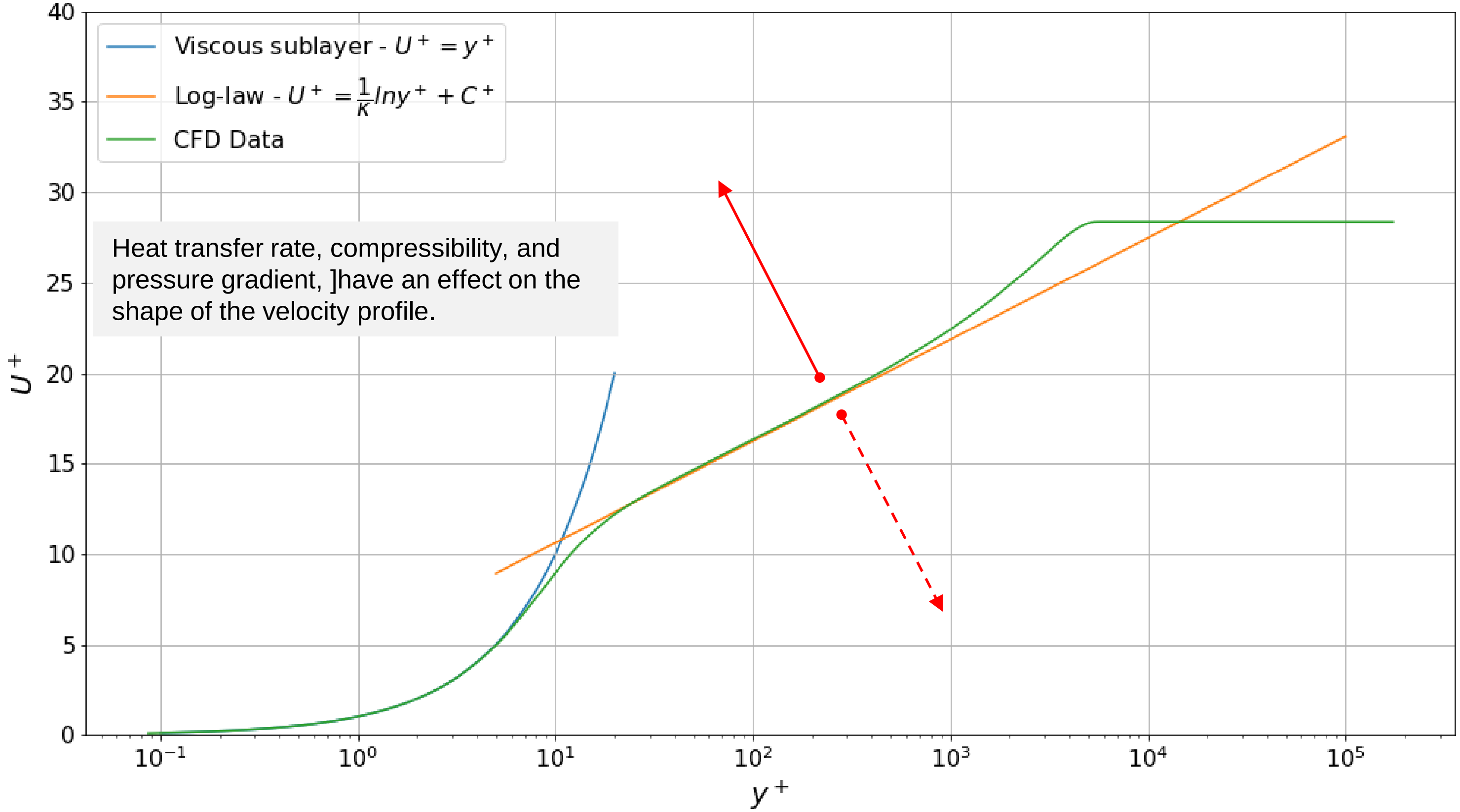
Wall functions – Additional observations

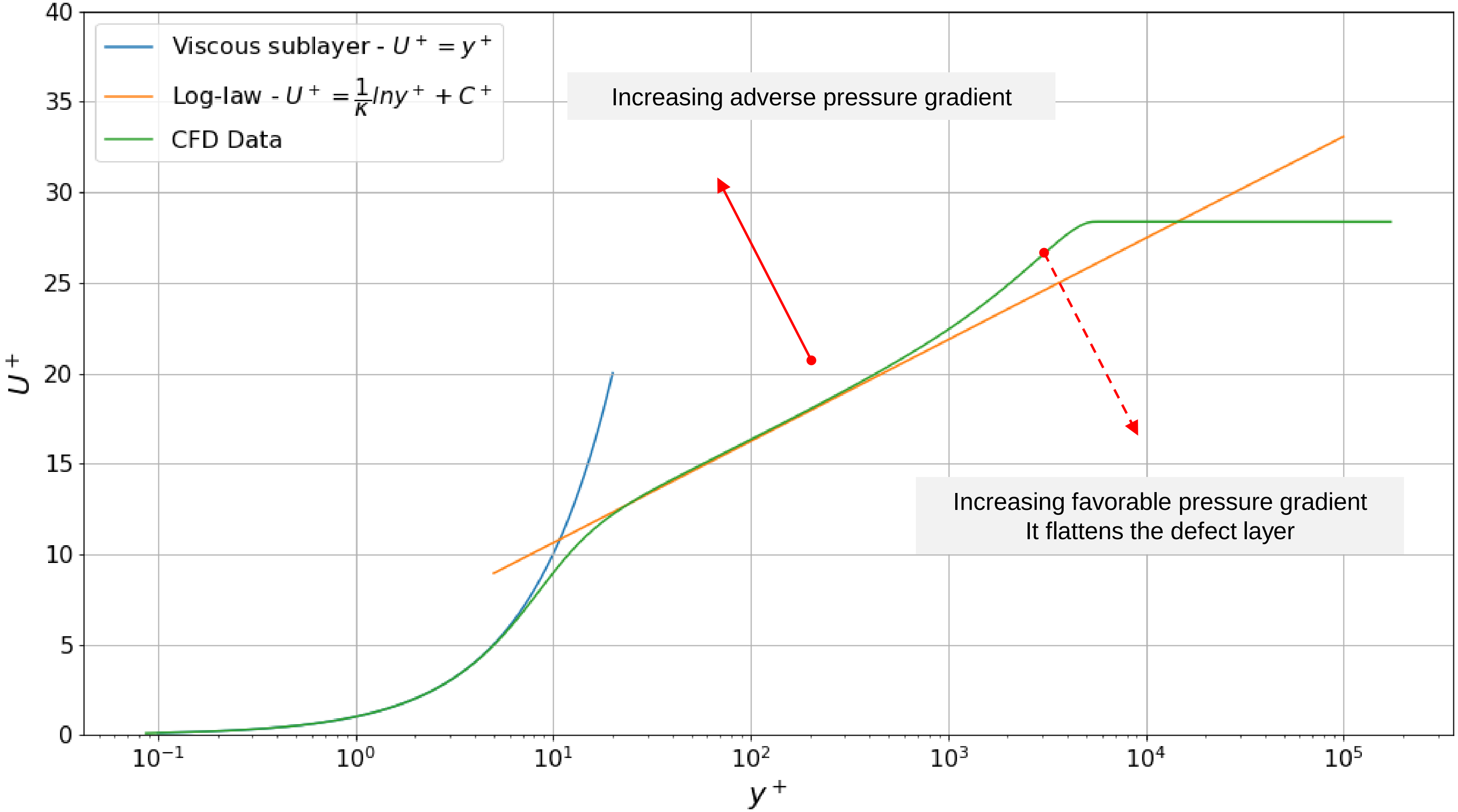
Wall functions – Additional observations

- When studying the boundary layer, heat transfer rate, compressibility, pressure gradient, mach number, and surface roughness (among many factors), they all have an effect on the velocity profile.
- It is not obvious what effect each of these parameters have on the velocity profile.
- Hereafter, we present a few observations or general guidelines (use with care).



Wall functions – Additional observations

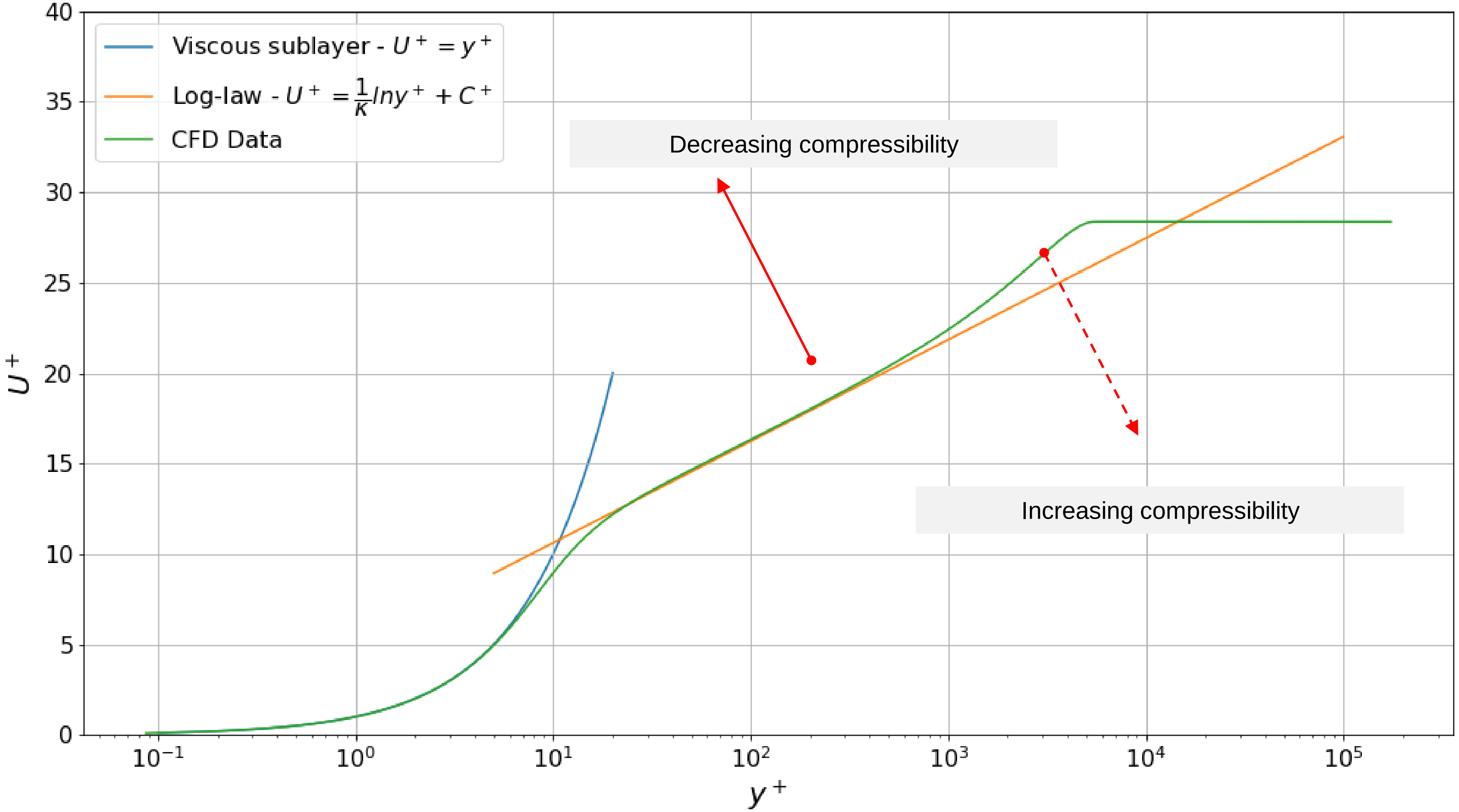
- Effect of adverse pressure gradient on the law of the wall.



- The boundary layer thickens, and the skin friction decreases as the adverse pressure gradient is increased.

Wall functions – Additional observations

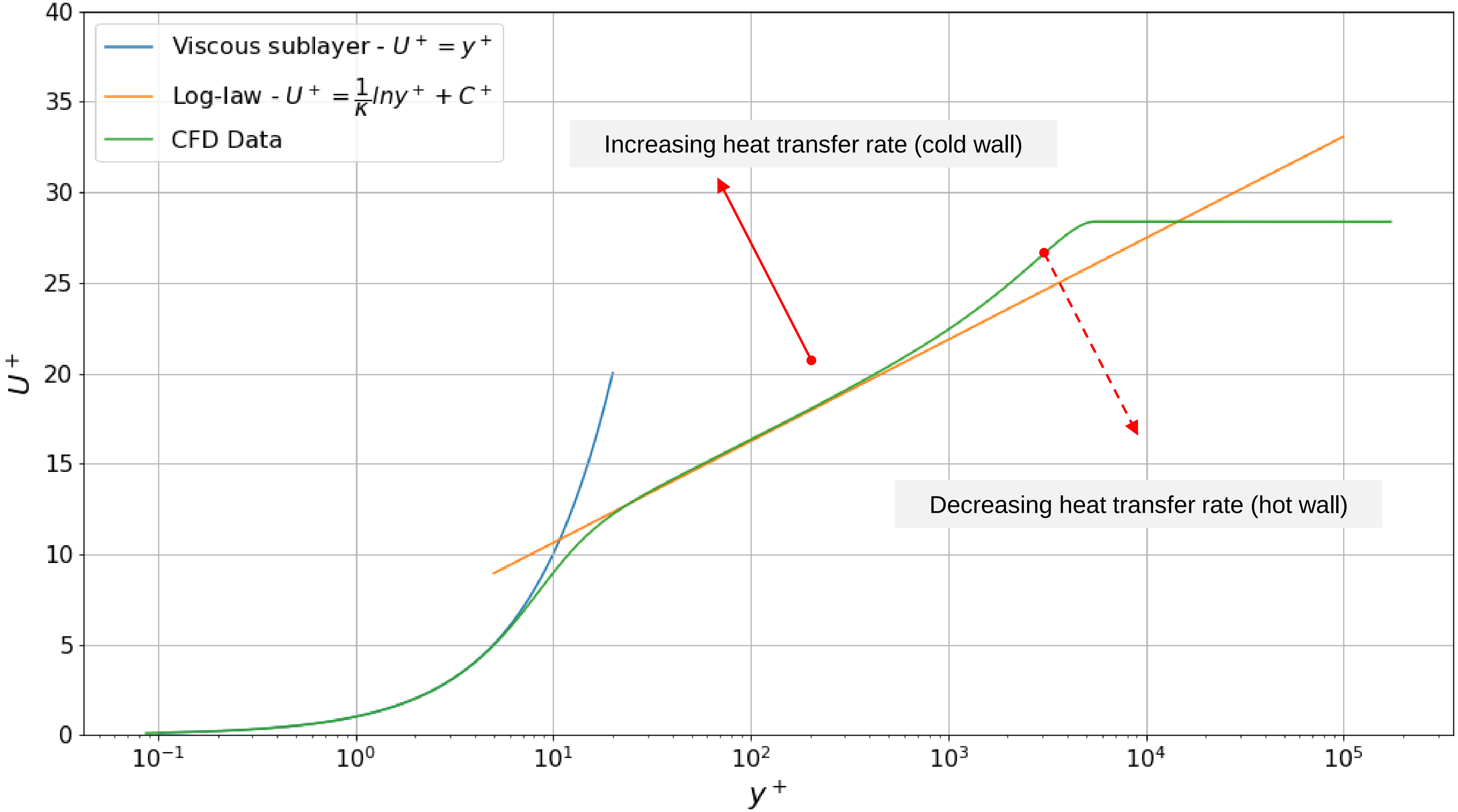
- Effect of compressibility on the law of the wall.



- Increasing compressibility causes the skin friction to decrease.

Wall functions – Additional observations

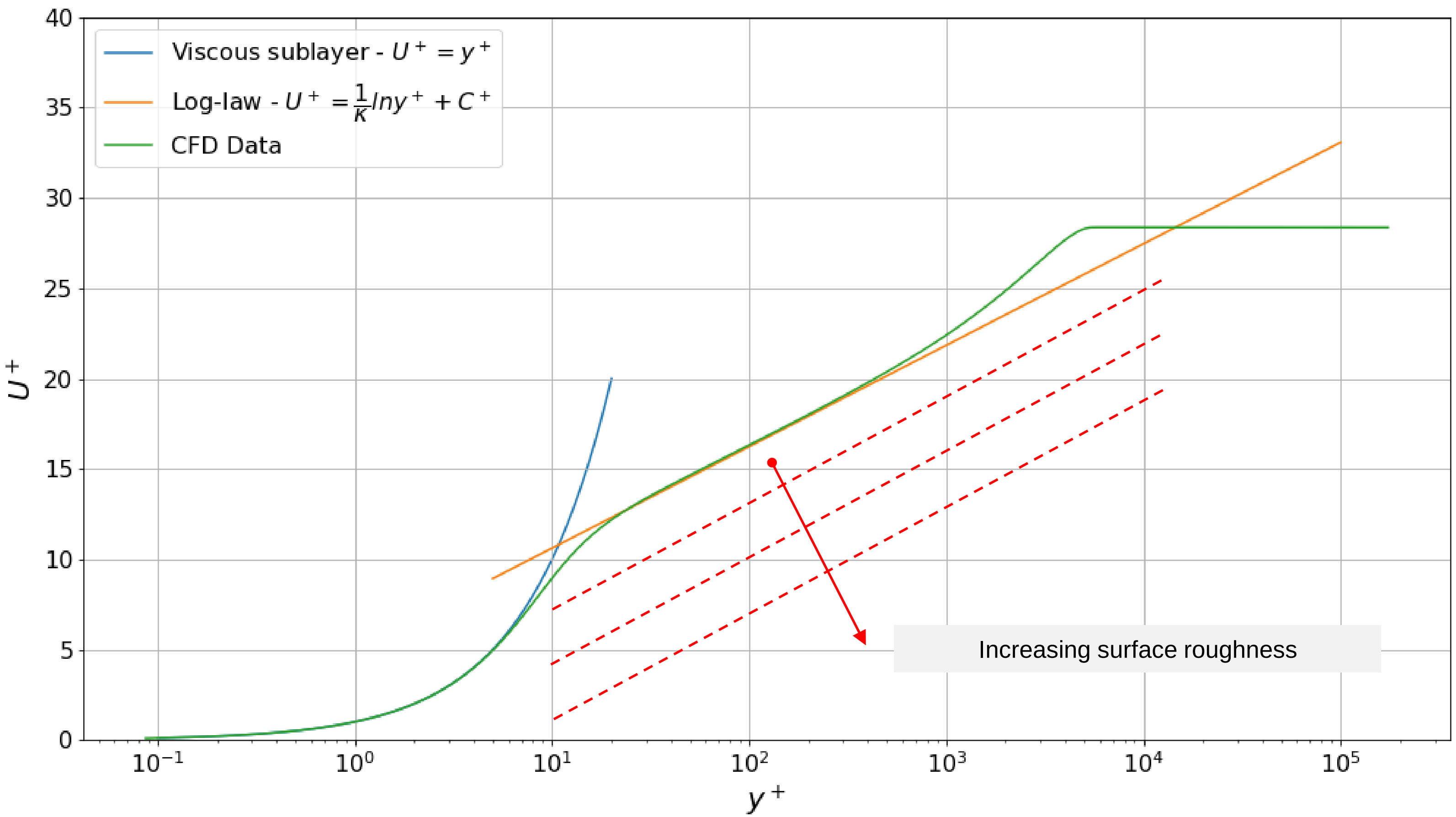
- Effect of heat transfer on the law of the wall.



- Notice that depending on the Prandtl number, the boundary layer can be thicker or thinner.
- A cold wall creates a thinner boundary layer and increases the skin friction
- A hot wall thickens the boundary layer and decreases the skin friction.

Wall functions – Additional observations

- Effect of surface roughness on the law of the wall.



- Surface roughness increases the skin friction.
- It shifts the velocity profile downwards.

Wall functions – Additional observations

- When dealing with high-speed flows and heat transfer, the coefficients appearing in the law of the wall relations and closure relations, have a strong dependence on the Mach number and the heat transfer rate.
- No need to mention that the physical properties also depend on the Mach number and heat transfer rate.

$$u^* = \begin{cases} y^* & y^* < 11.225 \\ \dfrac{1}{\kappa}\ln\left(y^*\right) + C & y^* > 11.225 \end{cases}$$

$$T^* = \begin{cases} Pr\, y^* & y^* < 5 \\ Pr_t\left[\dfrac{1}{\kappa}\ln\left(Ey^*\right) + P\right] & y^* > 30 \end{cases}$$

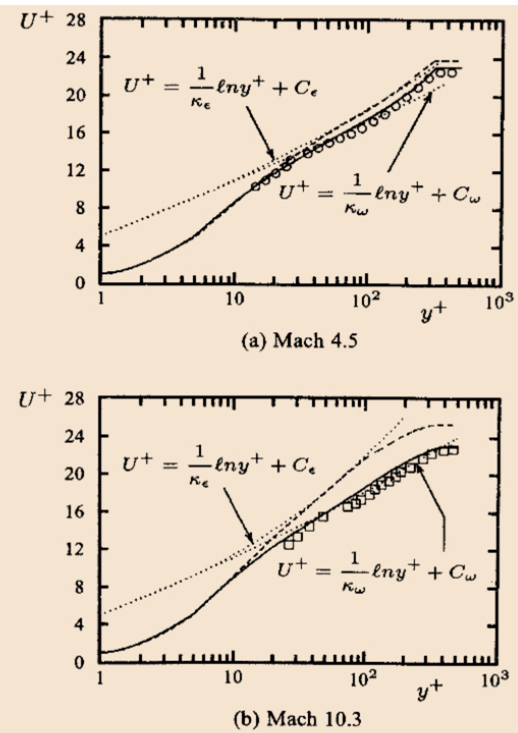


Figure 5.3: *Computed and measured velocity profiles for compressible flat-plate boundary layers:* —— *Wilcox (2006) k-ω; - - - Chien k-ε; ○ Coles; □ Watson.*

Wall functions – Additional observations

- In the case of pressure gradients and flow separation (non-equilibrium conditions), the law of the wall can be sensitized to pressure gradients effects [1].

$$\frac{\tilde{U} C_\mu^{1/4} k^{1/2}}{\tau_w/\rho} = \frac{1}{\kappa} ln \left(E \frac{\rho C_\mu^{1/4} k^{1/2} y}{\mu} \right)$$

- Where,

$$\tilde{U} = U - \frac{1}{2}\frac{dp}{dx}\left[\frac{y_v}{\rho\kappa k^{1/2}} ln\left(\frac{y}{y_v}\right) + \frac{y - y_v}{\rho\kappa k^{1/2}} + \frac{y_v^2}{\mu}\right]$$

$$y_v = \frac{\mu y_v^*}{\rho C_\mu^{1/4} k_p^{1/2}} \qquad\qquad y_v^* = 11.225$$

- This correction is recommended for use in complex flows involving separation, reattachment, and impingement where the mean flow and turbulence are subjected to pressure gradients and rapid changes.
- In such flows, improvements can be obtained, particularly in the prediction of wall shear (skin-friction coefficient) and heat transfer (Nusselt or Stanton number).

[1] S. Kim, D. Choudhury. A Near-Wall Treatment Using Wall Functions Sensitized to Pressure Gradient. In ASME FED Vol. 217,Separated and Complex Flows. ASME. 1995.

Wall functions – Additional observations

- The non-dimensional temperature T*, can be further improved by adding the viscous heating contribution [1],

$$T^* = T_c^* + \frac{D}{q}$$

- Where,

$$T_c^* = \begin{cases} Pr\, y^* & y^* < y_l^* \\ Pr_T \left[\frac{1}{\kappa} \ln\left(E y^*\right) + P \right] & y^* > y_l^* \end{cases}$$

$$D = \begin{cases} \rho u^* \frac{1}{2} Pr U_p^2 & y^* < y_l^* \\ \rho u^* \frac{1}{2} \left[Pr_t U_p^2 + \left(Pr - Pr_t\right) U_c^2 \right] & y^* > y_l^* \end{cases}$$

- Where U_c is the mean velocity at the intersection point y_l^*.
- This correction is recommended for highly compressible flows, where heating by viscous dissipation can have a strong influence in the temperature distribution in the near-wall region.

[1] J. R. Viegas, M. W. Rubesin, C. C. Horstman. On the Use of Wall Functions as Boundary Conditions for Two-Dimensional Separated Compressible Flows. Technical Report AIAA-85-0180. 1985.