Supplement

Predictions of confidence-resolution

Here we show simulations for confidence based on: i) integration to boundary + post-decision evidence, ii) vs Signal-Detection for total evidence when the number of samples is fixed. The algorithm of integration to boundary was simulated by sampling pairs of values from two normal distributions: *Left* ~ $N(52, 15^2)$ and *Right* ~ $N(48, 15^2)$, and accumulating the differences of each pair ($\sum_{i=1}^{N} Li - Ri$). Two types of decision mechanisms were contrasted: i) Free-response/integration to a fixed boundary, in which trials were terminated when the accumulated evidence reached a predefined decision boundary, and ii) Interrogation, in which decision was made after a fixed number of samples. The fixed boundary was set to 43 to equalize the mean accuracy of both decision mechanisms (accuracy = 0.72). The mean number of samples was 6.6 for the Free-response protocol, and was set to 10 for the interrogation protocol 10. In both simulations evidence values higher than the .995 quantiles or lower than the .005 quantiles were considered outliers and excluded than the analyses. Including these values did not change the results.

Free Response. Confidence for the free-response decision mechanism was determined following the 2DSD mechanism (Pleskac & Busemeyer, 2010, see Fig. S1A). For each simulated trial an extra sample was drawn from the Left & Right distributions. Confidence was computed based on the consistency between the extra-sample and choice. For example, if a participant chose the left alternative and the extra-sample had a high positive value (i.e., the left number was higher than the right one), then the confidence level would be high. As opposed to that, if the extra-sample had a high negative value (i.e., the left number is lower than the right number), the confidence level would be low. Specifically, we calculated the normalized confidence using the following formula:

$$Normlizedconfidence_{Freeresponse} = \begin{cases} \frac{ES - \min(ES)}{\max(ES) - \min(ES)}, & Left\ choices \\ \frac{(ES - \max(ES))}{\max(ES) - \min(ES)}, & Right\ choices \end{cases}$$

where ES is the value of the extra-sample (i.e., L_{ES} - Y_{ES}), and max(ES)/ min(ES) are the maximum/minimum values of the extra-samples across all trials.

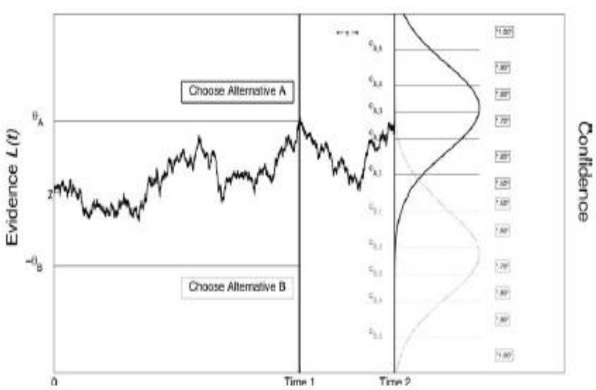
Interrogation. Confidence for the interrogation protocol was determined based on the SDT, using the value of the integrated evidence at the moment of response (see Fig. S1B). The H_1 distribution corresponds to the case in which $\mu_{Left} > \mu_{Right}$ (i.e., the correct response is left), and the H_2 distribution corresponds to the case in which the $\mu_{Left} < \mu_{Right}$ (i.e., the correct response is right).

Without loss of generality, we consider here only the case in which t$\mu_{Left} > \mu_{Right}$, as the other case is symmetrical. For each trial, Confidence was calculated as the absolute value of the difference between the integrated evidence at the moment of response of the criterion (i.e., 0). These values were normalized as follows:

$$Normlizedconfidence_{Interrogation} = \frac{|AC - \min(AC)|}{\max(AC)}$$

where AC is the value of the accumulated evidence at the moment of response ($\sum_{i=1}^{N} Li - Ri$), and max(AC)/min(AC) are the maximum/minimum values of the integrated evidence at the moment of response across all trials.

(A)



(B)

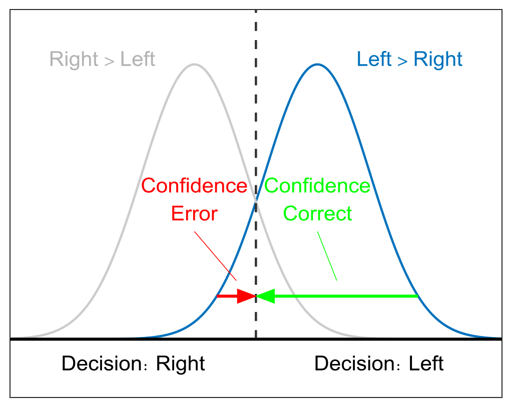


Fig. S1. Confidence resolution in the Free-response and interrogation protocols. (A) Confidence for the free-response protocol was determined by drawing an extra-sample and computing its consistency with choice following the 2DSD mechanism (Pleskac & Busemeyer, 2010). (B) Confidence was for the interrogation protocol was determined based on the SDT framework by computing the absolute distance of the accumulated evidence at the moment of decision from the criterion. Note that in Fig. 3 in the main text we only present the H_1 distribution ($\mu_{Left} < \mu_{Right}$), as the results are symmetrical for the H_2 distribution ($\mu_{Left} < \mu_{Right}$).

Additional computational results

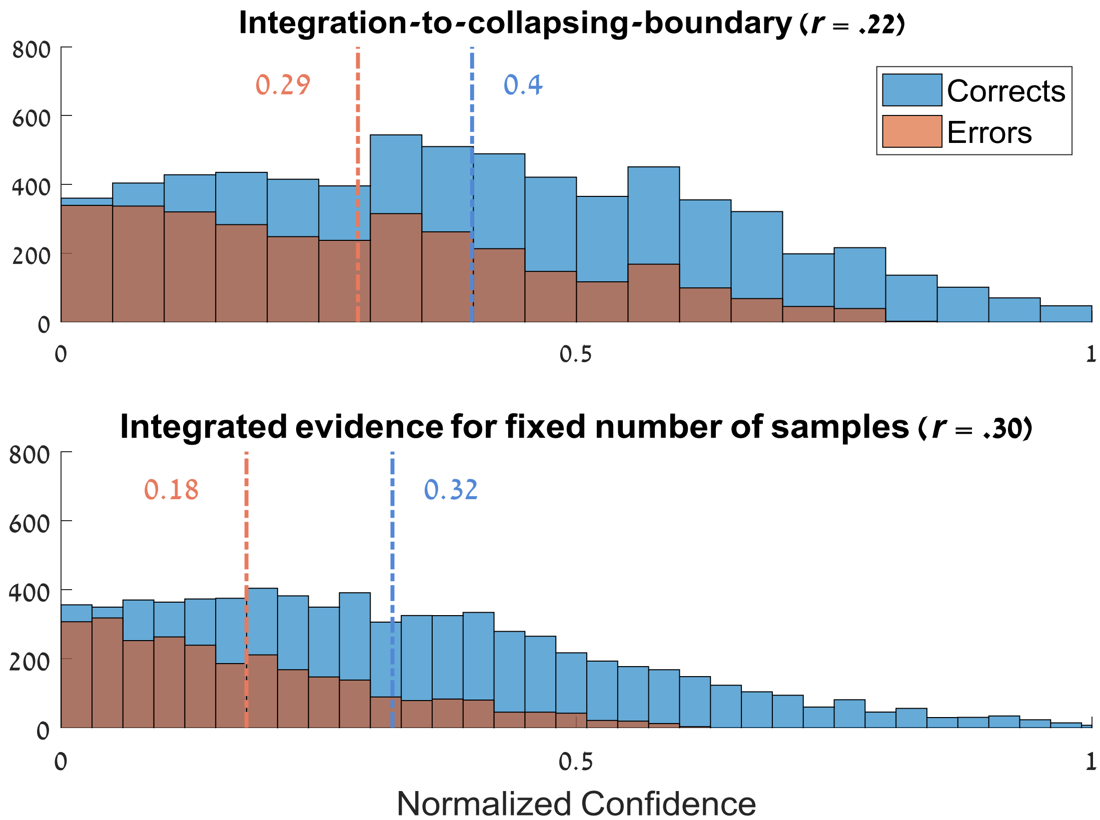


Fig. S2. *Confidence distributions for free response and interrogation protocols. As in Fig 3, but here the fixed boundary is replaced with a collapsing one* (Hawkins et al., 2015). *The figure shows the distribution of confidence levels for correct (blue) and incorrect (red) responses*.

Computational methods

Model-selection (choice)

For each participant in session 1 we fitted 3 models to their decisions (conditional on actual evidence samples and on RT):

i) Diffusion with fixed boundary (the evidence difference was integrated subject to external noise) to two fixed boundaries.

ii) Diffusion with collapsing boundary (we used a 3 parameter Weibull parametrization of the collapsing boundary (Hawkins et al., 2015):

$$u(t) = a - \left[1 - exp\left(-\left(\frac{t}{\lambda}\right)^k\right)\right] \cdot (a - a')$$

iii) Vickers (1970) model. Here if the difference is favors the left stream it is accumulated in a left-accumulator, and if it favors the right stream it is accumulated in a separate right-accumulator. RT is determined by the race of the two accumulators.

All the models included additional external noise (Normally distributed). Models ii) and iii) allow accounting for positive confidence-resolution in session 1 (free response). For session 2 (interrogation) we assumed that decisions were based on signal-detection theory, which is applied to the total-evidence (criterion 0). The model classification strongly favors model ii (Diffusion with collapsing boundary) for the free response session. Only 1 subject was classified as better supported by the Vickers model (see Table 3 in Supplement).

***Model-selection* (confidence)**

We used several models to predict confidence using a regression analysis. The models were consisted of different combinations of a few predictors (See Tables 4 and 5 in Suppl for model comparison):

1. Accumulated evidence = $\sum_{i=1}^{n} x_i - y_i$

Where for each sample i, x= the sequence selected by the participant; y= the unselected sequence.

Leaky accumulated evidence = $\sum_{i=1}^{n} (x_i - y_i)\ \lambda^{n-i}$

2. RT (number of samples in each trial)
3. Last item evidence = $x_n - y_n$
4. Last item evidence-1= $x_{n-1} - y_{n-1}$
5. Stop boundary point. Each participant's data was fitted to a collapsing boundary diffusion model. This predictor is correlated with RT and indicates the point on the boundary in which the participant took the decision in every trial.
6. Split evidence= $b_1 x - b_2 y$

We used BIC measure fits for model classification.

Additional data in Experiment 2

i)RT for correct and incorrect trials

To understand sources of confidence resolution in session 1 we tested if correct trials were faster than incorrect trials; we created two RT histograms (for correct and incorrect trials) for each participant and averaged quantiles 10, 30, 50, 70 and 90 across participants. A t-test showed that the median RT was significantly faster for correct trials compared to incorrect trials ($t(34)=4.3 p<0.001$).

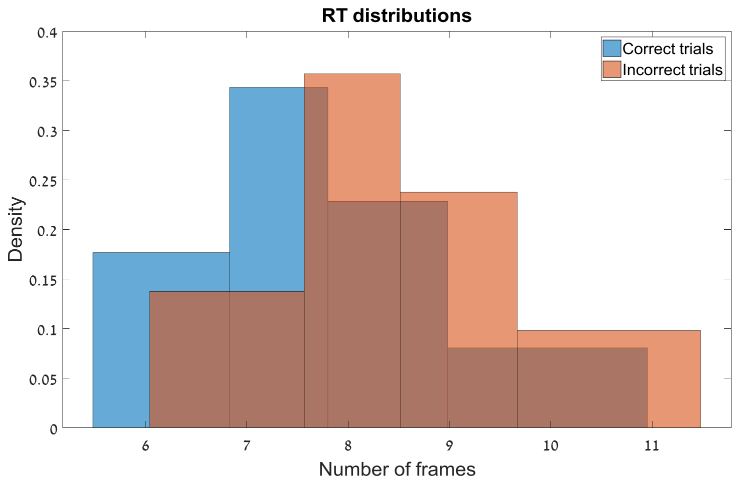


Fig S3. RT distributions (number of frames) for correct and incorrect responses in the free-response session. The faster RT for correct choices explains how a strategy that uses RT to inform confidence can generate some degree of confidence resolution.

ii) Multiple regression of confidence from RT and accumulated evidence in Exp-2.

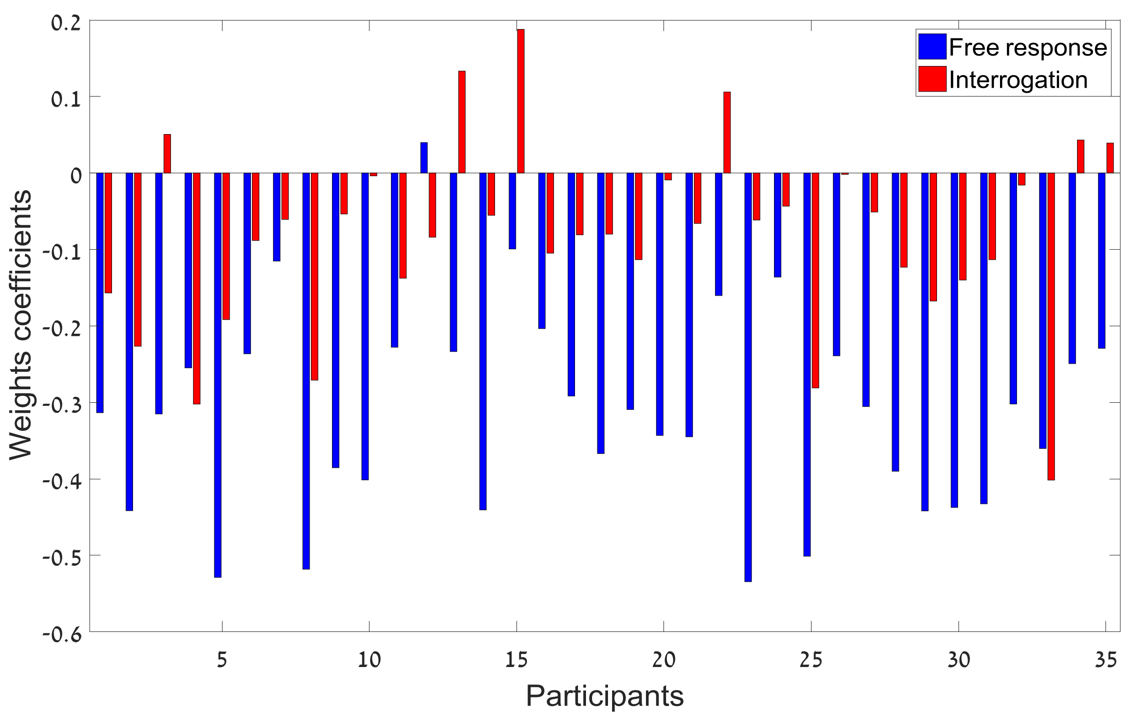


Fig S4. RT coefficients as a confidence predictor in a multiple regression that includes accumulated evidence as a second predictor, for each participant in the free-response and the interrogation session in Experiment 2. Group coefficients is -.31 (SD=.13) for the free response, and -0.08 (SD=.12) for the interrogation task.

iii) Reversed correlation for selected/unselected input with choice in Exp-2, interrogation session.

To examine if participants rely on an implicit boundary in their choices in the interrogation session (Kiani et al., 2008), we carried out a reverse correlation of the presented evidence based on choices made in all trials (see Kiani et al., for methological details). While implicit boundary models predict primacy (early evidence has more impact on the decision, as late evidence may arrive after boundary is reached), the signal detection model based on integrated evidence (or leaky integrated evidence) predicts either uniform temporal weights or recency. As shown in Fig S4, the data shows no primacy and possibly a small recency.

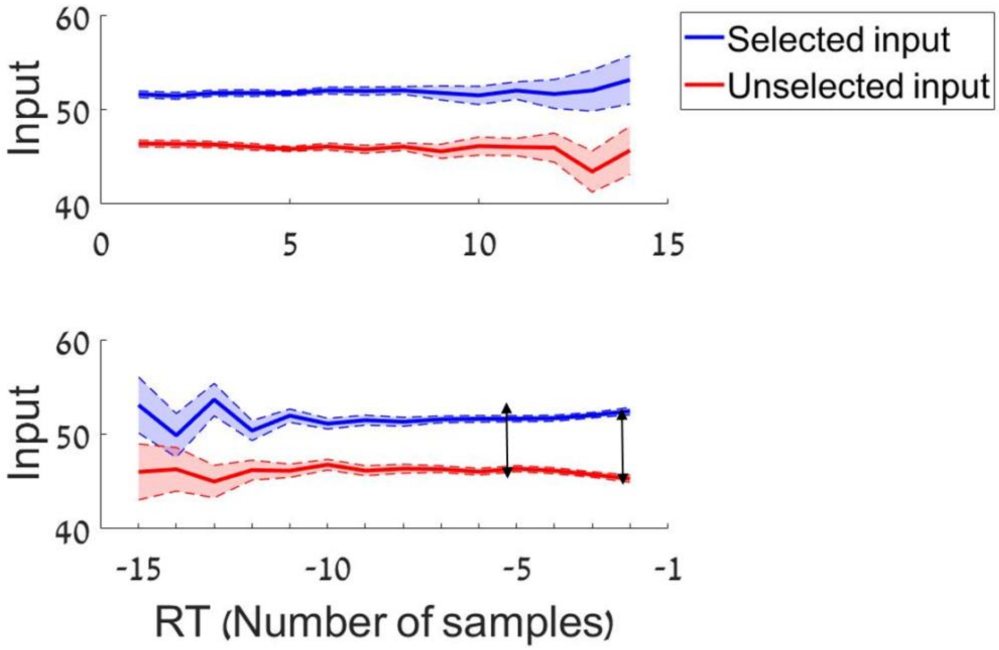


Fig S5. Reverse correlation of choices based on the two evidence streams (red-selected and blue-unselected) in Experiment 2, session-2. The lines correspond to the average of the two streams across all trials and the shade corresponds to confidence intervals. Top panel: the evidence streams are time locked to the start of the trial. Bottom panel, the evidence streams are time locked to the response. The data indicates a small recency (the two black arrows have the same height).

Choice models comparison:

Table-3: Free-response task

Subject	Diffusion model Fixed	Diffusion model Collapsed	Accumulator model
1	1108	**800**	1022
2	947	**784**	893
3	1073	**831**	1074
4	1041	**835**	1012
5	908	**783**	849
6	1281	**1057**	1318
7	958	**907**	1044
8	1063	**819**	1010
9	1042	**863**	1041
10	1053	**773**	869
11	981	**848**	977
12	920	**808**	946
13	1028	**813**	1037
14	981	**762**	931
15	1044	**887**	1088
16	943	**799**	931
17	981	**877**	995
18	778	699	**675**
19	1006	**742**	956
20	933	**757**	909
21	998	**730**	841
22	1091	**861**	1104
23	1006	**818**	929
24	1091	**757**	977
25	983	**865**	906
26	976	**822**	1065
27	973	**789**	956
28	947	**731**	845
29	872	**680**	817
30	1020	**741**	912
31	858	**702**	856
32	1015	**601**	1023
33	953	**853**	979
34	1047	**787**	1007
35	1094	**838**	1067
Average	**1000**	**801**	**967**

CONFIDENCE

Free response
Table-4

Model	BIC	R	(-2)*LL
LastItemEvidence_RestEvidence_RT	346.50	0.57	326.88
LastItemEvidence_RT	352.42	0.53	337.71
SplitLastItem_RT	354.97	0.53	335.35
LastItemEvidence_StoppingPointBoundary	355.14	0.51	340.42
AccumulatedEvidence_RT	355.30	0.51	340.59
LastItemEvidence_StoppingPointBoundary_RT	355.70	0.53	336.08
LeakyIntegratedEvidence	356.86	0.51	342.00
SplitLastItem_StoppingPoint	357.64	0.52	338.02
LastItemEvidence_RestEvidence_RT	360.60	0.48	345.89
LastItemEvidence_LastItemEvidence-1_Rest	361.79	0.50	342.17
AccumulatedEvidence	368.16	0.41	358.35
SelectedSamples_UnselectedSamples	374.37	0.50	340.04

Interrogation
Table-5

Model	BIC	R	(-2)*LL
AccumulatedEvidence	357.65	0.48	347.84
LeakyIntegratedEvidence	358.3	0.50	343.59
LastItemEvidence_RestEvidence	359.32	0.49	344.6
AccumulatedEvidence_RT	359.86	0.49	345.14
LastItemEvidence_RestEvidence_RT	361.84	0.50	342.23
LastItemEvidence_LastItemEvidence-1_RestEvidence	362.42	0.50	342.81
LastItemEvidence_LastItemEvidence-1_RestEvidence_RT	361.13	0.50	340.61
LastItemEvidence_RT	389.41	0.25	374.70
SelectedSamples_UnselectedSamples	392.82	0.37	358.50
SplitLastItem_RT	393.08	0.25	373.47

In the free response session, we see that the models that best account for confidence include RT, and the Last-item evidence (in some of the trials the latter involves post-decision integration). Both RT and post-decision integration can indicate correct choices and can mediate the smaller confidence resolution in the free response session. We also examined if there is an asymmetry in the weight given to chosen vs. unchosen evidence in confidence (Zylberberg, Barttfeld & Sigman, 2012). While the weight appeared higher for the chosen evidence this was not significant and it did not win in terms of BIC. In the interrogation session, the accumulated evidence appears to be the best predictor for confidence. Leaky-integration comes second (in terms of BIC, but higher in Log-Likelihood).

Wagering predictions

We consider a wagering version of our task. In this wagering version, the participants would view samples from the same distributions as in our experiments [see Fig. 2; normal distributions: *Left* ~ $N(52, 10^2)$ and *Right* ~ $N(46, 10^2)$] and choice accuracy is potentially rewarded in the following way: 1 point for each correct response, -3 points for each incorrect response. The subjects, however, have the possibility to wager on their choice based on their internal decision confidence. If they feel confident they wager (which means that their potential reward will count and accumulate towards the total reward), while if they are not confident they do not wager (which means that no reward will be accumulated on that trial). [1] Since this is based on a binary level of confidence, we transformed model-confidence to two levels based on a median split (across trials). Thus, in each trial a subject could accumulate the following number of points: i) +1, in case of a correct choice and confidence was higher than median confidence, ii) -3 in case of an incorrect choice and confidence which was higher than median confidence, iii) 0, in case confidence was lower than the median confidence. The total wagering score is the total number of accumulated points over all trials.

We compared three choice and confidence mechanisms on their wagering performance. The mechanisms we considered are: i) integration to a fixed boundary (for choice), followed by 1 postdecision sample (for confidence), ii) integration to a collapsing boundary (for choice and for confidence), iii) integration over a fixed number of samples (using SDT for both choice and confidence; see Fig S1). The boundaries (in the first two models, and the number of samples in the third model, were selected so as to obtain an equal average number of samples (RT), across models (this was selected to be 7, which is in the range of experimental data).

The algorithm for **choice** in each strategy was simulated by sampling pairs of values from the two distributions and accumulating the differences of each pair ($\sum_{i=1}^{N} Li - Ri$). Trials were terminated when the accumulated evidence reached a predefined decision boundary (integration to fixed\collapsing

[1] Since the rewards are asymmetric with losses higher than gains, it is not worth to wager on trials on which one is at chance of being correct.

boundaries) or after a fixed number of samples (fixed timer strategy). In order to equalize the mean number of samples for each strategy (7 samples), the fixed boundary was set to 67 and the collapsing boundary was defined by the Weibull function:

$$u(t) = a - \left[1 - exp\left(-\left(\frac{t}{\lambda}\right)^k\right)\right] \cdot (a - a')$$

where the intercept (a) was set to 205, the asymptote (a') was set to 20 and the scale parameter (λ) was set to 6.8.

The confidence mechanism was simulated exactly the same as described above in the predictions of confidence resolution section (extra sample for integration to fix boundary and total evidence for collapsing boundary and random timer strategies). As shown in Fig. S6, a collapsing boundary mechanism (dark blue) which gives a compromise between the fixed boundary and fixed number of samples mechanisms (light blue and red, respectively), in accuracy (left panel) and in confidence resolution (middle panel), results here in the highest wagering score (right panel). This is because wagering combines accuracy and confidence resolution.

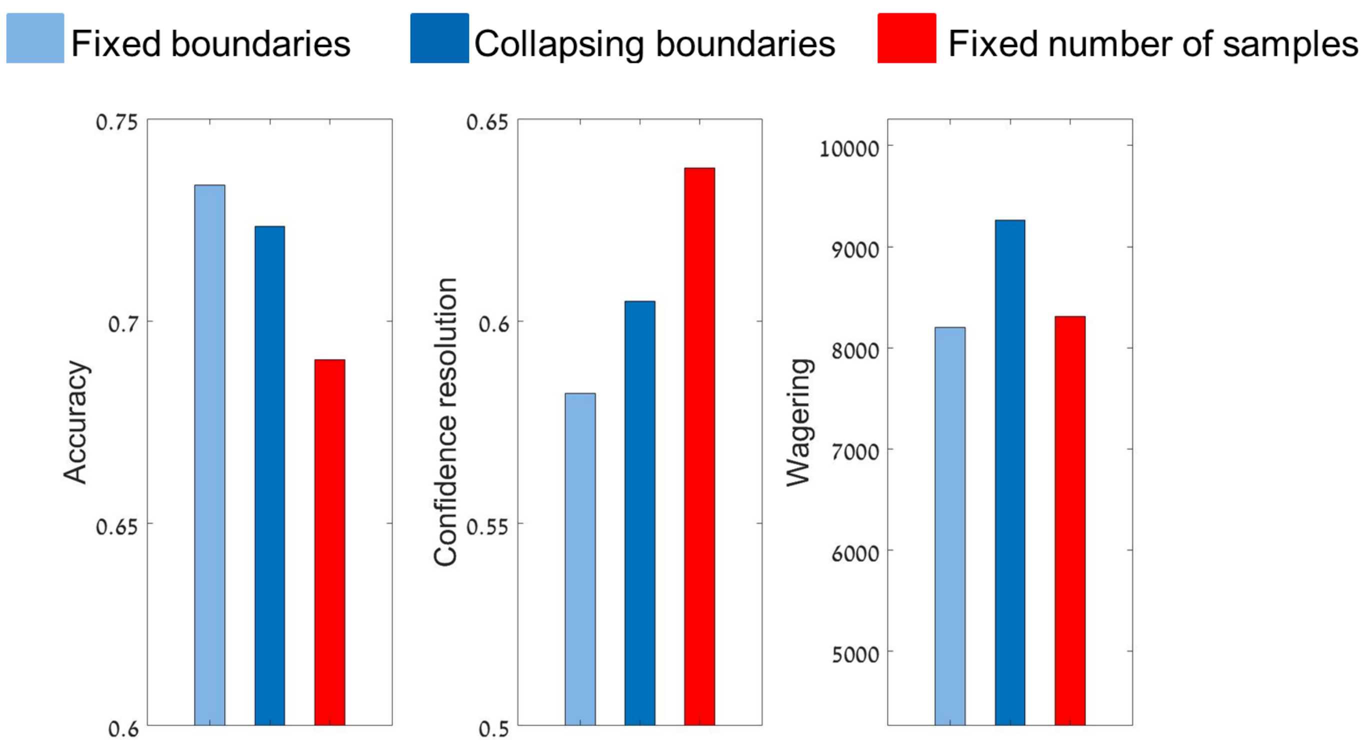


Fig. S6. Simulation for the three strategies: i) Integration to fixed boundaries ii) Integration to collapsing boundaries iii) Fixed timer. Left panel. Accuracy rate for each strategy. Middle panel. Confidence resolution for all strategies measured in type 2 AUROC. Right panel. Wagering score for each strategy which takes into consideration both accuracy and confidence resolution.

Control experiment

The aim of this study was to rule out a possible confound of a practice effect in metacognition performance, which could give an alternative explanation for the performance improvement in experiment 3.

Methods

Participants. 35 undergraduates from Tel-Aviv University (21 females; age: M=22, range 21-26 years) participated in the experiment. The participants received course credit in exchange for taking part in the experiment. The experiment was approved by the ethics committee at TAU.

Procedure and design. The task was similar to experiment 3 with one exception: I) The two sessions were a self-terminated task (Free response). The number of trials for each session was 80 (same as in experiment 3). In this case, if the metacognition improvement was due to a practice effect, we would expect an improved metacognition performance in the second session, in this experiment as well. See Results in the main text (Discussion section).