Supplementary Material

1 INITIAL CONDITIONS

Sensitivity to initial conditions was tested by varying each initial state variable. In Figure S1 the sensitivity to each variable (a_0 in **A**, n_0 in **B**, c_0 in **C**) is plotted against the initial area showing that at low initial areas the model is insensitive to all initial conditions. In Figure S2 the change of state variables in time is shown with a range of values for the initial conditions. In **A** the initial area varies, in **B** the nitrate reserve, and **C** the carbon reserve. It shows that over the range of initial areas all of the variables converge after around 10 months. For the full range of initial nitrogen and carbon reserves all variables converge in less than 4 months.

2 SENSITIVITY

The temperature dependence of the growth is captured by the piecewise Equation S1 modified from the originally model where $T_h = 19$. In order to capture this uncertainty and possibility of frond death T_h was varied by ± 2°C.

$$f_{temp}(T) = \begin{cases} 0 & T \leq -1.8 \\ 0.08T + 0.2 & -1.8 < T < 10 \\ 1 & 10 \leq T \leq 15 \\ \frac{T_h - T}{T_h - 15} & 15 < T \leq T_h \\ 0 & T > T_h \end{cases} \tag{S1}$$

In this section summed carbon/nitrogen refers to the sum of the variable scaled to be equivalent to one line with a vertical density of 100 frond per meter, every kilometer square. Varying the photosynthetic efficiency parameter, α, by $\pm 25\%$ resulted in a -10% and $+8\%$ change in the maximum summed carbon respectively. Figures S5**A** and S5**B** show that the increase in summed carbon is due to an increase over the most of the area where the kelp grows successfully.

Varying the maximum temperature tolerated for growth, t_m, by ± 2°C resulted in a $\pm 3\%$ change in summed carbon. As is shown in Figures S5**C** and S5**D**, this change largely results from movement of the southern boundary. Where the tolerance is increased (Figure S5**D**) growth increases on the west side of the boundary where the growth is temperature modulated (as discussed above). Comparing to Figure 1**A** it can be seen that the movement on the west side was parallel to the direction of the temperature gradient. On the east side there is little change in growth as it is largely nitrate modulated. When the tolerance is decreased (Figure S5**C**) the decrease occurs more evenly across the whole boundary, although more so on the west side.

An ANOVA analysis (Fox, 2008) was conducted using the model in Equation S2 where t_m, and α are as previously defined, y is forcing data start year (i.e. between 2002 and 2017 inclusive), and in this run x is the maximum summed carbon. Therefore the null hypothesis is that the maximum C_{sum} of a run is independent of the variables. The ANOVA results (shown in Table S1) show that the null hypothesis can be rejected with a very high significance level ($p < 2 \times 10^{-7}$). Regression with the model then revealed the nature of the dependence showing a strong proportionality to α, proportionality to t_m, and negative proportionality to y. These results support the inference related to the α and t_m dependence in the main text. An additional analysis of the ratio of maximum C_{sum} to N_{sum} on the same day was conducted using

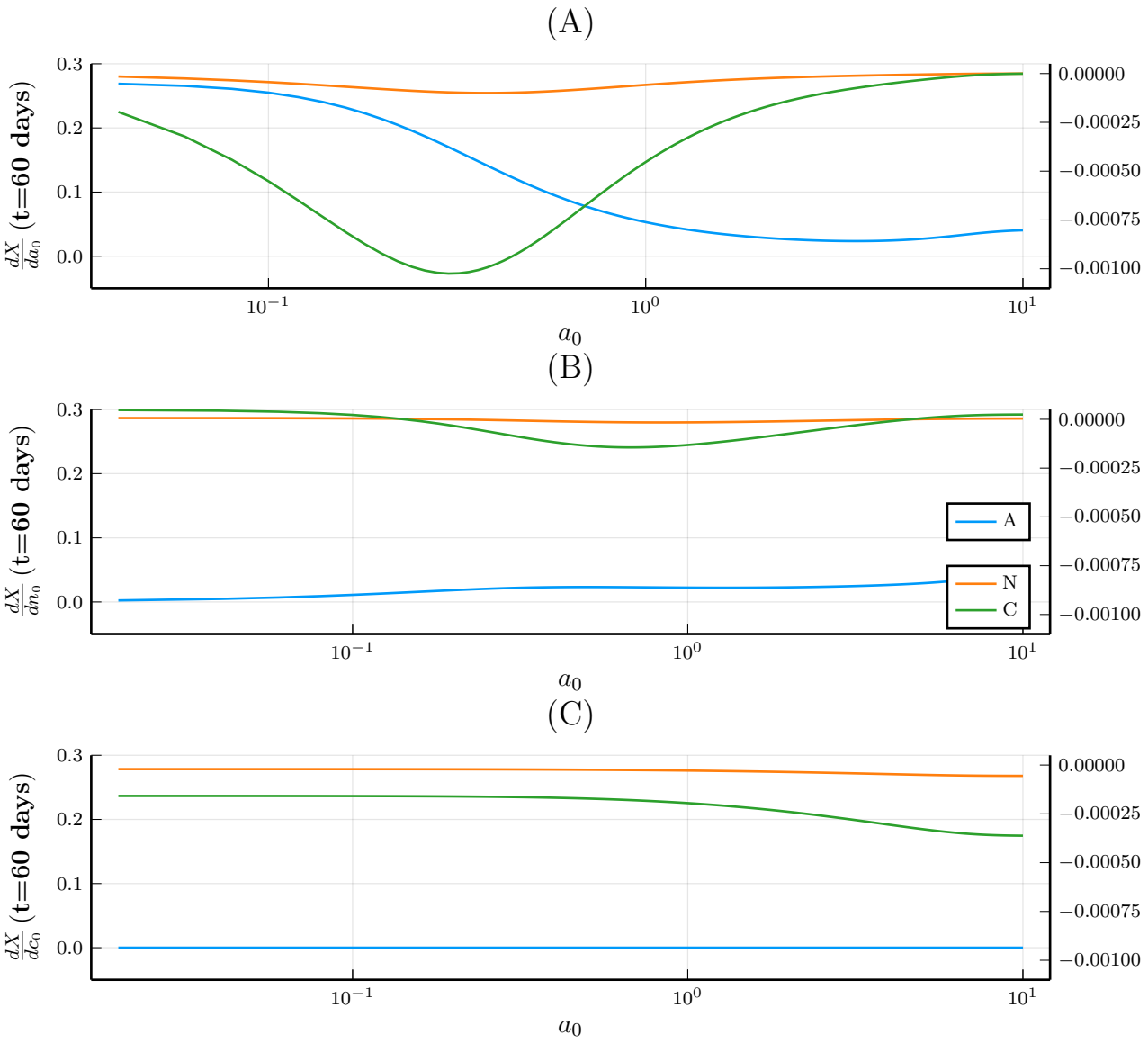

Figure S1: The sensitivity of the variables A, N, and C to the initial value of each variable is plotted. In **A** the sensitivity to initial area shows reduced sensitivity of N and C with sufficiently low initial area. In **B** and **C** the sensitivity to N_0 and C_0 is shown to be low across the full range of initial areas.

the same model with x as the ratio (results in Table S2) and a significant ($p < 2 \times 10^{-22}$) relation was found between the ratio and α. There was a weak relation between t_m ($p = 0.08$) and the ratio, likely because the highest ratios occur near the southern border, which is pushed northwards by t_m.

$$x \sim t_m + \alpha + y \tag{S2}$$

It is interesting to note that every iteration in the ensemble finished within a 6 day range on day 344.6 ± 1.2, showing that the exact timing of seeding and the seasonal changes to the forcing variables has little effect.

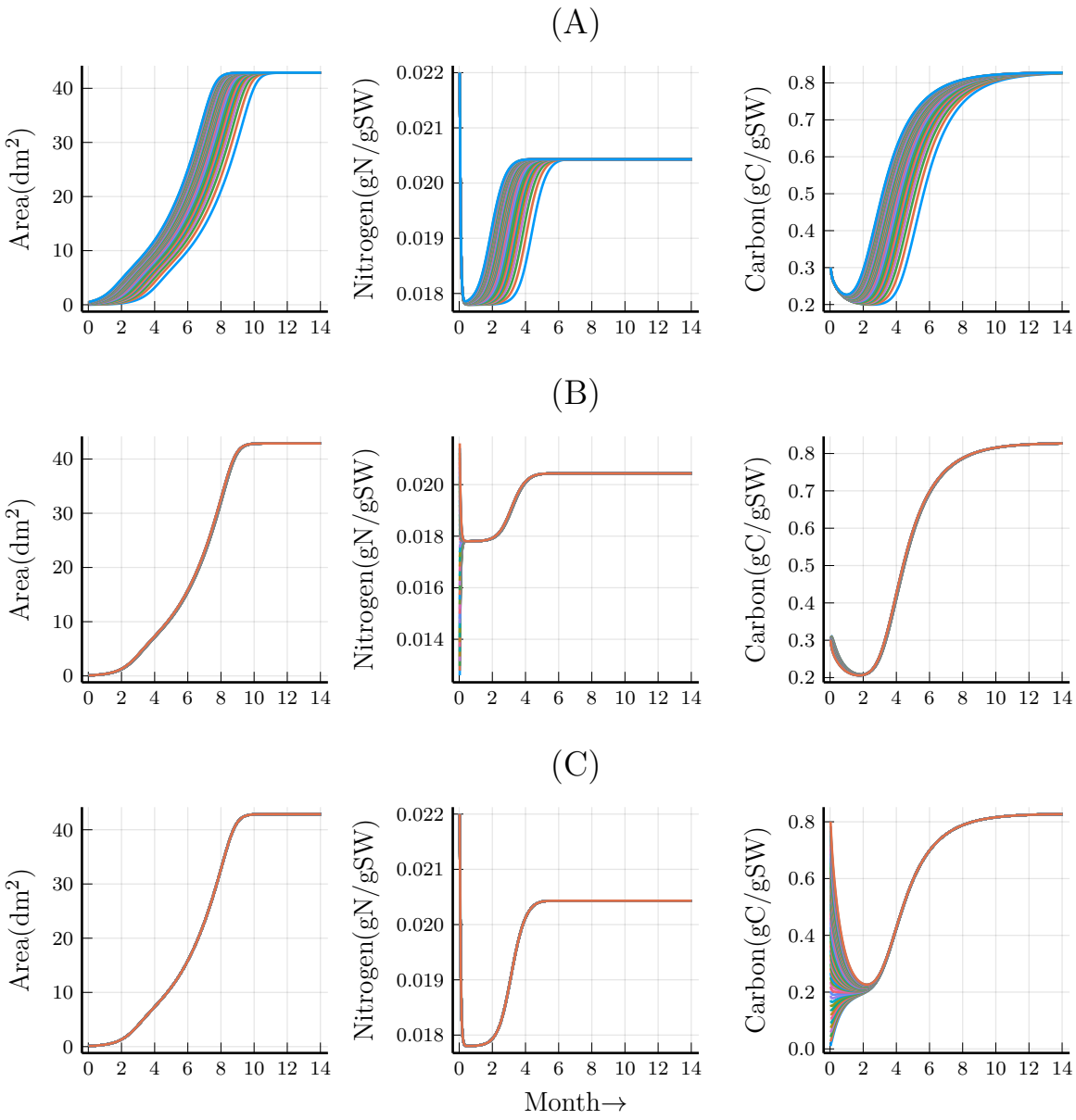

Figure S2: Initial conditions (A_0, N_0 and C_0) have a minimal effect on the outcome of the model run with stationary forcing. **(A)** shows that when initial area is varied by a factor of four (i.e. $A_0 \in [0.02, 0.5]$) all state variables converge to the same result after approximately one year. **(B)** shows that when the initial nitrogen reserve is varied across all valid values (i.e. $N_0 \in [N_{min}, N_{max}]$) that the convergence is far faster, around one month, with almost no effect on the area and carbon reserve. **(C)** shows that when the initial carbon reserve is varied from the valid minimum, C_{min}, to $2.5\times$ the value used in the main model runs (i.e. $C_0 \in [C_{min}, 0.8]$) the area and nitrogen reserve are unaffected and the carbon reserve converges in around four months. The sensitivity of total carbon and total nitrogen are indistinguishable from the sensitivity of the area as they depend most strongly on it. For all runs the forcing variables were help constant at 8mol photons/m^2/day, 5°C, and 4mmol/m^3. The change in day length effect (f_{photo}) was also removed.

REFERENCES

Fox, J. (2008). *Applied Regression Analysis and Generalized Linear Models* (SAGE Publications)
[Dataset] White, C., N. Marshall (2007). Saccharina latissima sugar kelp

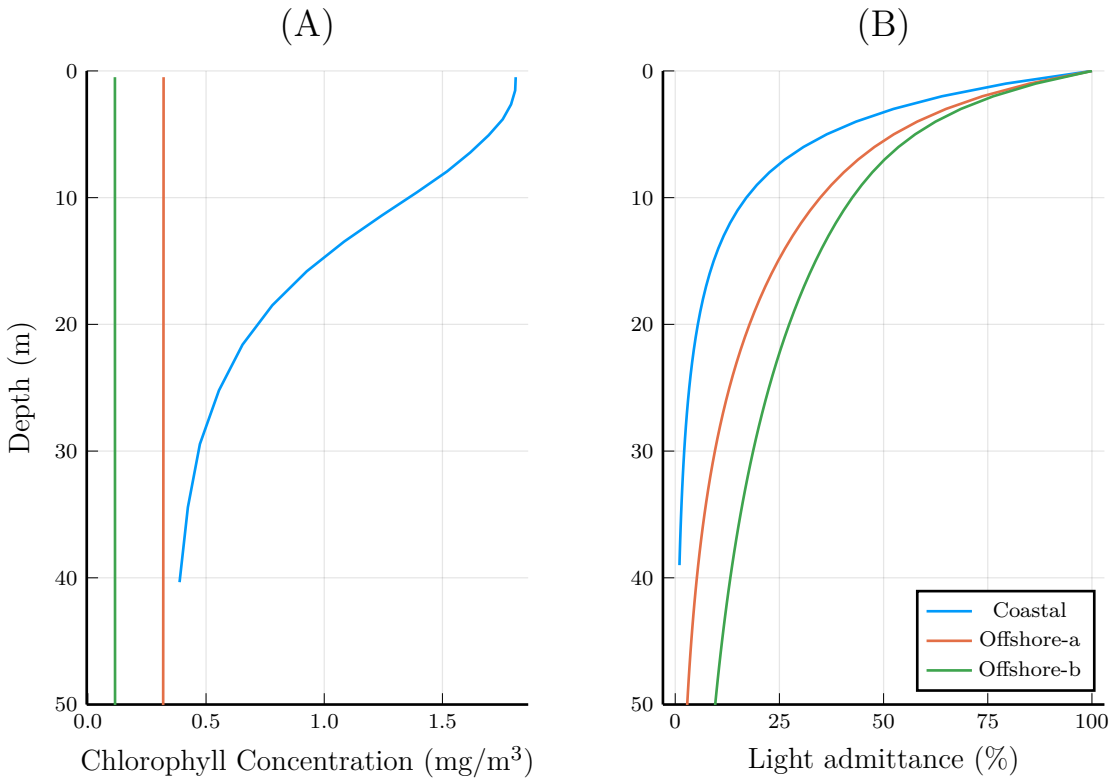

Figure S3: Panel A: Model chlorophyll profiles at three representative locations including one coastal site and two locations offshore for 1/1/2019. The representative coastal location is 47°N 2.75°W which is the nearest cell to the outflow of the Loire, France, in a region where S. Latissima grows naturally in near-shore waters (White, 2007). Location Offshore-a is at 47°N 11.75°W, and Offshore-b is at 65°N 11.75°W. Note that the mixed layer is deeper than the region shown at the offshore sites and hence the chlorophyll is nearly uniform in depth in the upper 50m at these locations. Panel B: The light admittance profiles, calculated from the chlorophyll profiles in Panel (A) as described in section 2.1 (Methods-Model).

Source	SS	MSS	F	p
α	5.40949×10^{23}	5.40949×10^{23}	267.932	2.48595×10^{-43}
t_m	6.3176×10^{22}	6.3176×10^{22}	31.2911	5.05534×10^{-8}
y	5.73252×10^{22}	5.73252×10^{22}	28.3932	1.96493×10^{-7}
Residual	5.97617×10^{23}	2.01897×10^{21}	0.0	0.0

Table S1. The results of the ANOVA analysis of the relation between maximum C_{sum} and the varied parameters. The p values show that the null hypothesis (maximum C_{sum} independent of each variable) can be rejected at a 2×10^{-7} significance level.

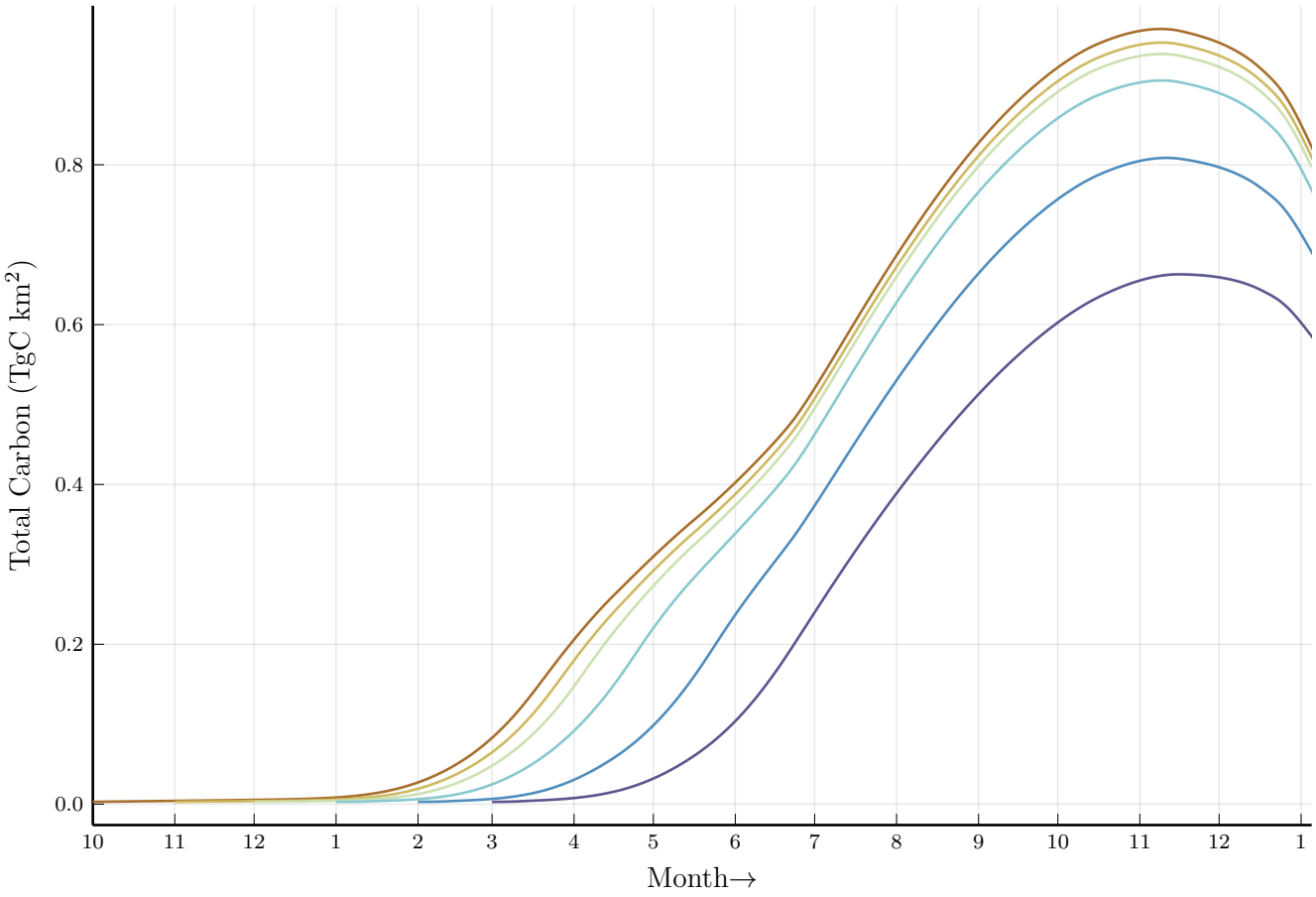

Figure S4: The total carbon content for model runs starting on the first of the months between October and March (2019/2020) shows a very small difference in the maximum carbon for the first three months ($> 3.4\%$), all reaching their maximum on the same day. For runs stating in January or later there is a significant decrease and delay. The growth rate appears to occur in two distinct phases, the first starting around February and ending around June, the second starting after June. In the first period the growth is modulated by the kelp's existing size, and in the second all runs grow at approximately the same rate. The first three runs are large enough to take advantage of the early period to almost the same extent where as the later ones were not. It was therefore chosen to start all model runs at the beginning of December as this causes a minimal reduction in growth after one year while taking the least time to compute.

Source	SS	MSS	F	p
α	31.9015	31.9015	110.683	3.35698×10^{-22}
t_m	0.862025	0.862025	2.99082	0.0847801
y	0.46066	0.46066	1.59827	0.207143
Residual	296.0	0.288223	0.0	0.0

Table S2. The results of the ANOVA analysis of the relation between the ratio of maximum C_{sum} and N_{sum} on the same day and, the varied parameters. The p values show that the null hypothesis (maximum C_{sum} independent of variable) can only be rejected for α at a 3×10^{-22} significance level.

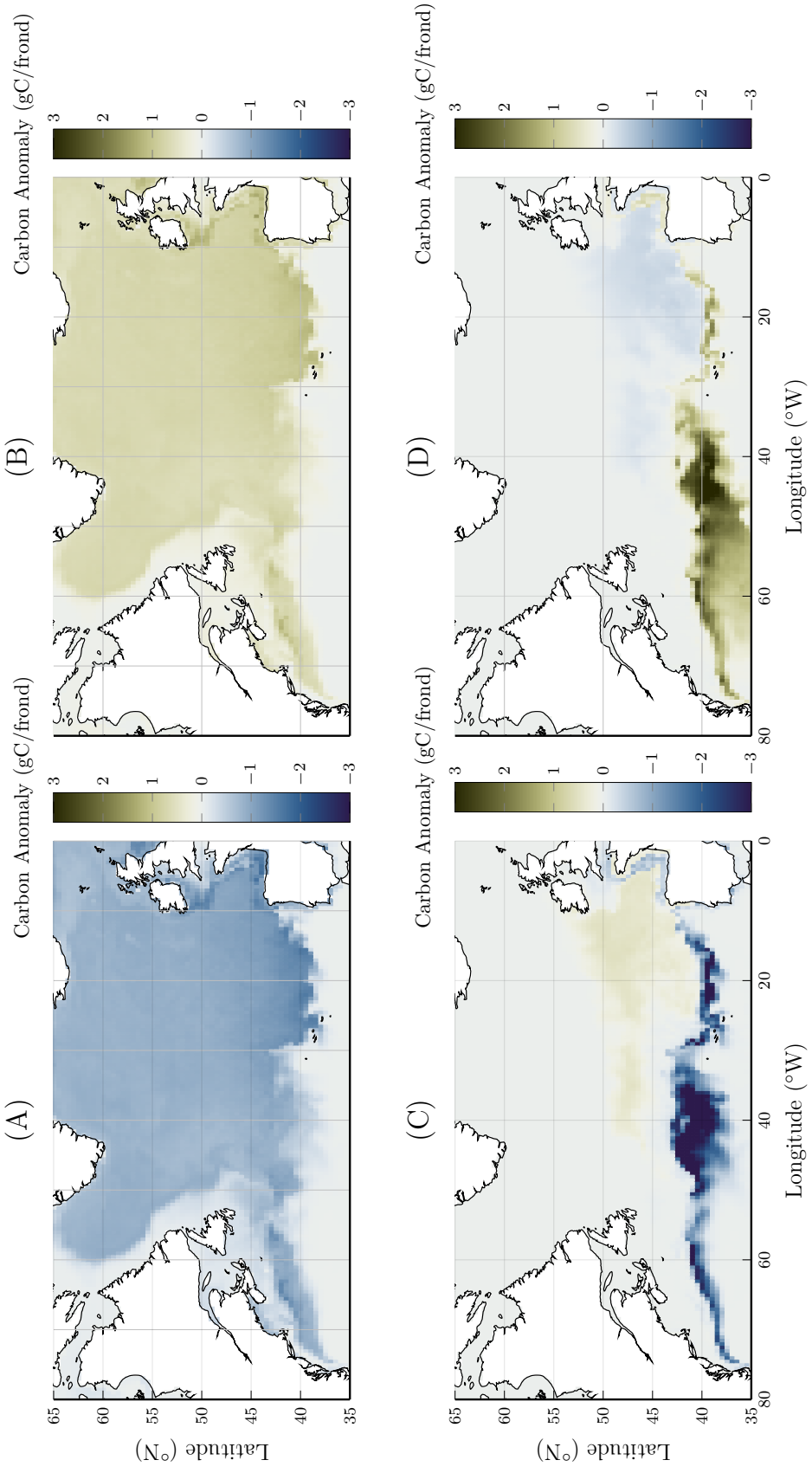

Figure S5: Depth averaged carbon content anomaly...(Continues on following page.)

Figure S5: Depth averaged carbon content anomaly when total carbon is highest (early December) with parameter variations in the model. In **(A)** the high temperature cutoff in f_{temp} is reduced by $2°$C, and in **(B)** it is increased by $2°$C. In **(C)** the photosynthetic efficiency is reduced by 25%, and in (D) it is increased by 25%. The difference between **(C)** and (D) shows that the variation in photosynthetic efficiency does not change the geographic distribution of the kelp but effects the amount of growth. The difference between (A) and **(B)** shows that the southern boundary in the model is caused by the high temperature cutoff in f_{temp} and that with changed temperature tolerance the boundary moves into areas with different sea temperatures.

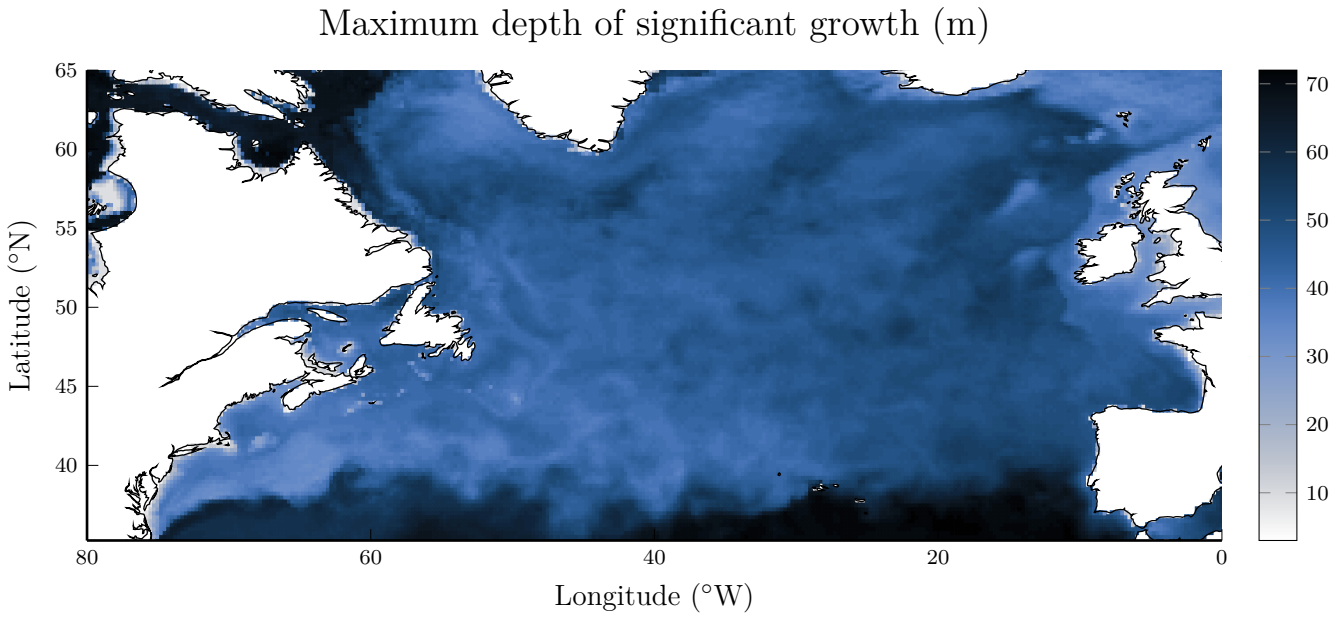

Figure S6: The maximum depth of significant growth, defined as the depth above which 90% of the lines growth has already occurred.