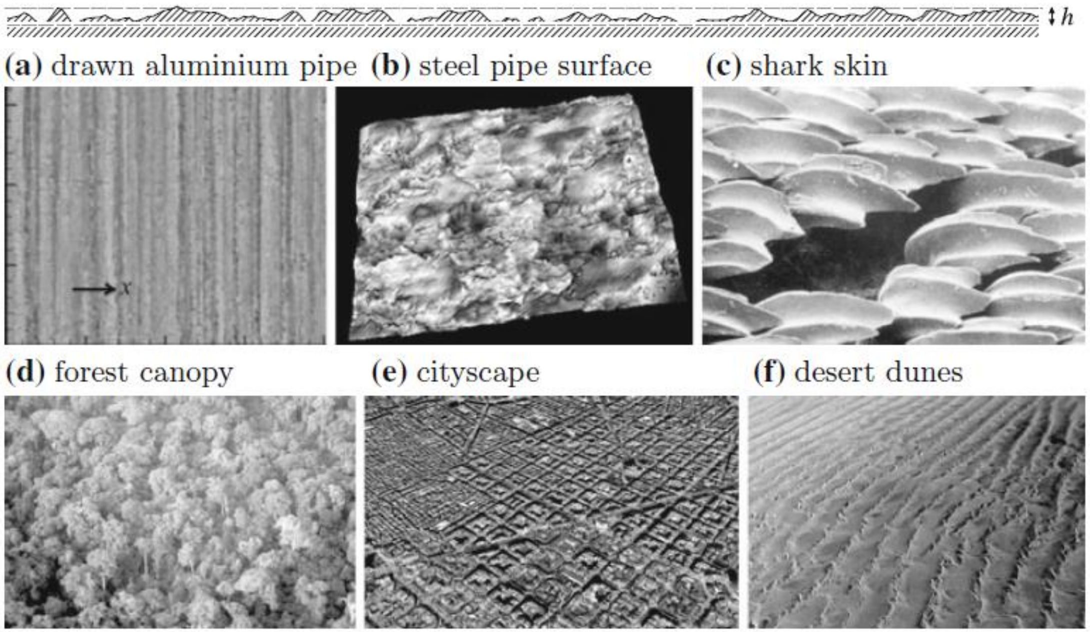
Surface roughness

Surface roughness

- So far, we have only considered smooth walls.
- In reality, every wall and material is characterized by small irregularities, which we refer to as roughness.
- The roughness has a characteristic height *h*, as shown in the figure below.
- Wall roughness increases the wall shear stress and heat transfer rate.

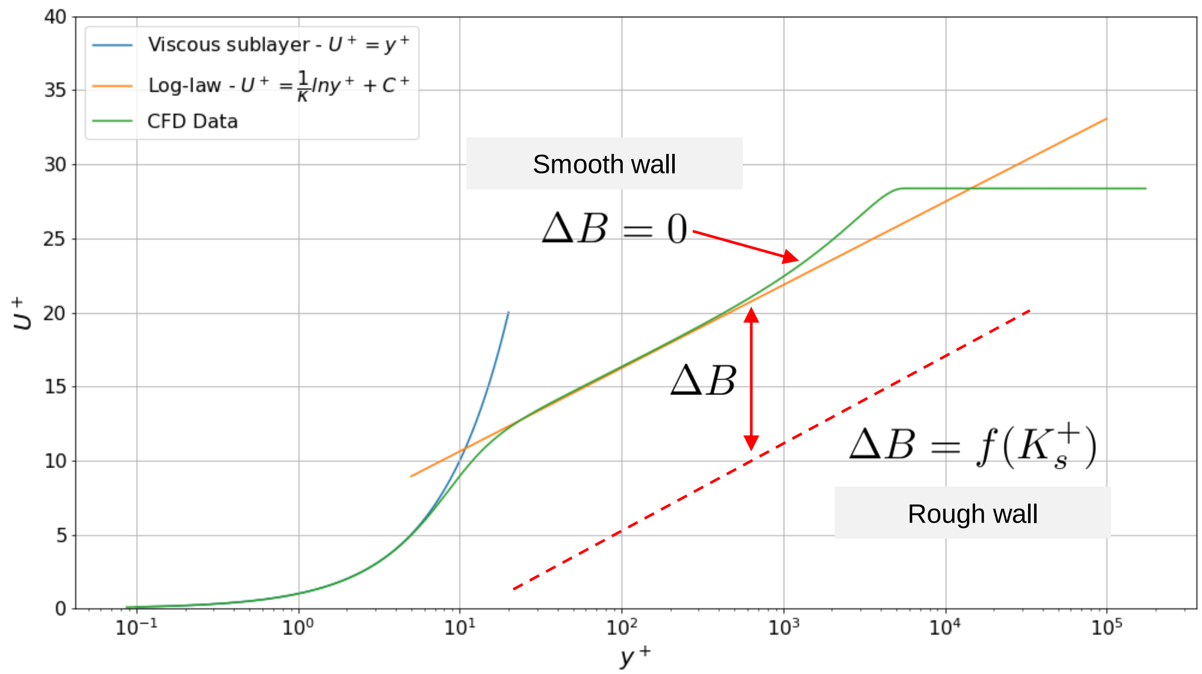


Examples of wall roughness. **(a)** Surface of an aluminum pipe: $h_{rms} \approx 0.16$ µm. **(b)** Scanned surface (1.4 x 1.1 mm^2) of a non-rusted commercial steel pipe: $h_{rms} \approx 5$ µm. **(c)** Scales of the great white shark: $h_{rms} \approx 0.1$ mm. **(d)** Aerial views of tropical forest in Gabon (h ≈ 0.1-10 m). **(d)** Barcelona landscape (h ≈ 10-100 m). **(f)** The Namib desert (h ≈ 10-500 m). Adapted from reference [1].

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.

Surface roughness

- In CFD, the roughness must be modeled.
- In theory, it can be resolved with very fine meshes but the computational requirements are too high.
- Wall roughness increases the wall shear stress and heat transfer rate.
- It also breaks up the viscous sublayer.
- By looking at the nondimensional velocity plot, the wall roughness shifts the nondimensional velocity downwards by a factor of ΔB.



Surface roughness

- There are many ways to add roughness to the solution.
- Let us study maybe the most common wall function for roughness.
- This implementation is based on the standard wall functions.

STEP 1.

$$U^+ = \frac{1}{\kappa}\ln\left(Ey^+\right) - \Delta B$$

Roughness correction coefficient

STEP 2.

$$U^+ = \frac{1}{\kappa}\ln\left(Ey^+\right) - \ln\left(e^{\Delta B}\right)$$

STEP 3.

$$U^+ = \frac{1}{\kappa}\ln\left(\frac{Ey^+}{e^{\Delta B}}\right)$$

- Where we used the following logarithm rules to derive the previous relations,

$$\ln\left(e^x\right) = x \qquad \ln\left(x\right) - \ln\left(y\right) = \ln\left(\frac{x}{y}\right)$$

Surface roughness

- If we introduce the following relation,

$$E' = \frac{E}{e^{\Delta B}}$$

- Into the relation corrected for wall roughness,

$$U^+ = \frac{1}{\kappa} \ln \left(\frac{E y^+}{e^{\Delta B}} \right)$$

- We obtain the following equation,

$$U^+ = \frac{1}{\kappa} ln \left(E' y^+ \right)$$

- Which is identical to the standard wall function formulation (except for the variable E').
- We now have a way to work with smooth and rough walls using the same log-law relation.

Surface roughness

- Let us now address the roughness correction factor ΔB.
- This factor can be computed as follows [1, 2],

$$\Delta B = 0 \qquad K_s^+ < 2.25$$

Hydraulically smooth

$$\Delta B = \frac{1}{\kappa}\left(\frac{K_s^+ - 2.25}{87.75} + C_s K_s^+\right) \times \dots$$

$$\dots \times \sin\left[0.4258\left(\ln K_s^+ - 0.811\right)\right] \qquad 2.25 < K_s^+ < 90$$

Transitional

$$\Delta B = \frac{1}{\kappa}\ln\left(1 + C_s K_s^+\right) \qquad K_s^+ > 90$$

Fully rough

- Notice that ΔB is a function of the nondimensional roughness height K_s^+ and the roughness constant C_s.

[1] T. Cebeci, P. Bradshaw. Momentum Transfer in Boundary Layers. Hemisphere Publishing Corporation. 1977.
[2] P. Ligrani, R. Moffat. Structure of transitionally rough and fully rough turbulent boundary layers. J. of Fluid Mechanics, 162, 69-98. 1986.

Surface roughness

- The hydraulically smooth condition exists when roughness heights are so small that the roughness is buried in the viscous sublayer.
- The fully rough flow condition exists when the roughness elements are so large that the sublayer is completely eliminated, and the flow can be considered as independent of molecular viscosity; that is, the velocity shift is proportional to $\ln(K_s^+)$.
- The transitional region is characterized by reduced sublayer thickness, which is caused by diminishing effectiveness of wall damping.
- Because molecular viscosity still has some role in the transitional region, the geometry of roughness elements has a relatively large effect on the velocity shift.

Surface roughness

- The nondimensional roughness height can be computed as follows,

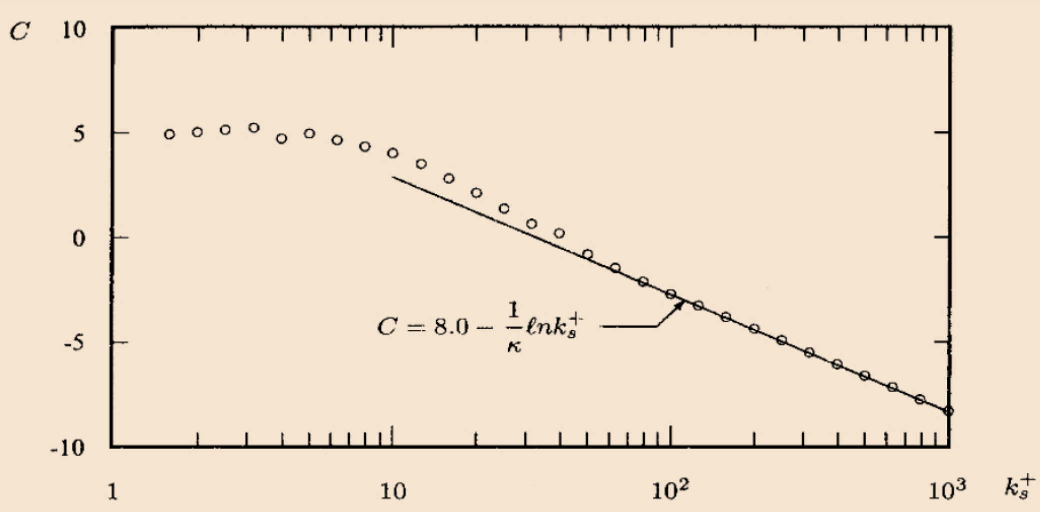
$$K_s^+ = \frac{\rho K_s u^*}{\mu}$$

Similar to the approach taken for y^+

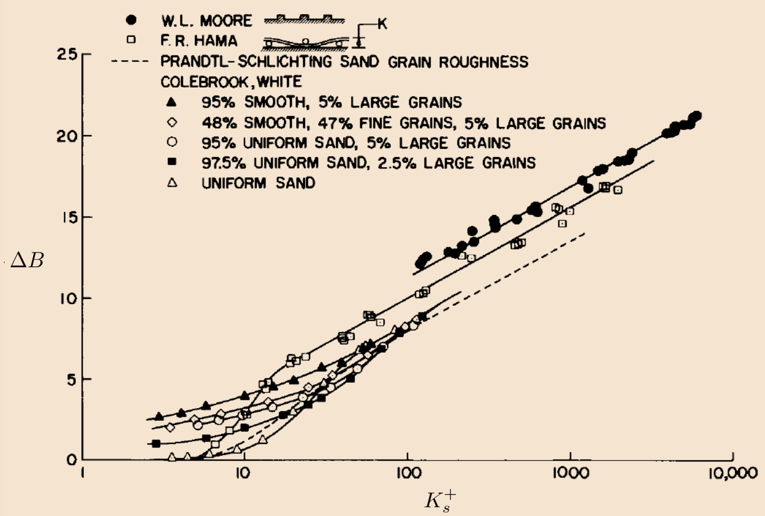
- Where K_s is the typical roughness height (sand grain diameter).
- The roughness constant C_s is often equal to 0.5. This constant represents the shape and distribution of the roughness elements (sand grains).
- It is recommended to fix C_s and adjust K_s^+.
- Remember, in our discussion, the roughness regime is subdivided into the three regimes.
    - Hydraulically smooth.
    - Transitional.
    - Fully rough.

Surface roughness

- The subdivision of the roughness regime and the dependence of the constant *C* (law of the wall) and the roughness correction factor ΔB are supported on experimental data (Nikuradse's data [1]).
- The constant *C* and the roughness correction factor depend on the roughness parameters K_s^+ and C_s.



Constant in the law of the wall as a function of surface roughness. Based on measurements of Nikuradse [1]. Adapted from [2].

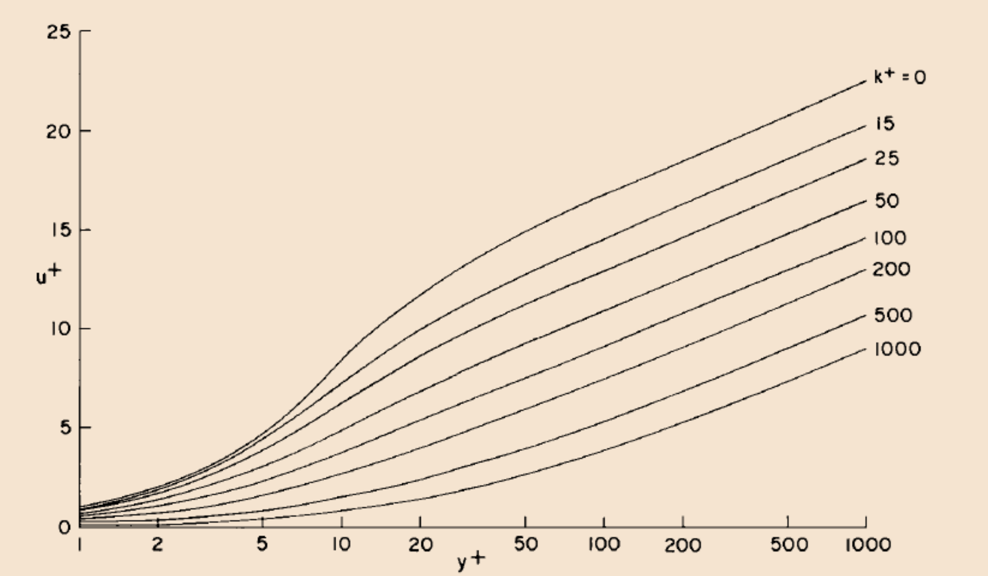


Effect of wall roughness on the roughness correction factor and universal velocity profiles [3].

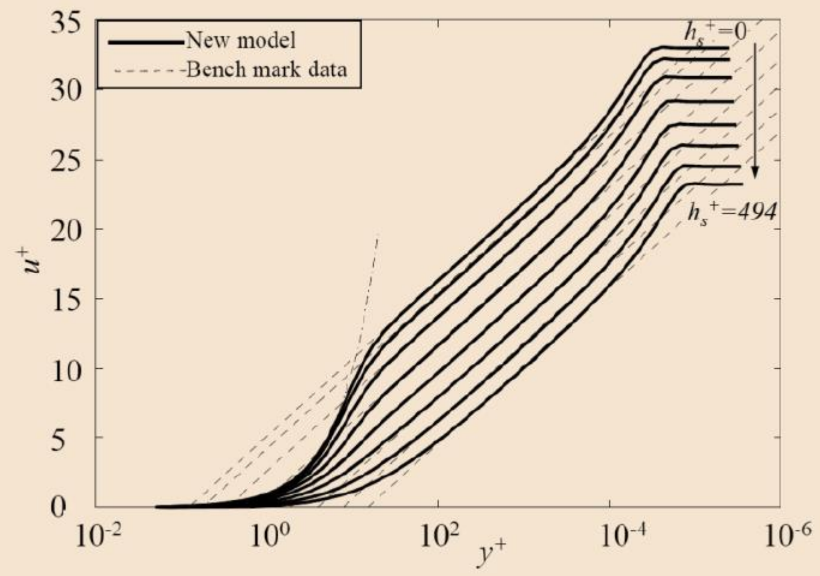
[1] J. Nikuradse. Law of Flow in Rough Pipes. Technical Memorandum 1292, National Advisory Committee for Aeronautics. 1950.
[2] D. Wilcox. Turbulence modeling for CFD. DCW Industries, Inc. 2006.
[3] F. Clauser. The turbulent boundary layer. Advan. Appl. Mech. 4, 1. 1956.

Surface roughness

- Plots of mean velocity distribution for uniform roughness at several K_s^+ values.



Plots of mean velocity distribution for uniform roughness at several values of nondimensional roughness height [1].



Plot of roughness mean velocity profiles [2].

[1] T. Cebeci, A. M. O. Smith. Analysis of turbulent boundary layers. Academic Press. 1974.

[2] U. Oriji, X. Yang, P. Tucker. Hybrid RANS/ILES for Aero Engine Intake. Proceedings of the ASME Turbo Expo 2014. Paper No: *GT2014-26472*. 2014.

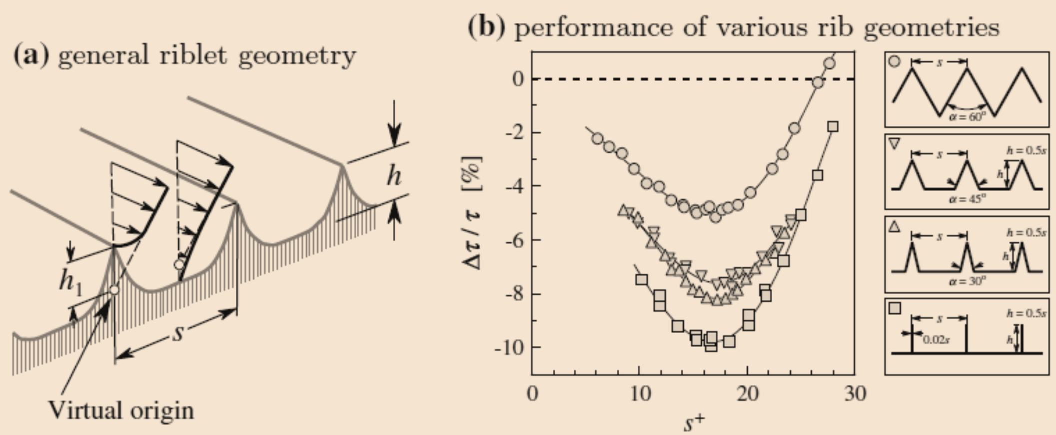
Surface roughness

- There is no universal roughness function valid for all types of roughness.
- Many methods to take into account the surface roughness are available in the literature, just to name a few,

- [1] T. Cebeci, P. Bradshaw. Momentum transfer in boundary layers. Hemisphere Publishing Corporation. 1977.
- [2] P. Ligrani, R. Moffat. Structure of transitionally rough and fully rough turbulent boundary layers. J. of Fluid Mechanics, 162, 69-98. 1986.
- [3] B. Aupoix, R. Spalart. Extensions of the Spalart-Allmaras turbulence model to account for wall roughness. Int. J. of Heat and Fluid Flow. 24. 2003.
- [4] D. Wilcox. Turbulence modeling for CFD. DCW Industries, Inc. 2006.
- [5] U. Oriji, X. Yang, P. Tucker. Hybrid RANS/ILES for aero engine intake. Proceedings of the ASME Turbo Expo 2014. Paper No: *GT2014-26472*. 2014.
- [6] B. Krishnappan. Laboratory verification of a turbulent flow model. J. of Hydraulic Eng. 110. 1984.
- [7] J. Yoon, V. Patel. A numerical model of flow in channels with sand-dune beds and ice covers. IIHR Report no. 362. 1993
- [8] J. Rotta. Turbulent boundary layers in incompressible flows. Progress in Aerospace Science. 1982.

Surface roughness

- Finally, as we have seen, a rough wall increases friction.
- But it happens that special rough surfaces can increase drag reduction.
- Well designed riblets can shift the nondimensional velocity profile upwards, and closer to the viscous law non-dimensional profile.
- It appears that small riblets that are aligned with the flow direction can achieve a significant drag reduction up to 10%.
- The maximum drag reduction occurs for a rib spacing s between 14 and 20 viscous wall units, i.e., $14 < s^+ < 20$, as illustrated in the figure.
- This is an example of biomimetics, since the development and application of ribbed surfaces has been inspired by the presence of ribbed scales on sharks

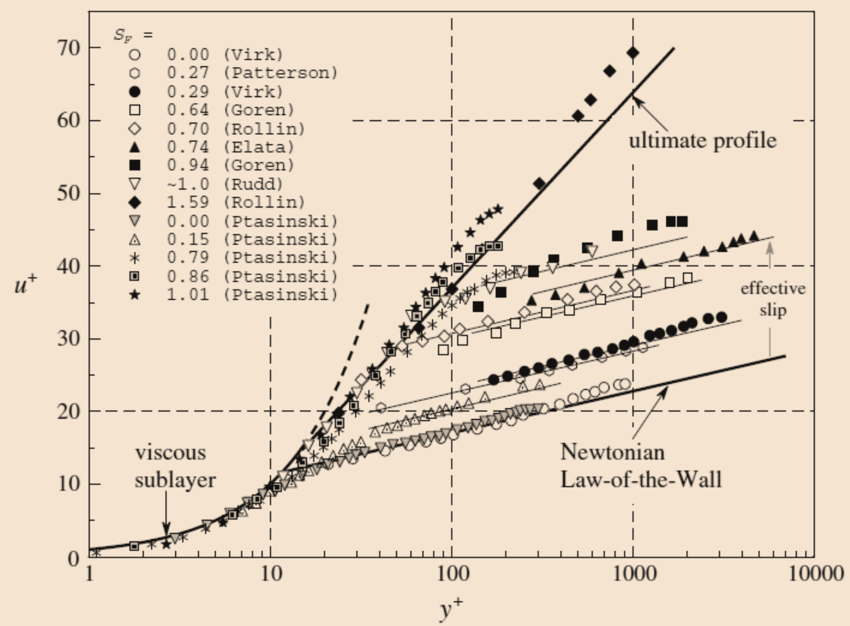


a. General geometry of a ribbed surface with a rib spacing s and rib height h. b. Drag reduction performance of various rib geometries. [1]

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.

Surface roughness

- Besides using riblets for drag reduction, a similar effect can be achieved by polymer additives.
- Small amounts of certain polymer additives to fluids can achieve a significant reduction of friction drag, which is know as the Toms effect [1, 2, 3].
- The figure below shows experimental data of the measured velocity profiles for various flows with polymer additives.
  - The profiles for drag-reducing flows have logarithmic profiles with the same slope as for a Newtonian fluid, but with an offset, or effective slip.



The mean velocity profiles for various cases of polymer drag reduction.

[1] B. Toms. Some observations on the flow of linear polymer solutions through straight tubes at large Reynolds numbers. In Proc. 1st. Int. Congr. Rheol., vol. 2, pp. 135–141. 1948.
[2] P. Virk. Drag reduction fundamentals. AIChE J. 21, 625–656. 1975.
[3] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.