN_1}{dt} &= \omega_{21}N_2 - \omega_1 N_{Yb1}N_1 - A_1 N_1 \\ \frac{dN_2}{dt} &= \omega_0 N_{Yb1}N_0 - \omega_{21}N_2 - \omega_2 N_{Yb1}N_2 - \omega_C N_4 N_2 - A_2 N_2 \\ \frac{dN_3}{dt} &= \omega_1 N_{Yb1}N_1 + 2\omega_C N_4 N_2 - A_3 N_3 \\ \frac{dN_4}{dt} &= \omega_2 N_{Yb1}N_2 - \omega_C N_4 N_2 - A_4 N_4 \end{aligned}$	$\frac{-A_4\omega_2 + \omega_c\omega_0N_0}{2A_3\omega_2} + \frac{\sqrt{A_4^2\omega_2^2 + 6\omega_c\omega_0\omega_2A_4N_0 + \omega_c^2\omega_0^2N_0^2}}{2A_3\omega_2}$
ET2	$\begin{split} \frac{dN_1}{dt} &= \omega_{21}N_2 - \omega_C N_4 N_1 - \omega_1 N_{Yb1} N_1 - A_1 N_1 \\ \frac{dN_2}{dt} &= \omega_0 N_{Yb1} N_0 + \omega_C N_4 N_1 - \omega_{21} N_2 - \omega_2 N_{Yb1} N_2 - A_2 N_2 \\ \frac{dN_3}{dt} &= \omega_1 N_{Yb1} N_1 + \omega_C N_4 N_1 - A_3 N_3 \\ \frac{dN_4}{dt} &= \omega_2 N_{Yb1} N_2 - \omega_C N_4 N_1 - A_4 N_4 \end{split}$	$\frac{A_4\omega_{21}}{A_3\omega_2N_{Yb1}-A_3\omega_{21}}$
ET3	$\begin{aligned} \frac{dN_1}{dt} &= \omega_c N_4 N_0 - \omega_1 N_{Yb1} N_1 - A_1 N_1 \\ \frac{dN_2}{dt} &= \omega_0 N_{Yb1} N_0 + \omega_{2'2} N_{2'} - \omega_2 N_{Yb1} N_2 - A_2 N_2 \\ \frac{dN_2}{dt} &= \omega_c N_4 N_0 - \omega_{2'2} N_{2'} - A_2 N_2 \\ \frac{dN_3}{dt} &= \omega_1 N_{Yb1} N_1 - A_3 N_3 \\ \frac{dN_4}{dt} &= \omega_2 N_{Yb1} N_2 - \omega_c N_4 N_0 - A_4 N_4 \end{aligned}$	$rac{\omega_c N_0}{A_4}$
ET4	$\begin{aligned} \frac{dN_1}{dt} &= \omega_b N_{Yb0} N_4 - \omega_1 N_{Yb1} N_1 - A_1 N_1 \\ \frac{dN_2}{dt} &= \omega_0 N_{Yb1} N_0 - \omega_2 N_{Yb1} N_2 - A_2 N_2 \\ \frac{dN_3}{dt} &= \omega_1 N_{Yb1} N_1 - A_3 N_3 \\ \frac{dN_4}{dt} &= \omega_2 N_{Yb1} N_2 - \omega_b N_{Yb0} N_4 - A_4 N_4 \end{aligned}$	$rac{arphi_b N_{Yb0}}{A_4}$
ET5	$\begin{aligned} \frac{dN_1}{dt} &= \omega_b N_{Yb0} N_4 + \omega_{21} N_2 - A_1 N_1 \\ \frac{dN_2}{dt} &= \omega_0 N_{Yb1} N_0 - \omega_2 N_{Yb1} N_2 - \omega_{21} N_2 - A_2 N_2 \\ \frac{dN_3}{dt} &= \omega_1 N_{Yb1} N_1 + \omega_{43} N_4 - A_3 N_3 \\ \frac{dN_4}{dt} &= \omega_2 N_{Yb1} N_2 - \omega_b N_{Yb0} N_4 - \omega_{43} N_4 - A_4 N_4 \end{aligned}$	$\frac{\underline{A_1\omega_{43} + \omega_1\omega_bN_{Yb0}N_{Yb1}}}{\underline{A_1A_3}}$

 Table S1 Rate equations of five ET processes.

 a Neglecting ETU from N_1 in ET1 and ET2 and radiation from N_1 and $N_{2(2^\prime)}$ in ET1-4.

 N_1 , N_2 , N_2 , N_3 , N_4 , N_{Yb0} and N_{Yb1} are the populations of the Er³⁺ ⁴I_{13/2}, ⁴I_{11/2}, ⁴I_{9/2}, ⁴F_{9/2}, ²H_{11/2}/⁴S_{3/2}, Yb³⁺ ²F_{7/2} and ²F_{5/2} manifolds, respectively. ω_0 , ω_1 and ω_2 are ET parameters between Yb³⁺ ²F_{7/2} \rightarrow ²F_{5/2} and Er³⁺ ⁴I_{15/2} \rightarrow ⁴I_{11/2}, ⁴I_{13/2} \rightarrow ⁴F_{9/2} and ⁴I_{11/2} \rightarrow ⁴F_{7/2}, respectively. ω_{21} and ω_{43} are MPR rates from Er³⁺ ⁴I_{11/2} \rightarrow ⁴I_{13/2} and Er³⁺ ²H_{11/2}/⁴S_{3/2} \rightarrow ⁴F_{9/2}, respectively. ω_C is the CR rate for ET1(⁴F_{7/2} + ⁴I_{11/2} \rightarrow ⁴F_{9/2} + ⁴F_{9/2})/ET2(⁴F_{7/2} + ⁴I_{13/2} \rightarrow ⁴F_{9/2} + ⁴I_{11/2})/ET3(⁴I_{15/2} + ⁴S_{3/2} \rightarrow ⁴I_{13/2} + ⁴I_{9/2}). ω_b is the EBT rate to the Yb³⁺ ions. A_1 , A_2 , A_2 , A_3 and A_4 are radiative rates of Er³⁺ ⁴I_{13/2}, ⁴I_{11/2}, ⁴I_{9/2}, ⁴F_{9/2} and ²H_{11/2}/⁴S_{3/2} manifolds, respectively. The above ET1-ET4 rate equations are proposed based on high Yb³⁺ concentrations. Hence, many radiative and nonradiative processes, such as N_1/N_2 radiative emissions, MPR processes and back-energy-transfer from Er³⁺ ⁴I_{11/2} \rightarrow ⁴I_{15/2} transition to Yb³⁺ ²F_{7/2} \rightarrow ²F_{5/2} transition and so on, can be neglected. As to ET5, N_1 emission is considered as major depletion of population. And MPR is considered due to the high phonon-energy groups attached to the surface of nanoparticles.

Of all rate equations, the population density of Yb³⁺ ions excited state can be generally described as following

$$\frac{dN_{Yb1}}{dt} = \sigma \rho N_{Yb0} - \sum_{i} \omega_{i} N_{i} N_{Yb1} - A_{Yb1} N_{Yb1}$$
(1)

where σ is the absorption cross-section of Yb^{3+ 2}F_{5/2} manifold. ρ is pump rate of the NIR laser. The incoming rate of Yb^{3+ 2}F_{5/2} manifold is mainly considered to be the NIR laser pumping rate. Then N_{Yb1} can be expressed as follows under steady-state condition

$$N_{Yb1} = \frac{\sigma \rho N_{Yb0}}{A_{Yb1} + \sum_{i} \omega_{i} N_{i}} \propto \rho$$
⁽²⁾

From the luminescence spectra results, we contribute the ET mechanism to ET4 or ET5, both of

which EBT process is the main mechanism to depopulate Er^{3+} green-emitting manifolds and populate Er^{3+} red-emitting manifold. To compare the two mechanisms (ET4 and ET5), the corresponding rate equations are solved and the values of N_1 , N_3 and N_4 are obtained. In ET4, upconversion (UC) rate is considered as dominant depletion for ${}^{4}\text{I}_{13/2}$ manifold. In low Yb³⁺ dose samples, linear decay (LD) rate is considered as primary depletion for ${}^{4}\text{I}_{11/2}$ manifold. By solving the equations, we have

$$N_{1} = \frac{\omega_{0}\omega_{2}\omega_{b}N_{0}N_{Yb0}N_{Yb1}}{A_{2}\omega_{1}\left(A_{4} + \omega_{b}N_{Yb0}\right)} \propto \rho$$
(3)

$$N_{3} = \frac{\omega_{0}\omega_{2}\omega_{b}N_{0}N_{Yb0}N_{Yb0}^{2}}{A_{2}A_{3}(A_{4} + \omega_{b}N_{Yb0})} \propto \rho^{2}$$
(4)

$$N_{4} = \frac{\omega_{0}\omega_{2}N_{0}N_{_{Yb1}}^{2}}{A_{2}\left(A_{4} + \omega_{b}N_{_{Yb0}}\right)} \propto \rho^{2}$$
(5)

In high Yb^{3+} dose samples, UC is considered as primary depletion for ${}^{4}I_{11/2}$ manifold. Under this situation, we have

$$N_1 = \frac{\omega_0 \omega_b N_0 N_{Yb0}}{\omega_1 \left(A_4 + \omega_b N_{Yb0} \right)} \propto \rho^0 \tag{6}$$

$$N_{3} = \frac{\omega_{0}\omega_{b}N_{0}N_{\gamma b0}N_{\gamma b1}}{A_{3}\left(A_{4} + \omega_{b}N_{\gamma b0}\right)} \propto \rho^{1}$$
(7)

$$N_4 = \frac{\omega_0 N_0 N_{Yb1}}{\left(A_4 + \omega_b N_{Yb0}\right)} \propto \rho^1 \tag{8}$$

In ET5, LD is considered as dominant depletion for ${}^{4}I_{13/2}$ manifold. In low Yb³⁺ dose samples,

LD is considered as primary depletion for ${}^{4}I_{11/2}$ manifold. Hence,

$$N_{1} = \frac{\omega_{0}\omega_{21}N_{0}N_{Yb1}}{A_{1}(A_{2} + \omega_{21})} + \frac{\omega_{0}\omega_{2}\omega_{b}N_{0}N_{Yb0}N_{Yb0}^{2}}{A_{1}(A_{2} + \omega_{21})(A_{4} + \omega_{43} + \omega_{b}N_{Yb0})} \propto a\rho^{1} + b\rho^{2}$$
(9)

$$N_{3} = \frac{\omega_{0} \left(A_{4} \omega_{1} \omega_{21} + A_{1} \omega_{2} \omega_{43} + \omega_{1} \omega_{21} \omega_{43} + \omega_{1} \omega_{21} \omega_{b} N_{Yb0}\right) N_{0} N_{Yb1}^{2}}{A_{1} A_{3} \left(A_{2} + \omega_{21}\right) \left(A_{4} + \omega_{43} + \omega_{b} N_{Yb0}\right)} + \frac{\omega_{0} \omega_{1} \omega_{2} \omega_{b} N_{0} N_{Yb0} N_{Yb1}^{3}}{A_{1} A_{3} \left(A_{2} + \omega_{21}\right) \left(A_{4} + \omega_{43} + \omega_{b} N_{Yb0}\right)} \propto a \rho^{2} + b \rho^{3}$$

$$N_{4} = \frac{\omega_{0} \omega_{2} N_{0} N_{Yb1}^{2}}{\left(A_{2} + \omega_{21}\right) \left(A_{4} + \omega_{43} + \omega_{b} N_{Yb0}\right)} \propto \rho^{2}$$
(10)

In high Yb^{3+} dose samples, UC is considered as primary depletion for ${}^{4}I_{11/2}$ manifold. Hence

$$N_{1} = \frac{\omega_{0}\omega_{21}N_{0}}{A_{1} + \omega_{2}} + \frac{\omega_{0}\omega_{b}N_{0}N_{Yb0}N_{Yb1}}{A_{1}\left(A_{4} + \omega_{43} + \omega_{b}N_{Yb0}\right)} \propto a\rho^{0} + b\rho^{1}$$
(12)

$$N_{3} = \frac{\left(A_{4}\omega_{1}\omega_{21} + A_{1}\omega_{2}\omega_{43} + \omega_{1}\omega_{21}\omega_{43} + \omega_{1}\omega_{21}\omega_{b}N_{Yb0}\right)\omega_{0}N_{0}N_{Yb1}}{A_{1}A_{3}\omega_{2}\left(A_{4} + \omega_{43} + \omega_{b}N_{Yb0}\right)} + \frac{\omega_{0}\omega_{1}\omega_{2}\omega_{b}N_{0}N_{Yb0}N_{Yb1}^{2}}{A_{1}A_{3}\omega_{2}\left(A_{4} + \omega_{43} + \omega_{b}N_{Yb0}\right)} \propto a\rho^{1} + b\rho^{2}$$

$$N_{4} = \frac{\omega_{0}N_{0}N_{Yb1}}{A_{4} + \omega_{43} + \omega_{b}N_{Yb0}} \propto \rho^{1}$$
(13)

All the above results are summarized in Table 1, showing the difference between ET4 and ET5.

From the above results, the more saturation part of N_1 ($a\rho^1$ in Eq. (9) or $a\rho^0$ in Eq. (12)) and N_3 ($a\rho^2$ in Eq. (10) or $a\rho^1$ in Eq. (13)) in ET5 are due to the MPR rates, i.e., ω_{21} and ω_{43} , indicating that MPR process also contributes to the ET mechanism.



Figure S3 Excitation power density dependence of RGR for different Yb^{3+} concentration doped GdF_3 :2% Er^{3+} NPs.

RGR increases faster in power dependence along with increasing Yb^{3+} concentration, demonstrating the RGR is related to two factors: Yb^{3+} concentration and pump power. This result also strengthens the point view that ET mechanism of our samples mainly conforms to ET5.



Figure S4 Normalized DS emission spectra of $GdF_3:20\%Yb^{3+}$ codoped with different Er^{3+} concentration. Inset shows the magnification of $Er^{3+} {}^4I_{13/2}$ NIR emission with different Er^{3+} concentration.

NIR emission relative to red emission increases along with increasing Er^{3+} concentration, which plainly demonstrates that CR process exists in our samples. Generally, CR is considered to be predominant when the average distance between activators is small enough,¹ which means the activator concentration should be adequately large. In our case, 5 mol% is large enough as the concentration quenching effect occurs when doping concentration is more than 2 mol%.² According to the results, the CR process may occur as $Er^{3+2}H_{11/2}/^4S_{3/2} + Er^{3+4}I_{15/2} \rightarrow Er^{3+4}I_{9/2} +$ $Er^{3+4}I_{13/2}$, which simultaneously depopulate the green-emitting manifolds and populate NIR-emitting manifold.



Figure S5 Excitation spectra monitored at 1 μ m and emission spectra of GdF₃:20% Yb³⁺/2% Er³⁺ NPs under 670 nm excitation. Inset shows the DC energy transfer process between Yb³⁺-Er³⁺.

The above spectra show that ET occur between Er^{3+} and Yb^{3+} with $Yb^{3+} {}^{4}F_{5/2}$ emission, only with $Er^{3+} {}^{4}F_{9/2}$ and $Er^{3+} {}^{4}I_{9/2}$ radiation. However, there is no excitation band of $Er^{3+} {}^{4}G_{11/2}$ manifold, suggesting that there is no Yb^{3+} NIR emission under $Er^{3+} {}^{4}G_{11/2}$ excitation.



Figure S6 DS ET processes in Yb^{3+} - Er^{3+} ion pair in low dopant concentration. Solid lines, dashed lines and dotted lines represent radiative transition, cross-relaxation and multiphonon relaxation processes, respectively.

For samples with low dopant concentration, ET mechanism for DS process should not involve Yb^{3+} , which can be summarized in the above ET scheme. Er^{3+} green and red emissions are mainly due to MPR process from ${}^{4}G_{11/2}$ manifold. $Er^{3+} {}^{4}I_{13/2}$ emission is majorly due to a CR process.

Hence, the corresponding rate equations can be established as follows

$$\frac{dN_{1}}{dt} = \omega_{c}N_{0}N_{4} - A_{1}N_{1}$$

$$\frac{dN_{3}}{dt} = \omega_{53}N_{5} + \omega_{43}N_{4} - A_{3}N_{3}$$

$$\frac{dN_{4}}{dt} = \omega_{54}N_{5} - \omega_{43}N_{4} - \omega_{c}N_{0}N_{4} - A_{3}N_{3}$$

$$\frac{dN_{5}}{dt} = \sigma\rho N_{5} - \omega_{54}N_{5} - \omega_{53}N_{5}$$
(15)

By solving the equations, population of N_i can be obtained

$$N_{5} = \frac{\sigma \rho N_{0}}{\omega_{54} + \omega_{43}} \propto \rho$$

$$N_{4} = \frac{\omega_{54}}{A_{4} + \omega_{43} + \omega_{C} N_{0}} N_{5} \propto \rho$$

$$N_{3} = \frac{\omega_{53}}{A_{3}} N_{5} + \frac{\omega_{43}}{A_{3}} N_{4} \propto \rho$$

$$N_{1} = \frac{\omega_{C} N_{0}}{A_{1}} N_{4} \propto \rho$$
(16)

Thus one can find that red emission N_3 is also linear proportional to pump power, which is in good agreement with results from Figure 8.

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[2] Haase, M.; Schäfer, H., Upconverting Nanoparticles. Angew. Chem. Int. Ed. 2011, 50, 5808-5829.