Wall functions for heat transfer

Wall functions for heat transfer

- As for the momentum (or viscous) wall functions, there is also a treatment for the thermal wall functions.
- Depending on the value of y*, the value of the non-dimensional temperature T* (equivalent to the concept of u*), can be computed as follows,

Thermal viscous sublayer where conduction is important

$$T^* = \begin{cases} Pr\, y^* & y^* < 5 \\ Pr_t \left[\dfrac{1}{\kappa} \ln\left(E y^*\right) + P \right] & y^* > 30 \end{cases}$$

Logarithmic law for the turbulent region where effects of turbulence dominate conduction

Pr

Molecular (or laminar) Prandtl number

Pr_t

Turbulent Prandtl number

Wall functions for heat transfer

- The following nondimensional temperature relation (in the log-law region),

$$T^* = Pr_t \left[\frac{1}{\kappa} \ln \left(E y^* \right) + P \right]$$

- It is used to relate the temperature at the cell center T_P to the heat flux at the wall q_w.
- This is similar to what we did with the viscous wall function, where we related U_P to τ_w.

Wall functions for heat transfer

- When using thermal wall functions, we are interested in computing the thermal diffusivity used to approximate the wall heat transfer q_w,

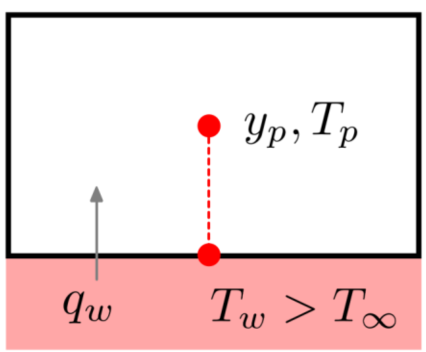
Thermal conductivity

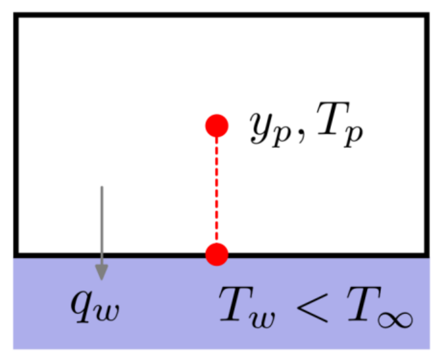
Specific heat

$$q_w = k\frac{\partial T}{\partial y} = \rho c_p \alpha \frac{\partial T}{\partial y} = \rho c_p \alpha \frac{T_w - T_p}{y_p - y_w}$$

Thermal diffusivity

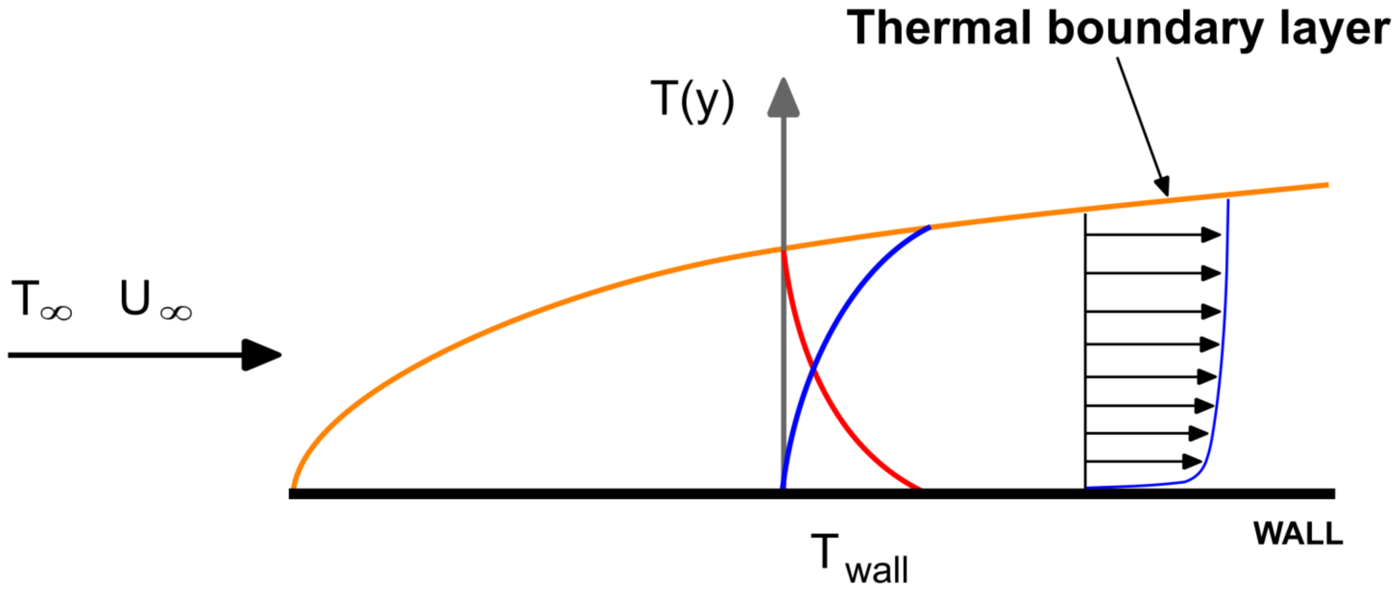
- We need to relate the temperature at the cell center T_P to the heat flux at the wall q_w.





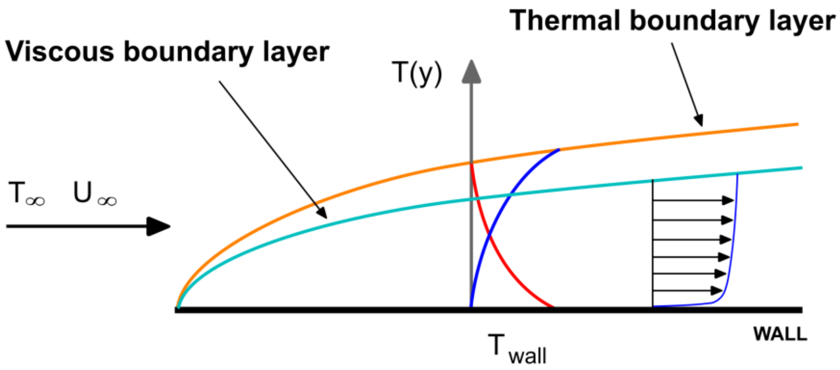
Wall functions for heat transfer

- Recall that in the momentum boundary layer, the velocity at the walls is zero.
- In the thermal boundary layer, the temperature at the wall can be higher or lower than the freestream temperature.
- Therefore, we can have very different temperature profiles growing from the walls.
- The temperature at the walls also has an influence of the wall shear stresses.

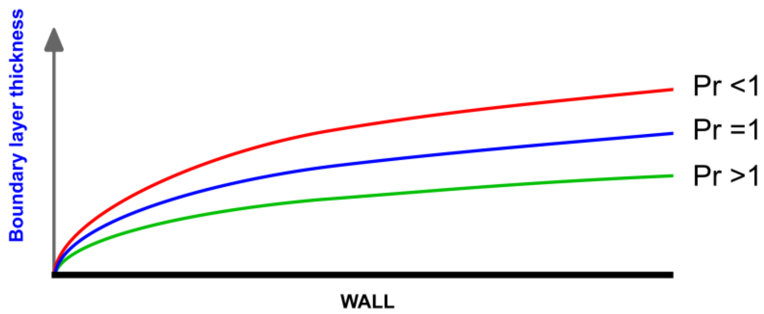


Wall functions for heat transfer

Momentum and thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection

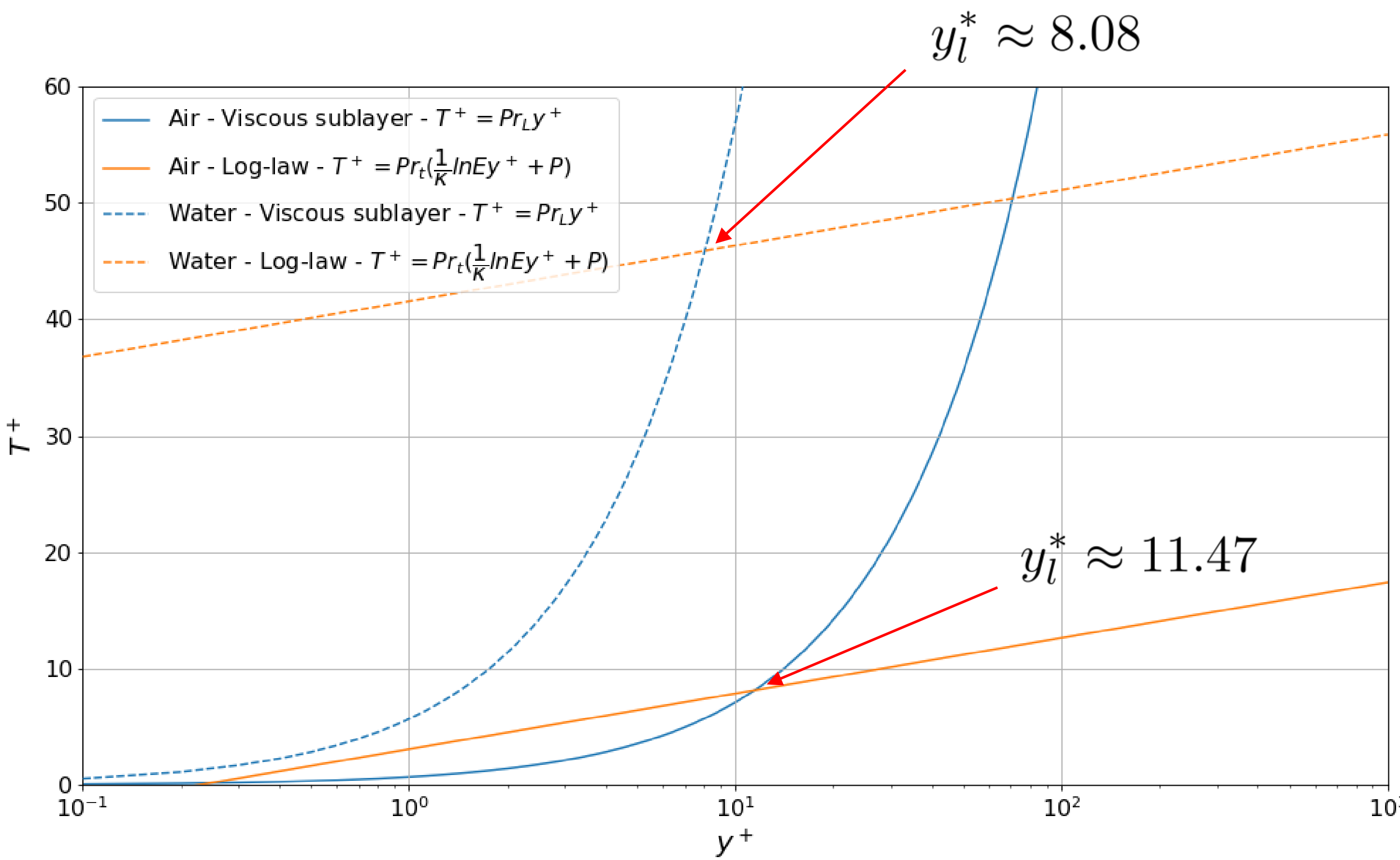


Thermal boundary layer in function of Prandtl number (Pr)

- Just as there is a viscous boundary layer in the velocity distribution (or momentum), there is also a thermal boundary layer.
- Thermal boundary layer thickness is different from the thickness of the viscous sublayer (momentum) and is fluid dependent.
- The thickness of the thermal sublayer for a high Prandtl number fluid (*e.g.*, water) is much less than the momentum sublayer thickness.
- For fluids of low Prandtl numbers (*e.g.*, air), it is much larger than the momentum sublayer thickness.
- For Prandtl number equal 1, the thermal boundary layer is equal to the momentum boundary layer.

Wall functions for heat transfer

- The normalized temperature plot (T^+ vs. y^+), depends on the Prandtl number.
- Different values of Prandtl number will result in different plots, with different intersection points between the viscous sublayer and the log-law region.
- The intersection point of the viscous sublayer and the log-law region is known as the non-dimensional thermal sublayer thickness or y_l^*.



Wall functions for heat transfer

- The thermal diffusivity coefficient α is computed as follows,

$$\alpha_w = \begin{cases} \alpha & y^* < y_l^* \\ \dfrac{u_\tau y_p}{Pr_t \left[\dfrac{1}{\kappa} ln \left(E y^* \right) + P \right]} & y^* > y_l^* \end{cases}$$

- Remember, the value of y_l^* depends on the Prandtl number.
- At this point, the heat flux q_w at the wall can be computed as follows,

$$q_w = \rho c_p \alpha_w \frac{T_w - T_p}{y_p - y_w}$$

- The turbulent Prandtl number Pr_t is usually equal to 0.85, but it highly depends on the flow properties and the flow physics.

Wall functions for heat transfer

- Let us revisit the nondimensional temperature function T^*,

$$T^* = Pr_t \left[\frac{1}{\kappa} \ln \left(E y^* \right) + P \right] \qquad y^* > 30$$

- The term *P* appearing in this relation, has a strong dependence on the Prandtl number (molecular and turbulent).
- One way to approximate this term is by using Jayatilleke function [1],

$$P = 9.24 \left[\left(\frac{Pr}{Pr_t} \right)^{3/4} - 1 \right] \left[1 + 0.28 e^{-0.007 Pr/Pr_t} \right]$$

- Kader [2] and Patankar and Spalding [3] also proposed alternative formulations for computing P.

[1] C. Jayatilleke. The influence of Prandtl number and surface roughness on the resistance of the laminar sublayer to momentum and heat transfer. Prog. Heat Mass Transfer, 1, 193-329. 1969.
[2] B. Kader. Temperature and concentration profiles in fully turbulent boundary layers. Int. J. Heat Mass Transfer. 24(9). 1981.
[3] S. Patankar and D. Spalding. A calculation procedure for heat, mass and momentum transfer in three dimensional parabolic flows. Int. J. Heat Mass Transfer, 15(10). 1972.

Wall functions for heat transfer

- Let us summarize the steps needed to compute the wall heat flux q_w using wall functions,
  - Compute y*.
  - Compute the Prandtl number,

$$Pr = \frac{\nu}{\alpha}$$

  - Compute the intersection point y_l^*.
  - Compute the thermal diffusivity at the wall.

$$\alpha_w = \begin{cases} \alpha & y^* < y_l^* \\ \dfrac{u_\tau y_p}{Pr_t \left[\dfrac{1}{\kappa} ln \left(E y^* \right) + P \right]} & y^* > y_l^* \end{cases}$$

  - Compute the heat flux at the wall.

$$q_w = \rho c_p \alpha_w \frac{T_w - T_p}{y_p - y_w}$$