ATTACHMENT

THE DERIVATION OF THE CONGESTION RENT

A. The Internalizing reliability ED model

The I-RED model can be completely formulated as follows:

$$(\text{I-RED})\ \psi := \min_{P_{i,t},\Delta p_{i,t}} \sum_{i=1}^{I}\sum_{t=1}^{T} C_{i,t} \tag{1}$$

$$\left(C_{i,t}, P_{i,t}, u_{i,t}^{IRUC}\right) \in \mathcal{X}_{i,t}\ ,\ \forall i,t \tag{2}$$

$$\left[\underline{\sigma}_{k,t}^{b}, \overline{\sigma}_{k,t}^{b}\right]\ f_{k,\min} \le \sum_{n} \Gamma_{k,n} \times P_{n,t}^{inj,b} \le f_{k,\max}, \forall k,t \tag{3}$$

$$\left[\lambda_t^{b}\right] \sum_{i} P_{i,t} = \sum_{n}\left(D_{n,t}^{b}\right), \forall i,n,t \tag{4}$$

$$\left(C_i, P_{i,t} + \Delta p_{i,t}, u^{IRUC}\right) \in \mathcal{X}_{i,t}, \forall i,t \tag{5}$$

$$\left[\underline{\sigma}_{k,t}^{r}, \overline{\sigma}_{k,t}^{r}\right]\ f_{k,\min} \le \sum_{n} \Gamma_{k,n} \times P_{n,t}^{inj,r} \le f_{k,\max}, \forall k,t \tag{6}$$

$$\left[\lambda_t^{r}\right]\ \sum_{i} \Delta p_{i,t} = \sum_{n}\left(D_{n,t}^{r} - D_{n,t}^{b}\right), \forall i,n,t \tag{7}$$

The net injection power for the flow limits is defined below:

$$P_{n,t}^{inj,b} = D_{n,t}^{b} - e_{n,i} P_{i,t} \tag{8}$$

$$P_{n,t}^{inj,r} = D_{n,t}^{r} - e_{n,i}\left(P_{i,t} + \Delta p_{i,t}\right) \tag{9}$$

B. The Potential Transmission Capacity

The potential transmission capacity reveals the market value of the forecasted load $D_{n,t}^{r}$ and has a potential connection with the net injection power (under bid conditions). For the sake of illustration, the residual transmission capacity is defined for different conditions below.

$$\begin{aligned} \Delta f_{k,t}^{b,+} &= f_{k,\max} - \sum_{n} \Gamma_{k,n} P_{n,t}^{inj,b}, \Delta f_{k,t}^{b,-} = f_{k,\min} + \sum_{n} \Gamma_{k,n} P_{n,t}^{inj,b} \\ \Delta f_{k,t}^{r,+} &= f_{k,\max} - \sum_{n} \Gamma_{k,n} P_{n,t}^{inj,r}, \Delta f_{k,t}^{r,-} = f_{k,\min} + \sum_{n} \Gamma_{k,n} P_{n,t}^{inj,r} \end{aligned} \tag{10}$$

With the LMP mechanism, the transmission capacity is priced through Lagrangian multipliers and covered by the congestion rent. Based on Ref. [1], this paper further derives the constituents of the *whole congestion rent* (WCR), *normal congestion rent* (NCR) and *potential congestion rent* (PCR).

In the I-RED model, the *whole congestion rent* (WCR) is calculated as follows:

$$\begin{aligned} WCR_t &= \sum_{n} \pi_{n,t}^{b} P_{n,t}^{inj,b} \\ &= \sum_{k} (\overline{\sigma}_{k,t}^{b} f_{k,\max} + \underline{\sigma}_{k,t}^{b} f_{k,\min}) \\ &\quad + \sum_{k}\left[(\overline{\sigma}_{k,t}^{r} f_{k,\max} + \underline{\sigma}_{k,t}^{r} f_{k,\min}) - (\overline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,-})\right] \end{aligned} \tag{11}$$

The derivation of Eq. (11) is provided in the Attachment. C. The second equation reveals the source of the WCR, including the *normal congestion rent* (NCR) and *potential congestion rent* (PCR).

The first term in the second equation, NCR, represents the congestion rent under the bid-in conditions. More specifically, if any system-wide constraints (3) are binding, it means the transmission capacities are scarce under bid-in conditions and the NCR appears. The NCR reveals the regional contribution of net injection power for the bid-in situation.

$$NCR_t = \sum_{k} (\overline{\sigma}_{k,t}^{b} f_{k,\max} + \underline{\sigma}_{k,t}^{b} f_{k,\min}) \tag{12}$$

The potential transmission capacity is reflected in the second term (covered by the PCR). The PCR is calculated as follows:

$$\begin{aligned} PCR_t &= \sum_{k}\left[(\overline{\sigma}_{k,t}^{r} f_{k,\max} + \underline{\sigma}_{k,t}^{r} f_{k,\min}) - (\overline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,-})\right] \\ &= \sum_{n} \pi_{n,t}^{r} P_{n,t}^{inj,b} \end{aligned} \tag{13}$$

The derivation of Eq. (13) is provided in Attachment. D. Different from the NCR, the PCR is more complex because the payment is not only associated with system-wide constraints (6) but also a coupled impact with the bid-in condition. The PCR reflects a coupled market impact on both the bid-in conditions and the forecasted condition after the internalization.

PCR is the quantified energy payment for the potential transmission capacity incurred by the RUC requirement. In the equation (13), the intermediate price signal $\pi_{n,t}^{r}$ is introduced. It is defined as the marginal cost of the next production of the units under the forecasted load. Based on Eq. (13), the payment of the potential transmission capacity is connected with the generation and load (under bid conditions) via the price signal $\pi_{n,t}^{r}$. Therefore, the PCR for each node ultimately reveals the regional contribution of the net injection power (under bid conditions) for the forecast situation. This is the root reason that the price component $\sigma_{k,t}^{r}$ can provide guidance for the generation and load consumption.

C. Derivation of WCR

According to (10), the WCR is calculated as follows:

$$\begin{aligned} WCR_t &= \sum_{n} \pi_{n,t}^{b} P_{n,t}^{inj,b} \\ &= \sum_{k}\left(\overline{\sigma}_{k,t}^{b} + \overline{\sigma}_{k,t}^{r} - \underline{\sigma}_{k,t}^{b} - \underline{\sigma}_{k,t}^{r}\right) \times \Gamma_{k,n} P_{n,t}^{inj,b} \\ &= \sum_{k}\left[\left(\overline{\sigma}_{k,t}^{b} + \overline{\sigma}_{k,t}^{r}\right)\left(f_{k,\max} - \Delta f_{k,t}^{b,+}\right) - \left(\underline{\sigma}_{k,t}^{b} + \underline{\sigma}_{k,t}^{r}\right)\left(\Delta f_{k,t}^{b,-} - f_{k,\min}\right)\right] \\ &= \sum_{k}\left[\left(\overline{\sigma}_{k,t}^{b} + \overline{\sigma}_{k,t}^{r}\right) \times f_{k,\max} + \left(\underline{\sigma}_{k,t}^{b} + \underline{\sigma}_{k,t}^{r}\right) \times f_{k,\min}\right] \\ &\quad - \sum_{k}\left(\overline{\sigma}_{k,t}^{b} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{b} \Delta f_{k,t}^{b,-}\right) \\ &\quad - \sum_{k}\left(\overline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,-}\right) \end{aligned}$$

According to the complementary slackness conditions, the WCR is derived as follow:

$$\begin{aligned} WCR_t &= \sum_{k}\left[\left(\overline{\sigma}_{k,t}^{b} + \overline{\sigma}_{k,t}^{r}\right) \times f_{k,\max} + \left(\underline{\sigma}_{k,t}^{b} + \underline{\sigma}_{k,t}^{r}\right) \times f_{k,\min}\right] \\ &\quad - \sum_{k}\left(\overline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,-}\right) \\ &= \sum_{k}\left(\overline{\sigma}_{k,t}^{b} f_{k,\max} + \underline{\sigma}_{k,t}^{b} f_{k,\min}\right) \\ &\quad + \sum_{k}\left[\left(\overline{\sigma}_{k,t}^{r} f_{k,\max} + \underline{\sigma}_{k,t}^{r} f_{k,\min}\right) - \left(\overline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,+} + \underline{\sigma}_{k,t}^{r} \Delta f_{k,t}^{b,-}\right)\right] \end{aligned}$$

D. Derivation of the PCR

The additional net power injection is defined below.

$$\Delta P_{n,t}^{inj,r} = P_{n,t}^{inj,r} - P_{n,t}^{inj,b} = (D_{n,t}^{r} - D_{n,t}^{b}) - e_{n,i}\Delta p_{i,t}$$

The intermediate price signal $\pi_{n,t}^{r}$ is introduced.

$$\pi_{n,t}^{r} = \frac{\partial L}{\partial (D_{n,t}^{r} - D_{n,t}^{b})} = \lambda_{n,t}^{r} - \Gamma_{n,k} \times \left(\overline{\sigma_{k,t}^{r}} - \underline{\sigma_{k,t}^{r}}\right)$$

The first auxiliary expression is derived as follows:

$$\begin{aligned}
&\sum_n \pi_{n,t}^{r} P_{n,t}^{inj,r} \\
&= \sum_k \left(\overline{\sigma_{k,t}^{r}} - \underline{\sigma_{k,t}^{r}}\right) \times \Gamma_{k,n} P_{n,t}^{inj,r} \\
&= \sum_k \left[\overline{\sigma_{k,t}^{r}} \times \left(f_{k,\max} - \Delta f_{k,t}^{r,+}\right) - \underline{\sigma_{k,t}^{r}} \times \left(\Delta f_{k,t}^{r,-} - f_{k,\min}\right)\right] \\
&= \sum_k \left(\overline{\sigma_{k,t}^{r}} f_{k,\max} + \underline{\sigma_{k,t}^{r}} f_{k,\min}\right) - \sum_k \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,-}\right)
\end{aligned}$$

According to the complementary slackness conditions, the final first auxiliary expression is obtained.

$$\sum_n \pi_{n,t}^{r} P_{n,t}^{inj,r} = \sum_k \left(\overline{\sigma_{k,t}^{r}} f_{k,\max} + \underline{\sigma_{k,t}^{r}} f_{k,\min}\right)$$

The second auxiliary expression is derived as follows:

$$\begin{aligned}
&\sum_n \pi_{n,t}^{r} \Delta P_{n,t}^{inj,r} \\
&= \sum_k \left(\overline{\sigma_{k,t}^{r}} - \underline{\sigma_{k,t}^{r}}\right) \times \Gamma_{k,n} \Delta P_{n,t}^{inj,r} \\
&= \sum_k \left[\overline{\sigma_{k,t}^{r}} \times \Gamma_{k,n} \Delta P_{n,t}^{inj,r} - \underline{\sigma_{k,t}^{r}} \times \Gamma_{k,n} \Delta P_{n,t}^{inj,r}\right] \\
&= \sum_k \left[\overline{\sigma_{k,t}^{r}} \times \Gamma_{k,n} \left(P_{n,t}^{inj,r} - P_{n,t}^{inj,b}\right) - \underline{\sigma_{k,t}^{r}} \times \Gamma_{k,n} \left(P_{n,t}^{inj,r} - P_{n,t}^{inj,b}\right)\right]
\end{aligned}$$

According to (10), the second auxiliary expression is derived.

$$\begin{aligned}
&\sum_n \pi_{n,t}^{r} \Delta P_{n,t}^{inj,r} \\
&= \sum_k \Big[\overline{\sigma_{k,t}^{r}} \left(f_{k,\max} - \Delta f_{k,t}^{r,+} - \Gamma_{k,n} P_{n,t}^{inj,b}\right) \\
&\qquad - \underline{\sigma_{k,t}^{r}} \left(\Delta f_{k,t}^{r,-} - f_{k,\min} + \Gamma_{k,n} P_{n,t}^{inj,b}\right)\Big] \\
&= \sum_k \left[\overline{\sigma_{k,t}^{r}} \left(f_{k,\max} - \Gamma_{k,n} P_{n,t}^{inj,b}\right) - \underline{\sigma_{k,t}^{r}} \left(-f_{k,\min} + \Gamma_{k,n} P_{n,t}^{inj,b}\right)\right] \\
&\qquad - \sum_k \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,-}\right) \\
&= \sum_k \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,-}\right) \\
&\qquad - \sum_k \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{r,-}\right)
\end{aligned}$$

According to the complementary slackness conditions, the second auxiliary expression is derived

$$\sum_n \pi_{n,t}^{r} \Delta P_{n,t}^{inj,r} = \sum_k \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,-}\right)$$

With the above two auxiliary expressions, the PCR is proved as follows:

$$\begin{aligned}
&PCR_t \\
&= \sum_k \left[\left(\overline{\sigma_{k,t}^{r}} f_{k,\max} + \underline{\sigma_{k,t}^{r}} f_{k,\min}\right) - \left(\overline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,+} + \underline{\sigma_{k,t}^{r}} \Delta f_{k,t}^{b,-}\right)\right] \\
&= \sum_n \pi_{n,t}^{r} P_{n,t}^{inj,r} - \sum_n \pi_{n,t}^{r} \Delta P_{n,t}^{inj,r} \\
&= \sum_n \pi_{n,t}^{r} P_{n,t}^{inj,b}
\end{aligned}$$

REFERENCES

[1] H. Ye, Y. Ge, M. Shahidehpour, and Z. Li, “Uncertainty Marginal Price, Transmission Reserve, and Day-Ahead Market Clearing with Robust Unit Commitment,” *IEEE Trans. Power Syst.*, vol. 32, no. 3, pp. 1782–1795, 2017.