Non-linear eddy viscosity models

Non-linear eddy viscosity models

- One approach to achieving a more appropriate description of the Reynolds-stress tensor without introducing any additional transport equations (as in the RSM models) is to add extra high order terms to the Boussinesq approximation.

$$\rho\overline{u_i' u_j'} = \frac{2}{3}\rho k \delta_{ij} - 2\mu_t S_{ij} + f(S_{ij}, \Omega_{ij})$$

- Where $f(S_{ij}, \Omega_{ij})$ is a nonlinear function dependent on the mean strain rate S_{ij} tensor and spin tensor (rotation) Ω_{ij}.
- Recall that the mean strain rate tensor and spin tensor are defined as follows,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

- These models are known as nonlinear eddy viscosity models (NLEVM).
- This idea was originally proposed by Lumley [1,2], and many NLEVM has been proposed since then.

[1] J. Lumley. Toward a turbulent constitutive equation. Journal of Fluid Mechanics. 1970.
[2] J. Lumley. Computational modeling of turbulent flows. Advances in Applied Mechanics. 1978.

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- The NLEVM approach can be seen as a remedy to the deficiencies of the EVM.
- Where the main deficiencies of the EVM are:
  - Inability to proper describe the anisotropic behavior in shear layers. In the EVM the normal stresses are isotropic, $\overline{u'^2} = \overline{v'^2} = \overline{w'^2} = (2/3)k$.
  - Flow in ducts with secondary motions.
  - Overpredicting production of turbulent kinetic energy in stagnation points.
  - Failure to reproduce the asymmetric behavior of the velocity profiles in the presence of streamlined geometries (strong curvature).
  - Underpredicting turbulent viscosity in the presence of system rotation (strong vortices).
- In comparison to the EVM, the NLEVM models are more computational expensive (as they need to solve more terms and are wall resolving).
- They are also harder to convergence.
- However, they do offer improved prediction capabilities for certain complex turbulent flows.
- Despite the many apparent advantages of NLEVM, they are not widely used.
- EVM still are the workhorse of turbulence modeling.

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- The NLEVM are usually quadratic or cubic.
- Let us briefly discussed the NLEVM by Shih et al [1], which is cubic.
- In this model, the Reynolds stresses are computed as follows,

$$-\rho\overline{u_i'u_j'} = -\frac{2}{3}\rho k\delta_{ij} + \mu_t 2S_{ij}^* + A_3\frac{\rho k^3}{\epsilon^2}\left[\bar{S}_{ik}\bar{\Omega}_{kj} - \bar{\Omega}_{ik}\bar{S}_{kj}\right]$$

$$-2A_5\frac{\rho k^4}{\epsilon^3}\left[\bar{\Omega}_{ik}\bar{S}_{kj}^2 - \bar{S}_{ik}^2\bar{\Omega}_{kj} + \bar{\Omega}_{ik}\bar{S}_{km}\bar{\Omega}_{mj} - \frac{1}{3}\bar{\Omega}_{kl}\bar{S}_{lm}\bar{\Omega}_{mk}\delta_{ij} + I_s S_{ij}^*\right]$$

- Where I_s , S_{ij}^* , and S_{ij}^2 are given by the following relationships,

$$I_s = \frac{1}{2}\left[\bar{S}_{kk}\bar{S}_{mm} - \bar{S}_{kk}^2\right] \qquad S_{ij}^* = \bar{S}_{ij} - \frac{1}{3}\bar{S}_{kk}\delta_{ij} \qquad S_{ij}^2 = S_{ik}S_{kj}$$

[1] TH. Shih, J. Zhu, WW. Liou, K-H. Chen, N-S. Liu, J. Lumley. Modeling of turbulent swirling flows. NASA-TM-113112. 1997.

Non-linear eddy viscosity models

- This model is particularly suited for swirling flows.
- It was developed to deal with aircraft engine combustors that generally involve turbulent swirling flows in order to enhance fuel-air mixing and flame stabilization.
- The model includes third order terms, so it offers extra accuracy.
- The method also satisfy the constraints of rapid distortion theory (RDT) and realizability.
- All the coefficients appearing in the nonlinear constitutive equation are calibrated using DNS and experimental data.
- The coefficient C_μ is not constant, it depends on the strain rate tensor.
- The value of the turbulent kinetic energy k and the dissipation rate ϵ are obtained from low-RE $k-\epsilon$ turbulence models (wall resolving).
- Also, the value of the turbulent eddy viscosity is computed using the relations from low-RE $k-\epsilon$ turbulence models.