

In the one dimensional case, the solution to Eq. (5) with time dependent diffusivity is well known

$$C(x, t) = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left[\operatorname{erf} \left(\frac{x - (n+1)L}{\sqrt{4\Theta(t)}} \right) - \operatorname{erf} \left(\frac{x - nL}{\sqrt{4\Theta(t)}} \right) \right] + \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left[\operatorname{erf} \left(\frac{x+(n+1)L}{\sqrt{4\Theta(t)}} \right) - \operatorname{erf} \left(\frac{x+nL}{\sqrt{4\Theta(t)}} \right) \right]. \quad (\text{S1})$$

Eq. (6) can be further simplified by applying Gauss's theorem:

$$DSC(t) = \Delta H \int_0^L \frac{\partial}{\partial t} C(z, t) dz = 2D(t)\Delta H \left. \frac{\partial C(x, t)}{\partial x} \right|_{x=0} \quad (\text{S2})$$

Combining the above two equations yields Eq. (7). To obtain Eq. (9), an approximation must be made as shown in the following equation

$$\Theta(t) \approx \frac{RT^2}{\beta E_a} D(t). \quad (\text{S3})$$

which comes from the asymptotic expansion. This approximation results in less than a 1% error in the temperature range of $0^\circ\text{C} < T < 1300^\circ\text{C}$ and the activation range of $30\text{kJ/mol} < E_a < 300\text{kJ/mol}$. Substituting this approximation into Eq. (7) gives

$$DSC(t) = \left(1 - 2e^{-\frac{1}{4\Theta_t}} + e^{-\frac{1}{\Theta_t}} + \dots \right) \frac{\sqrt{\Theta_t} E_a}{RT} \quad (\text{S4})$$

$$\Theta_t \equiv \frac{E_a K_0}{R\beta L^2} \left(\frac{RT}{E_a} \right)^2 e^{-\frac{E_a}{RT}}.$$

To locate the DSC curve peak position, ξ_t is treated as an independent variable, and a value for ξ_p is found that makes $DSC'(\xi_p)$ zero, and then T_p is found from ξ_p .

$$DSC'(\Theta_t) = \frac{\partial}{\partial t} \left[\left(1 - 2e^{-\frac{1}{4\Theta_t}} + e^{-\frac{1}{\Theta_t}} + \dots \right) \frac{\sqrt{\Theta_t} E_a}{RT} \right] \approx \frac{\partial}{\partial t} \left[\left(1 - 2e^{-\frac{1}{4\Theta_t}} + e^{-\frac{1}{\Theta_t}} + \dots \right) \sqrt{\Theta_t} \right] \cdot \frac{E_a}{RT} \quad (\text{S5})$$

An approximate is made because E_a/RT always changes slowly compared to Θ_t and therefore it can be viewed as a constant in the derivatives. The root of the above equation is independent of any system parameters can be found numerically as:

$$\Theta_p \approx 0.0023 \quad (\text{S6})$$